

**The Mathematics Education into the 21<sup>st</sup> Century Project**  
The Future of Mathematics Education  
**Pod Tezniemi, Ciechocinek, Poland**  
**June 26<sup>th</sup> – July 1<sup>st</sup>, 2004**

**An Effective Identification and Response to Children's Learning Needs in Number.**

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*Why do New Zealand children have low achievement scores in Mathematics in comparison to many developed countries? How can their learning needs be identified, and what can be done to ensure they will be able to participate successfully in New Zealand's knowledge based economy? These questions form the basis of this paper which describes a significant and innovative New Zealand initiative aimed at raising children's achievements in mathematics by improving teachers' professional knowledge, skills and confidence. The Numeracy Development Project has been developed through the Ministry of Education's Literacy and Numeracy strategy which has three key foci: clarifying expectations, improving professional capability and involving the community.*

The word 'numeracy' is usually associated with basic principles of mathematics and science. However, in New Zealand, the Numeracy Development Project has as its reference the number strand in Mathematics in the New Zealand Curriculum (MINZC). That focus is the vehicle through which the professional development is delivered to teachers. It is envisaged that other aspects of numeracy will be developed in the future. Within the context of the Numeracy Development Project classroom teachers are able to effectively identify and respond to children's learning needs with regard to number.

#### Historical Background

When the results of the Third International Mathematics and Science Studies (TIMSS) were released in 1996, several countries were dismayed at their standing in the global mathematics community. Along with their counterparts from countries such as Scotland, England and Norway, New Zealand children achieved below the international average. Of special concern were the lower results of Maori and Pasifika children who are destined to become a significant component of the future workforce. At that time it was also noted that teachers in New Zealand were experiencing difficulties implementing the new reforms in MINZC. The response from the Ministry of Education (MOE) was to expediently establish a Mathematics and Science Taskforce (MAST) in 1997, whose recommendations have since shaped the focus of mathematics education in New Zealand.

The immediate findings of MAST confirmed what had been identified by several national and international researchers (Askew, Brown, Rhodes, Johnson & Wiliam, 1997; Britt, Irwin, Ellis & Ritchie, 1993; Fennema and Franke, 1992; Shulman, 1986); the most influential person in children's learning is the teacher. A critical factor in the teaching of mathematics is teachers' content knowledge and pedagogical content knowledge. Children's outcomes are an indication of the impact of teachers' practice and therefore, by implication, children's lack of achievement was associated with their teachers and pre-service programmes (MOE, 1997a; MOE, 1997b). The National Education Monitoring Project (NEMP) mathematics assessment results also established that there were gaps in children's knowledge about place value, subtraction, multiplication facts and fractions (Flockton & Crooks, 1997) and subsequently the Education Review Office found that a significant number of teachers did not have sufficient content knowledge required for quality teaching of mathematics (2000).

The MOE commenced a publicity programme, website development and advanced qualification papers for teachers of Year 3 classes. Within the MOE funded classes at Auckland College of Education, Hughes, a senior lecturer in Mathematics, implemented some of the work Wright had developed for Count Me In Too (CMIT) a teacher development programme in Australia; a Learning Framework in Number (LFIN) and a Schedule for Early Number Assessment (1998). Hughes initiative and motivation in trialling those aspects, was to prove fundamental to the development of the numeracy project. Hyland, curriculum facilitator (MOE) recognised features of the CMIT would coalesce with MOE's numeracy policy and strategy and precipitated the beginnings of the numeracy development project. His drive has seen the numeracy development project grow exponentially. Along with experienced practitioners, teacher educators, policy analysts and researchers a symbiotic relationship developed.

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In developing the numeracy project there was a significant move away from the linear model. Research, development and dissemination evolved into a more complex model closely resembling Begg's eight co-emerging activities in the educational process; researching, theorizing, developing policy, developing assessment, developing curriculum, reflecting on practice, growing professionally and developing resources (2002). His model advocates not for control from the top, nor a bottom-up process but rather movement both ways which is essential to the development of open and frequent dialogue between all parties. While Begg asserts that his model has not been tested when one looks at the interactions between parties over the last four years, it can be clearly seen within the project. Although the circumstances in developing the numeracy project were dynamically complex the result was not disparate groups without cohesion but a group bound together by a shared vision. Over the years the numeracy project has been owned by many people, who have woven their original ideas into the rich tapestry of numeracy development. It is because of the creative and innovative thinking alongside theoretical reasoning that the project has evolved into the exciting, dynamic paradigm that at times has driven policy writing rather than be driven by policy.

