Playing to win: using games for motivation and the development of mathematical thinking

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Abstract: Games provide a unique opportunity for integrating the cognitive, affective and social aspects of learning. They can be used to introduce new ideas and lay the foundations for processes and thinking strategies that will be formalised later as well as consolidate existing thinking. When they are structured around mathematical ideas, play is dependent on mathematical understanding. In this way, the manipulation of materials, use of diagrams, detection of patterns and verbalisation of actions, thoughts and interpretations while playing can assist in the construction of mathematical concepts.

Mathematics learning occurs through the construction of ideas, processes and understandings in a social setting, rather than by the transmission of pre-formed knowledge from teacher to student. Consequently, the teaching of mathematics can be viewed as a process of assisting the learner to reconstruct particular ways of thinking rather than re-invent pre-existing mathematical thoughts. The use of materials, diagrams and patterns that assist students in this construction and language which is appropriate to both the mathematics and the particular capabilities and needs of the student have become ever more critical in the development of mathematical understanding. Sequences of development which reflect a constructive process so that new learning relates to and calls upon well understood precedents are also critical since existing conceptions, whether gained from everyday experiences or previous learning, guide the understanding and interpretation of any new information or situation that is met (Booker, 1996).

Yet even the most well chosen materials, illustrative diagrams and activities are not sufficient in themselves to foster the development of understanding. Meanings do not reside in words, actions and objects independently. They need to reflect a sharing of mathematical thoughts among active interpreters of language and action (Cobb, Yackel & Wood, 1992). Learning situations need to encourage discussion among students and with their teacher in order for emerging meanings and understandings to be built up. Talking about what is happening along with reflection on the effect of the actions that are entailed not only describes concepts, but also helps them to take shape in each learner's mind (Usiskin, 1996). Thus, the development of mathematical understanding and ways of thinking require situations in which learners can discuss, negotiate, resolve and reflect on their constructions and the mathematical conventions that must be acquired along with them.

A classroom environment in which the teacher's role is to ask questions and the students expectation is to provide not only answers but also to justify the reasonableness of their responses and to put forward the thinking that led to their conclusions is one way of achieving this. Another is through the provision of activities which of themselves foster engagement and discussion. Many of the structured uses of materials and problem solving activities conducted in mathematics classrooms have this provision as their focus. Mathematical games provide a means of combining both of these aspects in the one educational setting. The games themselves foster discussion among the players; a teacher can draw out from the class the thinking that is required in coming to terms with and playing the games; and the games can involve the forms of materials, ideas and problems that promote effective learning.

Learning mathematics through instructional games

Games, then, contribute to teaching and learning through providing a background in which mathematical concepts can be developed and constructed. Problem solving ability is improved when the discovery and use of strategies is required and previously acquired skills are maintained through motivating practice. At the same time, an element of chance ensures

that each player has an opportunity to win and build self-esteem, so that games themselves are seen as fun, not only providing motivation but also ensuring the full engagement on which constructive learning depends. Social interactions conducive to learning are also fostered as children learn that without cooperation a game may not proceed and there will certainly not be any chance of winning. Listening to other players, talking about what is happening and even assisting others to understand and complete the tasks involved in the game come to be seen as critical playing behaviours. Students can then learn from one another as much as from the structured activities through sharing the method of play, consequences and needs of the game. Thus, instructional games provide a unique opportunity for integrating the cognitive, affective and social aspects of learning mathematics (Pulos & Sneider, 1994).

A framework for using instructional games

A number of factors concerning instructional games, methods of play and the manner in which games need to be integrated into a mathematics program should be considered when choosing, developing or adapting a game to promote learning (Booker 2000). A full understanding of the topic is needed so that a game will focus on all the essential aspects. Likely difficulties with a topic need to be examined as they may come to light in playing the game. They may also need to be the focus of a specific game designed to resolve them. The nature of the topic being taught will also lead to the form of game used. At a concept level, matching representations of the underlying idea would be needed; with a process, materials, diagrams and patterns could be used to develop a particular thinking strategy; with problem solving, moves should focus on analysing the problem for underlying meanings.

