

Graphic calculator in education at schools and in the educational training of future teachers.

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Extended Abstract: Graphic calculator is one of the tools of new information and communication technologies. It may be applied for various mathematical activities, which can be realized more efficiently without often-routine mathematical operations. In the paper [1] several tasks are presented on different levels, which are solved by a graphic calculator. We have applied the following didactic activities:

- a) To familiarize the students with basic functions of graphic calculator.
- b) To solve with students the sample tasks prepared in advance.
- c) To supervise the individual work of students (creation and solution of their own tasks)
- d) To record and analyze the results of individual work of students.

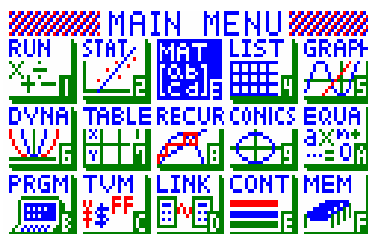
During the entire work, students mutually communicated and consulted their approaches with the teacher. In present workshop, we shall demonstrate education processes corresponding to mainly b) and d).

Modernization of education includes also educational training of future teachers of mathematics in the area of information and communication technologies [ICT]. We shall consider only one part of ICT, namely the graphic calculator CFX-9850 GB-PLUS from Casio.

Graphic calculator can help students in cognitive process - to construct separated models, to crystalline and to automatize knowledges, to make experiments in the course of problem solving.

We shall show a part of a lesson, in which a graphic calculator was used. The examples have been solved in **EQUATION** mode and **MATRIX** mode.

One communicates with the graphic calculator CFX-9850 GB PLUS which is provided with a color display after has been pressed **MENU**. The **MENU** has 15 icons. The appropriate mode is selected by the cursor keys, ◀ ▶ ▲ ▼ and by subsequent pressing **EXE**.



EQUA mode enables solving quadratic and cubic equations in the set of complex number and system of linear equations with two to six unknowns, which have just one solution. This approach will be demonstrated by the next example.

Example 1. Let us solve system of linear equations:

$$\begin{aligned} -x + 3y + 2z &= 3 \\ 3x + 2y - z &= 8 \\ 2x - y + 4z &= -4 \end{aligned}$$

Solution: Press **MENU**, **EQUA**, **EXE**. By choosing **F1(SIML)** one can solve system of linear equations. Press **F2(3)** because the linear equations being solved have three unknowns.

```
Simultaneous
Data For 3 Unknowns
In Memory
```

```
Number Of Unknowns?
2 3 4 5 6
```

$$a_n X + b_n Y + c_n Z = d_n$$

	a	b	c	d
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0

```
SOLV DEL CLR
```

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One inputs values of all coefficients

-1, **EXE**, 3, **EXE**, 2, **EXE**, 3, **EXE**,
 3, **EXE**, 2, **EXE**, -1, **EXE**, 8, **EXE**,
 2, **EXE**, -1, **EXE**, 4, **EXE**, -4, **EXE**,

$ax+by+cz=d_n$

	a	b	c	d
1	-1	3	2	3
2	3	2	-1	8
3	2	-1	4	-4

-4

SOLV **DEL** **CLR**

and by pressing **F1(SOLV)** the solution is obtained.

$ax+by+cz=d_n$

x	1
y	2
z	-1

1

REPT

If a system of linear equations possesses an infinite number of solutions or it has no solution at all, display shows **Ma ERROR**.

In these cases, one can solve system of linear equations in the **MATRIX** mode. The augmented matrix of system we brought to reduced form. By pressing **F1(R.OP)** the following menu appears on the display:

- F1 ... Swap** - row swap,
- F2 ... xRw** - scalar product of specified row,
- F3 ... xRw+** - addition of scalar product of specified row to another row,
- F4 ... Rw+** - addition of specified row to another row.

Let us transform the augmented matrix of system into the reduced triangular matrix by means **F1, F2, F3, F4**. In this described experiment, we have supposed that the entire procedure has been already theoretically known to students.

Example 2. Let us solve system of linear equations:

$$\begin{aligned} x + 2y - z &= 2 \\ 3x - y + 2z &= 7 \\ 2x + y + z &= 7 \\ -2x + 4y - z &= 3 \end{aligned}$$

Solution: Press **MENU, MAT, EXE, 4, EXE, 4, EXE** (matrix A , 4-row x 4-column);

```
Matrix
Mat A      : 4x4
Mat B      : None
Mat C      : None
Mat D      : None
Mat E      : None
Mat F      : None
DEL DEL
```

Let us input values of all coefficients the matrix A into the table in the same way as in example 1.

1, **EXE**, 2, **EXE**, -1, **EXE**, 2, **EXE**, ... 3, **EXE**.