Of significance in the initial developing stages of the Numeracy Development Project was not a theoretical claim but a practical one.

In every field of practice there are ideas that have never been confirmed by research and would in principle, be difficult to demonstrate. Nevertheless these maxims represent the accumulated wisdom of practice, and in many cases are as important a source of guidance for practice as the theory or empirical principles. (Shulman, 1986, p11).

Their names [practitioners] and ideas are often subsumed by the political agenda set by policy and curriculum makers. This is not an argument for all projects to be developed without the principles of research based theory, but rather an acknowledgement of those people who started with the pilot project in 2000 and brought their beliefs, ideas and creativity to meld with theoretical principles. They were the pioneers with only a few bits of paper, their passion for mathematics and a belief that what they were doing would make a difference.

#### Theoretical Approach

The theoretical approach encapsulates many of the views researchers and educators believe will enable children's mathematical understandings to develop (Hughes, 1995; Fennema & Franke, 1992; Mayers and Britt 1995; Thomas, 1998; Shulman, 1986; Steffe, Cobb with von Glaserfeld, 1988; Young-Loveridge, 1999). Although constructivism underpins the philosophy of Mathematics in the New Zealand Curriculum many classroom teachers, particularly in senior classes, could be seen teaching in a behaviourist style; dictated by the textbooks they used. The Numeracy Project has its roots in constructivism, through the use of a strategy teaching model adapted from the Pirie/Keiren (PK, 1994) recursive theory of mathematical understanding; along with principles of cognitively guided instruction (CGI). CGI is a programme which focuses on creating classrooms in which student inquiry and explanation of solution methods are encouraged (Carpenter, Fennema & Franke, 1996).

Facilitators, delivering the numeracy project professional development, scaffold teachers with their changing shifts in teaching practice. Teachers are encouraged, for example, to move away from their instrumental style of teaching formal algorithms and tap into children's thinking. The Fravillig, Murphy and Fuson pedagogical framework for advancing children's thinking is incorporated in the numeracy project and espoused through facilitators modeling the three components; eliciting children's solution methods, supporting children's conceptual understanding and extending children's mathematical thinking (1999).

Several well known researchers have put forward frameworks on the development of number concepts, that suggest a commonality of stages (Frank, 1989; Fuson, 1998; Steffe, Cobb with von Glaserfeld, 1988; Young-Loveridge, 1999). In establishing a number framework for the numeracy project, consideration was given to their works and in some instances adapted. Aspects of Fuson's forward and backward number word sequence reworked by Wright (1998) can be seen in the knowledge section of the framework, along with Wright's numeral identification model. The strategy section, which had its beginnings in Steffe's (1992) work in psychological models to explain children's thinking and

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development in number, has gone through several stages of metamorphosis. Firstly, a tabular five stages of early arithmetical learning, which grew to nine stages, initially through Hughes, enhanced by Wright (2000) and published by MOE in its present form (MOE, 2001).

While the work to be delivered had been shaped by theoretical and practical terms it was important that implementation follow sound professional development principles and that to effect change development needed to be over a considerable period of time (Fullan, 1999). The Numeracy Project is staggered over approximately 30 weeks with 15.5 hours (face to face delivery) per teachers at Years 1-6, and 19 hours for teachers at Years 7 and 8. The parameters in which facilitators work are broad and allow for flexibility and professional autonomy (Higgins et al, 2003). Schools are asked to participate in the project and a contractual commitment is undertaken to ensure all staff participate fully. Although visions imposed on an organisation command compliance not commitment it is the facilitators responsibility to take the MOE's vision and help schools make it their own.

**Professional Development**

The success of a pilot in 2000 for Yr 1-3 children (Thomas & Ward, 2001) contributed to further project development which now includes work in schools for Yr 1 -Yr 8 children; a small secondary component which offers information and development for Years 9-10; and Te Poutama Tau an initiative in Maori. Professional development is mostly delivered through face-to-face facilitator support and some through facilitator-supported workshops online. Each teacher participates in a series of six professional development workshops and is visited in their classrooms on at least four occasions. These visits involve some modelling of class and group lessons by the facilitator and also teacher led lessons followed by specific feedback discussion sessions. Pivotal to the success of the project in schools are the principals who are required to participate in initial workshops and to actively and fully participate in the professional development programme of their staff. Lead numeracy teachers are appointed in each school to further facilitate the project implementation and to act as the school liaison personnel. Ancillary teaching and support staff are also encouraged to take full part in training sessions.