Ideally, a game is played in a self-contained manner with little supervision by a teacher or other adult. The instructions need to be presented in a form and level suited to particular learners as well as being true to the mathematical content. To foster discussion among the players, it is often preferable for students to play in pairs so that one student makes a move at each play while the other watches and assists, with the roles alternating from move to move. How the mathematical requirements leading to a move can be judged appropriate also needs to be considered. A game could be made self-checking through using an answer sheet or a built in code; one child might use a calculator to assess the reasonableness of each result; or all children might complete the mathematics underlying each move and a consensus be required as to whether a move is acceptable.

Discussion during and following the playing of the game is needed in order to make explicit to all of the players the underlying mathematical thinking on which the game has been focussed. It is also essential to assess the effectiveness of the game in achieving the educational and mathematical ends for which it is intended. Thus, the selection, development or adaptation of an instructional game to suit the teaching and learning of mathematics requires:

- *Conceptual analysis of the content to be taught*: to seek out the underlying basis for the ways of thinking they require. (See, for example Anghileri 2001, Booker, et al 2004, Hiebert 1986, Ma 1999).
- *Determine potential difficulties or misconceptions:* an analysis of the content to be taught should also point to the aspects which are known to cause difficulties among learners (See, for example Booker 2000, Ma 1999)
- *Identify a game structure in which the concept or process can be embedded* Many mathematical concepts or processes require a matching among representations or between question and response. Include an aspect of the mathematical topic in the title of the game so that discussion about the game will serve as an anchor for the mathematical idea.

- *Focus the instructions and manner of play on crucial mathematical aspects*: include examples of the thinking or use of materials in the instructions. Ensure that only appropriate responses lead to a move.
- *Plan means to foster the concepts and thinking in discussion surrounding play*: relevant discussion provides the means for students to develop mathematical arguments and justifications, leading to robust concepts and thinking. If students only discus the state of play or likelihood of winning, the design of the game may be flawed or children's conceptual understanding may not be sufficiently developed.
- *Relate aspects developed in the game to the curriculum*: draw students' attention to the outcomes that they have achieved both in terms of new learning that has occurred and in the way a whole area of mathematics is being expanded and developed.
- *Assess the effectiveness of an instructional game*: determine how well a game meets the purpose for which it was designed or selected. Do students concentrate on relevant aspects or do they disregard important features? Can they complete the tasks required for each move in the intended manner or do they simply guess or seek an answer in a less efficient way? Do students enjoy the activity or do they simply see it as another teacher imposed task?

Thought also needs to be given to ways of managing the classroom environment to ensure students' engagement with both the games and underlying mathematics, and to allow a productive environment for this (Booker 2000, Pp109–117)

Games to develop naming and renaming among fraction concepts

Numeration is the key to mathematical thinking and to most of the mathematical processes that students will meet and need. Understanding how fractions are named, what the symbols used for them represent and how they are renamed in various equivalent forms underpins comparison, computation and every day applications (Booker 1998). Such an understanding also provides a basis for the generalisation of these ideas to algebraic processes, ratio and proportion needed for further mathematics.

The use of materials is as important in establishing an initial conception of a fraction as a part of something fraction, although one of something is now broken into parts rather than built up to form larger numbers. To allow students to acquire fraction understanding for themselves, they need to experience this partitioning rather than be given materials already formed into parts. Consequently, a model in which equal parts are completed and shaded is usually preferable to cutting something into equal pieces or to reconstructing 1 one from ready-made parts.

