Frameworks for Improving Mathematical Sophistication and Teaching Philosophies

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Abstract
The major aim of this paper is to entice you to attend my presentation and participate in a self-assessment of your beliefs about mathematics and its teaching. The self-assessment will help you become familiar with educational research theories that are designed to describe mathematical sophistication (Weinstein, 1998) and mathematics teaching philosophies (Ernest, 1991). As a mathematics teacher, your new knowledge and self-assessment skills will help you become more effective. As a mathematics teacher educator, you can use these frameworks to diagnose teachers' beliefs and, when needed, to help them change their beliefs in ways that will allow them to become more effective mathematics teachers.

Mathematical Sophistication

There are many perspectives on beliefs about mathematics. For example, Skemp (1976) discussed (often) opposing goals for mathematical learning: *instrumental understanding*, with a focus on rules and procedures, and *relational understanding*, with a focus on meaning and structure. Thompson (1984) observed in teachers three basic conceptions of the nature of mathematics: *instrumentalist*, as a set of unrelated but utilitarian rules and facts; *Platonist*, as a static body of certain knowledge to be discovered; and *problem-solving*, as a continually expanding cultural product. Similarly, Lerman (1990) provided a framework that presented two competing groups who hold different philosophies of mathematics: *Absolutists*, who believe that mathematical knowledge is certain and universal, and *Fallibilists*, who believe that mathematics is uncertain and must develop through conjecture and proof. The theoretical framework for understanding mathematical sophistication presented here resonates with those perspectives, but it has a different origin – it evolved from student intellectual development theory.

Student intellectual development theory has sought to answer the questions "With what are [college] students concerned, how do they make decisions, what is their personality, and what motivates them?" (Stage, 1991). The three most prominent theoretical frameworks are "Forms of Intellectual and Ethical Development" (Perry, 1970), "Women's Ways of Knowing" (Belenky, Clinchy, Goldberger & Tarule, 1986) and "Epistemological Reflection" (Baxter Magolda, 1992). An oversimplification of the different models of intellectual development is that adults should move from dualistic "black and white" views of truth, knowledge, and authority, through stages of greater acceptance of complexity and uncertainty, to final stages of constructive and relativist knowledge that incorporate and accept multiple viewpoints. Perry's scheme is the forefather of all others, but suffers from being derived from an exclusively white, male, upper and upper-middle class population of college students. In response, Belenky *et al.* developed a framework specifically from women of varying socioeconomic backgrounds and levels of education. Baxter Magolda provides a synthesis and extension of those theories, based on a gender-balanced mix of college students.

By asking and answering the question, "What does this mean in mathematics?" I created a framework for understanding students' "Ways of Knowing Mathematics" which provides descriptions of various levels of mathematical sophistication (Weinstein, 1998). These descriptions are applicable to adults with respect to their learning of mathematics, much as Piaget's stages of development are applicable to children. Follow-up studies (Wiersma & Weinstein, 2001; Sovak, 2004) have shown the effectiveness of this theory for "diagnosis" of the mathematical sophistication of mathematics teachers and teacher candidates. However, no longitudinal studies of the effects of professional development based on these theories have yet been conducted, so the effectiveness of this theory for “remediation” is not yet proven.

Mathematics Teaching Philosophies

Teachers’ beliefs about mathematics vary widely and those beliefs affect their teaching philosophies (Thompson, 1992). Many teachers have views of mathematics that are unsophisticated, which leads to teaching philosophies that are limiting (Brown & Borko, 1992; Cooney, 1985; Cooney, 1999; Cooney, Shealy, & Arvold, 1998; Cooney & Wilson, 1995). Ernest (1989) is careful to note that the espoused models (the philosophies) for teaching and learning mathematics are modified by the “constraints and opportunities provided by the social context of teaching” and become enacted models. Ernest’s sensitivity to social context shows in his theoretical framework (1991) for mathematics teaching philosophies in that the five philosophies he presents are grounded in five distinct interest groups with different ideological views and different sociological purposes expressed in their aims for mathematics education. Therefore, these philosophies are firmly rooted in the cultural heritage of his country (United Kingdom) and it is an ongoing project to see how much relevance they have when applied to teachers elsewhere –two small studies (Wiersma & Weinstein, 2001; Sovak, 2004) have shown these philosophies make sense when used to understand American teachers.

Self-Assessment

This is an activity to be conducted at the conference, where participants will explore the question: *What are your beliefs about mathematics, and how does that affect the way you teach?* Participants will explore their own beliefs about mathematics and its teaching and then reflect on how knowing their own beliefs can help them become more effective as mathematics teachers and/or teacher educators.

The presenter will show some text, without title, that describes one specific category of mathematical sophistication or mathematics teaching philosophy. Participants will score themselves -2, -1, 0, +1, or +2, where -2 indicates strong disagreement that the text describes themselves and +2 indicates strong agreement that it does. After the participant determines their score, the presenter will reveal the title of that specific category, which will be entered on the scoring sheet below.

Scoring Sheet

<u>Mathematics Teaching Philosophy</u> (Ernest, 1991)

___Industrial Trainer
___Technological Pragmatist
___Old Humanist
___Progressive Educator
___Public Educator

<u>Ways of Knowing Mathematics</u> (Weinstein, 1998)

<u>Learning Mathematics</u>	<u>Verifying Mathematics</u>
___Mimicking the Procedure	___Receiving Answers Alone
___Choosing among Procedures	___Verifying Answers Alone
___Understanding many Procedures	___Verifying Answers Together
___Understanding the Structure	___Verifying Structure Together
___Constructing the Concepts	___Agreeing on Social Structure

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