Building a Relationship Between Undergraduate Mathematics and Mathematics Teacher Education: Innovation in Teacher Education

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Introduction

In order to teach mathematics effectively, teachers must deeply understand the mathematics they are to teach (Committee on Science and Mathematics Teacher Preparation, 2001; Gay, 1994; Kessel, Epstein, & Keynes, 2001; National Commission on Mathematics and Science Teaching for the 21st Century, 2000; National Council of Teachers of Mathematics, 2000). At the secondary level this implies that teachers require at least a minor, and preferably a major in mathematics study. But as Graham and Fennell (2001) and commissions such as the Conference Board of the Mathematical Sciences (Kessel et al., 2001) note, simply studying advanced mathematics is not enough. Mathematics courses tailored to prospective teachers' needs should form part of their mathematics study. Teachers need to "be able to draw upon that knowledge with flexibility in their teaching tasks" (p. 17, National Council of Teachers of Mathematics, 2000). To do this, according to Cooney (1994), "teachers need to learn mathematics in a manner that is consistent with the way we expect them to teach" (p. 13). The Conference Board of the Mathematical Sciences (Kessel et al., 2001) recommends that for this to happen, mathematicians need to take a strong interest and play a significant role in the preparation of mathematics teachers (pp. 8-9).

The citations above are all from American sources, but their statements apply equally well to the Canadian context. It has been our experience, however, that mathematics teacher candidates often see little connection between their subject studies at the university level and what they believe they are expected to teach in secondary schools. To compound this disconnection, teacher education programs often only indirectly acknowledge the candidates' mathematics background by simply noting that they have met the requirements for entry into the program, and therefore should be capable, perhaps with some review, of doing the school mathematics they will teach. However, there is often considerable variation among the mathematics backgrounds of candidates because typically, numbers of acceptable mathematics courses rather than types of mathematics courses are the main criterion for admission (along with grades). Also, very few candidates have taken mathematics courses specifically designed for them as secondary mathematics teachers.

These statements apply particularly to "consecutive" teacher education programs in which candidates first complete (at least) a baccalaureate degree and then enter a teacher education program, but may potentially also apply to "concurrent" programs. This paper describes our plans at Nipissing University to build a stronger bridge between the undergraduate program and the teacher education program for mathematics majors who intend to become teachers. First we will describe the general teacher education environment in which Nipissing functions.

Teacher Education in Ontario

In Ontario there are two principal types of relationships between undergraduate and teacher education programs: concurrent and consecutive. The consecutive program is the dominant form among the current faculties of education although the number of concurrent programs is expanding.

In most concurrent programs students earn both an undergraduate degree (BA, BSc, BBA, etc) and an education degree (BEd), usually over a five-year period. The course of study will vary somewhat according to the requirements of the level of teacher education program (e.g., elementary, intermediate, senior). The respective faculties independently define most of the required courses, although a limited number of courses may be jointly

approved. Together the faculties have considerable control over a concurrent student's program of studies, but that does not necessarily mean there is much collaboration between the faculties, for example, to facilitate students' understanding of the place of their discipline in education.

The prevailing consecutive form of teacher education in Ontario is a "one-year" program, although in most cases it is actually shorter than 12 months. Students must first successfully complete an undergraduate degree (some have graduate degrees) with a minimum average, and, in the case of those wishing to teaching at the secondary level, a minimum number of courses in two disciplines recognized as teachable subjects in school. Although there is some variation in what the programs expect of secondary school teacher candidates among the faculties, typically five full course equivalents are required in the major or "first" teachable, and three full course equivalents are required in the second. As noted above, there are relatively few limitations placed on the types of undergraduate courses, and at the secondary level there are few if any mathematics courses specifically designed to prepare candidates for teaching secondary school mathematics.