1 one
3 fourths

2 ones
7 fourths or 1 and 3 fourths
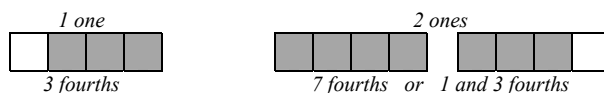
When 1 one is partitioned into equal-sized parts, this fraction is referred to as a *proper fraction*, since it denotes a number between 0 and 1. By extending this idea to fractions which are parts of more than 1 one, other fractions can be formed and seen as numbers in an extended number system rather than simply as a way of using already known whole numbers, separated by a point or comma (3.46 or 3,46) or one on top of another ($\frac{3}{4}$). Thus an *improper fraction* may be a part of 2 ones (eg $\frac{7}{4}$), 3 ones (eg $\frac{8}{3}$), 4 ones and so on and these can also be expressed as a *mixed number*, that is as a whole number together with a proper fraction (eg $1\frac{3}{4}$ or $2\frac{2}{3}$). Of course, each of these amounts could also be expressed as a decimal fraction or per cent, and fractions can be seen as numbers that lie between any two whole numbers:

proper fractions
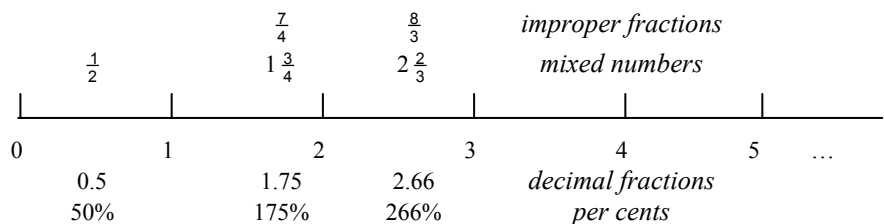
A number line shows that fractions are parts of 1 one, 2 ones, 3 ones, … and names them as proper and improper fractions, mixed numbers, decimal fractions and per cents

Games can be used to develop such a framework for numeration ideas across fractions of all kinds, decimal, common and per cents. In this way, a meaningful context is provided to build these new numbers that relate to, but are fundamentally different from, the whole numbers that have been the focus of previous number work. Fraction ideas can then relate to a learner's own experiences and understanding can develop from the language used to talk about the models and symbols used in play.

Spinners, dice or cards are used to display a fraction model, name or symbol to build up the various fraction concepts, or to rename as equivalent fractions and among the different fraction forms. Since moves in the game are based on a visualisation of the fractions that are used and the different ways they are represented, meaningful processes can be constructed in contrast to the reliance on rote rules that many students come to rely on. In this way, new ideas are engaged with while the most fundamental ideas underpinning them are kept to the fore, providing the basis for successful generalisation to new ways of thinking.

A game to introduce and consolidate that naming of common fractions. Common fractions are named by linking the use of a model to the notion of so many equal parts out of the total number of equal parts, using ordinal names to name these parts:

5 out of 8 equal parts → 5 out of *or* eighths → $\frac{5}{8}$
or 5 eighths 8
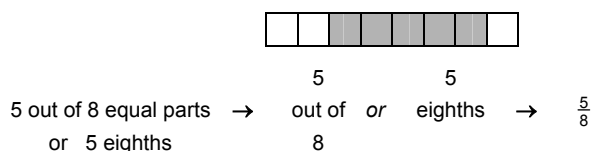
In this game, each player in turn spins the spinner and moves their marker to the first shaded region or name that matches the fraction symbol indicated by the spinner. The first player to make three circuits of the board and pass the start/finish position is the winner.
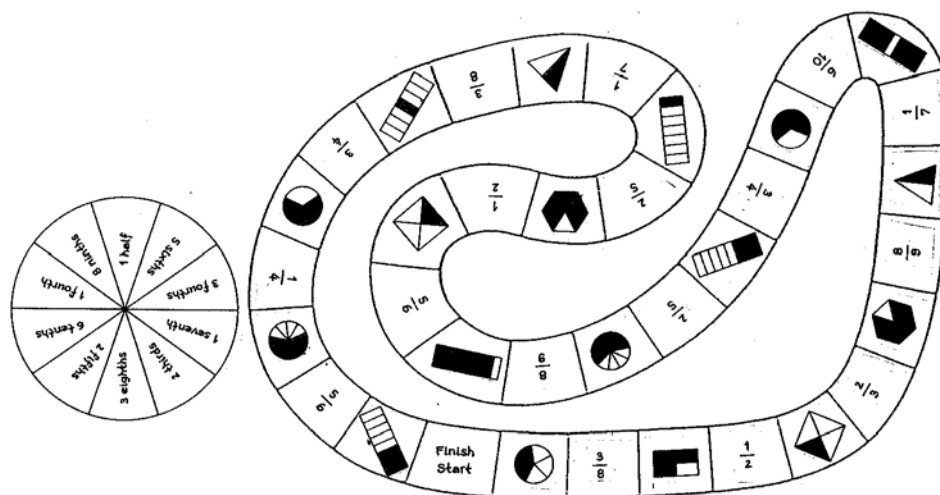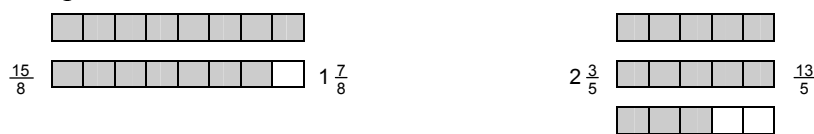
At each move, the spinner points to a fraction name. The player's marker is moved to the next space on the board that contains a model or symbol that matches.

A game to introduce and consolidate the renaming of improper fractions and mixed numbers.
The concept that fractions that are parts of more than 1 one can be named in two different ways is introduced using the region model that underpinned the naming of common fractions:
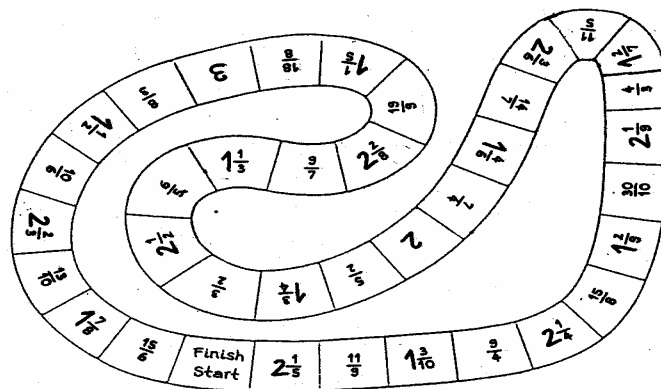

1 one   *3 fifths*   *1 and 3 fifths*

5 fifths   *3 fifths*   *8 fifths*
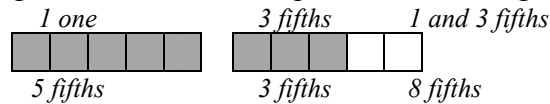
1 and 3 fifths or $1\frac{3}{5}$ is 8 fifths or $\frac{8}{5}$

In this game, Each player picks up a card, names the fraction indicated as both a mixed number and improper fraction and moves their marker to the first fraction symbol that matches the region model on the card. The first player to make two circuits of the board and pass the start/finish position is the winner.

$\frac{15}{8}$ $1\frac{7}{8}$ $2\frac{3}{5}$ $\frac{13}{5}$

Cards show fractions as mixed numbers and improper fractions

At each move, a player picks up a card that shows a fraction of 2 or 3 ones. The player moves his or her marker to the next space on the board that contains an improper fraction or mixed number that matches.

Conclusion

Involvement in instructional games like these induces students to make sense of their ideas and the interpretations of others. The dialogue engaged in while playing facilitates the construction of mathematical knowledge as each player's thinking is articulated. In this way, a conceptual framework can be built up through a process of reflection and social interaction. In turn, real mathematical issues arise out of the playing and engender an exchange of ideas as students strive to make sense of their actions and thoughts. Mathematics can then be seen as a social process of sense-making and understanding, rather than a set of rules handed down from some authority on high.

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