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$$A \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 2 & -1 & 2 \\ 2 & 3 & -1 & 2 & 7 \\ 3 & 2 & 1 & 1 & 7 \\ 4 & -2 & 4 & -1 & E \end{array}$$

R-OP ROW COL

Let us press **F1(R.OP)** (row calculation menu) for the row operations.
 Let us multiply the first row of the matrix A by -3 and add the result to row 2.
 Press **F3(xRw+)**.

Step 1: Let us input multiplier value -3, **EXE**,

$$A \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 2 & -1 & 2 \\ 2 & 3 & -1 & 2 & 7 \\ 3 & 2 & 1 & 1 & 7 \\ 4 & -2 & 4 & -1 & E \end{array}$$

k? -3

kxRow m+Row n→Row n

Step 2: Let us specify the number of row whose scalar product should be calculated 1, **EXE**,

$$A \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 2 & -1 & 2 \\ 2 & 3 & -1 & 2 & 7 \\ 3 & 2 & 1 & 1 & 7 \\ 4 & -2 & 4 & -1 & E \end{array}$$

m? 1

kxRow m+Row n→Row n

Step 3: Let us specify the number of row where result should be added 2, **EXE**,

$$A \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 2 & -1 & 2 \\ 2 & 3 & -1 & 2 & 7 \\ 3 & 2 & 1 & 1 & 7 \\ 4 & -2 & 4 & -1 & E \end{array}$$

n? 2

kxRow m+Row n→Row n

One has the result:

$$A \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 2 & -1 & 2 \\ 2 & 0 & -7 & 5 & 1 \\ 3 & 2 & 1 & 1 & 7 \\ 4 & -2 & 4 & -1 & E \end{array}$$

3

SWAP | xRw | RRw+ | Rw+

In this way, we have explained to students all the row operations. Further work of students was individually. We shall show a concise record of one possible procedure.

F4, 3, EXE, 4, EXE, (the third row is added to the fourth one)

F3, -2, EXE, 1, EXE, 3, EXE,

F1, 3, EXE, 2, EXE (the third row is swapped to the second one)

F2, (-1:3), EXE, 2, EXE, (the second row is multiplied by -1:3)

F3, 7, EXE, 2, EXE, 3, EXE

F3, -5, EXE, 2, EXE, 4, EXE

F2, -0,5, EXE, 3, EXE,

F3, -5, EXE, 3, EXE, 4, EXE,

F4, 3, EXE, 2, EXE,

F4, 3, EXE, 1, EXE,

F3, -2, EXE, 2, EXE, 1, EXE,

In this way the reduced matrix was obtained

$$A \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 \end{bmatrix}$$

[SWAP] [XRW] [RRW+] [RW+]

One can see that the system of equations has the solution $x = 1, y = 2, z = 3$. (This system of equations could not be solved in **EQUA** menu because it has other number of equations than is the actual number of unknowns).

Similarly, one can solve also a homogeneous system of linear equations, determine linear dependence or independence of vectors, as well as tasks of analytical geometry which lead to the solution of linear equations.

Conclusions The work with students has shown their interest in utilization also this form ICT. The educational training of students for teaching is made more effective because the identification of appropriate tasks is accelerated. In the paper [2] my Ph.D. student presented the alternative way of teaching L'Hospital rule. The graphic calculator CASIO RM – 9850 GB PLUS was used in his experiment. This approach was well accepted by pupils even if they did not directly used their own calculators.

WORKSHOP In the course of workshop, I wish to present a solution of equation and inequation containing a real parameter. The goal is to show how the graphic calculator can help to solve the tasks in the case when the numerical and also graphical solution is not entirely successful.

Mathematical tasks

Task 1. Equation $x^2 = a^x$, (1) can have in dependence on the parameter $a \in (1, \infty)$

- two, three or four solutions
- two or three solutions
- one, two or three solutions
- two, three, four or five solutions
- non, two or four solutions

The task 1 can be formulated also in the following way.

How many intersections can have the graphs of functions $f: y = x^2$ and $g: y = a^x$, where a is the real parameter $a \in (1, \infty)$?

- Let us solve the Task 1 in as many ways as you can.
- Let us find such value of parameter a , for which the equation (1) has just one solution.
- Let us perform experiments and describe your findings.

Task 2. For which value of the real parameter a , the solution of inequation $\log_a(x^2 + 2) > 1$ (2) is the set of all real numbers? Second interpretation of the Task 2: **For Which value of the real parameter a “lies” entire graph of the function $f: y = \log_a(x^2 + 2)$ above the graph of the function $g: y = 1$?** Let us try solving the Task 2 numerically or graphically. Record your findings anonymously on the enclosed sheet.

Using the calculator Graphs of the functions described in the previous can be drawn in the GRAPH mode. We shall choose concrete values of the real parameter and look for all intersections of graphs of given functions. When simultaneously displaying graphs of several functions, it can be helpful to use a different color for each. The color can be selected by highlighting a desired function and then selecting F4. We shall compare results of numerical or graphical solutions with results of solutions, at which the graphic calculator has been used.

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