Nipissing Context

Nipissing conforms very much to the general Ontario norm. Presently we have only a consecutive program although a concurrent program is being developed. Students enter the teacher education program with at least an undergraduate degree. A majority of applicants will have earned that degree from another university although a substantial number of students will have obtained it through study at Nipissing. (In fact, many students enrol as undergraduates at Nipissing with the intention of subsequently entering the faculty of education.) Students wishing to teach at the secondary school level must meet the minimum grade and number of courses requirements. These are the only conditions that have applied to those wishing to teach secondary mathematics, whether they enter the faculty of education from Nipissing or elsewhere.

This has meant that students are admitted with widely varying mathematics backgrounds. It has also meant that prospective secondary mathematics teachers enter the consecutive teacher education program with no preparation for teaching mathematics, other than as students themselves at one time in elementary and secondary school. Their awareness of the specific mathematics content of each grade is frequently limited, and they often feel "rusty" with some of the content. Their understanding of any relationship between the mathematics they study at university and that which they are expected to teach in schools is often tenuous. This situation, however, is beginning to change in a small way through various strategies. We will describe three strategies or events that are leading to change: the mathematics research project, the curriculum research project, and the introduction of a concurrent education program.

The Mathematics Research Project

This change has been precipitated by the recent implementation of a four-year honours undergraduate mathematics program. An independent research project is a component of the fourth year of this program, and for those planning to enter the consecutive education program, the intent is optionally to include a mathematics education component in the students' research to provide an early link to teaching.

This year (2003-2004) has been the first implementation and three students who have applied to enter the teacher education program as secondary mathematics teachers have been working on research projects. Two have been exploring fractals, and the third, projective geometry.

The students used the first semester of their research projects to gain some familiarity with their respective fields of study, and in December each gave a presentation to faculty. Their second semester projects explored in more detail a particular aspect of either fractals or projective geometry. Again, they are expected to give a presentation at the end of the semester reflecting their studies. As they complete their mathematics projects, we have asked them to consider ways in which the curriculum of the secondary (and even elementary) mathematics program can be connected to their studies.

A study of fractals lends itself very well to several school mathematics connections: geometry and proportionality (for example, similar figures, ratio, scale, scalars, dimensionality, area, perimeter), functions, iterative processes, the use of the computer and graphing calculators, a sense of the history of mathematics as a discipline that is "alive" with the opportunity to explore a newly developing mathematics field, and so on. Certainly the very visual nature of fractal displays and the application of fractals to a number of areas would be attractive to students. Books such as the *A Tool Kit of Dynamics Activities* series (Choate & Devaney, 2000; Choate, Devaney, & Foster, 1999a; Choate, Devaney, & Foster, 1999b; Devaney, 2004) and numerous websites provide activities to introduce fractals meaningfully to students at middle and secondary school levels.

A study of projective geometry also lends itself well to secondary mathematics connections. Euclidean and analytic geometries are developed throughout middle and secondary school curricula, and so the extension to modest explorations of projective geometry seems natural. The author of the text (Cederberg, 2001a) used by the student in her research project states that the book is appropriate for mathematics majors intending to become secondary teachers. Dynamic geometry software such *Geometer's Sketchpad* (Key Curriculum Press), with which students at both elementary and secondary mathematics levels have become familiar, would facilitate these initial forays into projective geometry. In fact, Cederberg (2001b) offers a companion set of dynamic geometry software explorations for mathematics majors using her text. Having these experiences in the research project would prepare the prospective teacher for using such software in his/her own classes.

Equally as important as recognizing where general content connections may be made between the university and secondary mathematics is the desire for the research project students to explore how they might teach certain aspects of what they are studying to secondary students. This would mean that as part of their research project, for example, they would prepare, with supervision, a lesson plan or lesson plans for a small secondary-level unit. This would in turn first require a close analysis of relevant school curricula in order to understand in some depth at what point(s) it would be appropriate to engage the students in the lesson or unit. Ideally this would also mean collaborating with a willing secondary mathematics teacher, and actually delivering the lesson or small unit.

It has been Franks' observation that mathematics teacher candidates often view teaching mathematics as equivalent to teaching a collection of topics. They appear tentative about the fundamental processes and concepts—the big ideas--of mathematics. They frequently lack a sense of what it is that a mathematician does when he or she does research, and so have a difficult time conveying that spirit and seeing what lessons might become if they incorporated more of that spirit in their own teaching. The fourth year research project provides these prospective teachers with an opportunity to get closer to that investigative spirit.

Ideally, the identification of a mathematics education element would occur near the beginning of each project. This being the first year, processes and structures and the timing of events are still being established. The students developed proposals in consultation with their supervisors in the spring of 2003 for their fall studies. They completed their projects with a presentation in December and began their more focused studies in the same topics in

January. The education collaboration did not begin until February and March 2004, by which time it was really too late to introduce a substantial education component to the students' projects. The three student researchers did discuss their projects individually with one of the authors (Franks) and some ideas for connections to school mathematics were shared.

This year has provided the authors with a clearer sense of the possibilities and potential for this joint undertaking to make a definite difference in the mathematical thinking and preparation of mathematics majors for entry into teacher education. It is clear that in future the mathematics/mathematics education connections must be made early so that students are fully aware of the education option; thus they and their mathematics and mathematics education faculty supervisors can begin collaborating with adequate time to realize more of the project's education potential. In the context of a consecutive education program in which opportunities to establish meaningful links between mathematics and mathematics education are almost by definition very limited, these projects are bringing mathematicians actively into teacher education (Kessel et al., 2001) and providing teacher candidates the chance to think and act as mathematicians do (Cooney, 1994).

Curriculum Research Project

The undergraduate mathematics research project holds long-term promise for students. A second, short-term curriculum research project also has substantial benefit for a mathematics student intending to enter the education program. The Nipissing mathematics/mathematics education group recently won an internal research grant to study the current Ontario mathematics school curriculum for the depth to which it supports, in its objectives or expectations, student growth in mathematical thinking through meaningful investigations and inquiry, problem solving, and proof. The current curriculum is new. Since 1997 the entire K-12 curriculum has been revised, with the grade 12 program implemented in 2002-03. Mathematical reasoning is promoted across all grades.

It is our hypothesis that the actual content expectations of the various curricula, and in particular those of the secondary grades 9 to 12, do not live up to the focus statements found in the commentary sections of the documents. Teachers concentrate on the "what" of curricula, not on the big ideas of mathematics espoused in "perspectives" sections. Evidence suggests (Tuncali) that first and even second year university mathematics students are not good at being able to develop logical arguments and communicate the reasoning behind their solutions and proofs.

This summer a promising mathematics student with intentions to enter teacher education will research the secondary mathematics curriculum. Under supervision, she will perform a close analysis of content expectations and document her findings, research the literature for alternative approaches to secondary mathematics, and create a set of recommendations for revisions to the current curriculum. This research will offer this student an excellent opportunity to understand more deeply the nature of mathematics and the nature of school mathematics. When she arrives in her classroom to start teaching in a few years, she will have an appreciation for the subject that few other beginning teachers will be able to match.

This project is a one-time event and thus only benefits one student. It is our hope, however, to make such funded internal research projects regularly available to students. There are numerous related topics that will provide capable students with opportunities to strengthen their mathematical understanding.

Concurrent Program

Finally, we are anticipating the establishment of a concurrent education program at Nipissing in which the connections between mathematics and mathematics education can be strengthened considerably. In 2004-05 we begin a small pilot concurrent program.

Mathematics is not one of the subjects in which concurrent students will be able to major initially, but that situation will change. The mathematics/mathematics education group sees great potential to fulfill many of the recommendations of commissions such as the Committee on Science and Mathematics Teacher Preparation (2001) and the Conference Board of the Mathematical Sciences (Kessel et al., 2001). This will only happen, though, if the program is permitted to develop thoughtfully and with recognition of both the mathematical and educational obligations and needs of the prospective teachers and the students they will eventually teach. We look forward to the challenge of establishing an effective companion to the consecutive program.

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