It is recognized that change can be met with resistance (Fullan, 1999). However, in this project knowledge of what teachers' value led to many practical changes and helped to effect change in schools more easily. For example, one of the major recommendations from MAST was that teachers receive supporting material. To this end the Ministry has provided 8 booklets for teachers, mathematical curriculum support "Figure it Out" books for children's use and a website which includes summarised professional readings, online facilitation, 6 online workshops, planning assistant and material masters. Another worthwhile aspect of the teacher professional development is the cognisance paid to time. Time is allowed for interviewing, scaffolding, analysing, reflection, observing, trialling and sharing professional knowledge.

**Key Tools**

The **Number Framework** has two main sections: knowledge and strategy. The distinction between the two is for teaching purposes and to help teachers focus on the strategy framework. Aspects of knowledge, in the number framework, have always been taught in classrooms, albeit in a spasmodic way. It is the strategy framework that has required teachers to reflect on their beliefs about how children learn and what children know about number concepts.

The *Knowledge* section describes the key items of knowledge that students need to learn and consists of five aspects; numeral identification, number sequence and order, grouping/ place value, basic facts and written recording. The *Strategy* section describes the mental processes children use to estimate answers and solve operational problems with numbers. It consists of a sequence of nine global strategy stages (see Figure 1) with three operational domains: addition and subtraction; multiplication and division and; proportions and ratios. In the first workshop teachers are introduced to the number framework and its use over time helps teachers become familiar with the behaviour indicators at each stage of development, and guide them to identifiable learning steps for the children.

<b>Figure 1. The Number Framework</b>	
<b>Counting Strategies</b> 0. Emergent	<b>Part-Whole Strategies</b> 5. Early Additive

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<ol style="list-style-type: none"> <li>1. One to one counting</li> <li>2. Counting from one on Materials</li> <li>3. Counting from one by Imaging</li> <li>4. Advanced Counting</li> </ol>	<ol style="list-style-type: none"> <li>6. Advanced Additive</li> <li>7. Advanced Multiplicative</li> <li>8. Advanced Proportional</li> </ol> <p>Source: Ministry of Education, 2004</p>
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The second key part of the Numeracy Development Project is the **Diagnostic Interview**. This assessment tool is an individual task-based oral interview in which carefully sequenced questions are couched to elicit a variety of responses. The problem  $24 \times 6$  may invite a range of strategies such as place value partitioning, or rounding and compensating or proportional adjustment. The move away from traditional pen and paper tests offers teachers insights into children's thinking about the operations they are performing. In the second workshop teachers learn how to administer this interview. It is critical that teachers closely observe children's behaviour, ask the children to explain *how* they reached the answer and to note these in concise detail. Children show their strategies in many ways ranging from touching counters 1 by 1, to counting on or verbalising a more sophisticated strategy. The teachers refer to the frameworks to identify the children's stages of development in number and use the results to group children for teaching purposes. As well as using this information to inform their learning programmes, teachers are required to enter this comprehensive baseline data on a secure national website. The interview process and data entry is repeated at the end of the first year of the professional development project. Comparison of the initial and final data informs teachers of the impact of their teaching practice and provides information for school wide use.

A final key component is the **Strategy Teaching Model** (see Figure 2.) Teachers in New Zealand, particularly of junior classes, are familiar with using materials when introducing new concepts in mathematics. However, it is questionable if children can make the step to abstraction with ease or even at all. The 'using imaging phase' is the bridge to help make that translocation. In 'using materials' a teacher may use number lines shaded in groups of 5 and transparent counters to help build visual images in the children's mind. As they become more proficient in solving problems, by using counters and number lines, the teacher may turn the lines over thus shielding the sequence from the children and encouraging them to image the materials operation. If children are having trouble imaging, the teacher provides the materials again. This is a fold back idea, based on the PK theory and the process can be repeated until the child is secure in imaging. When numbers are larger than usually could be tracked in their minds it is expected they will connect to number properties without the use of imaging. If children have difficulty the teacher decides, as before, to re-introduce the imaging stage or even the materials again. To move children from the use of naïve strategies to more sophisticated ones can be difficult and much depends on teachers' content knowledge and their sound pedagogical content knowledge.

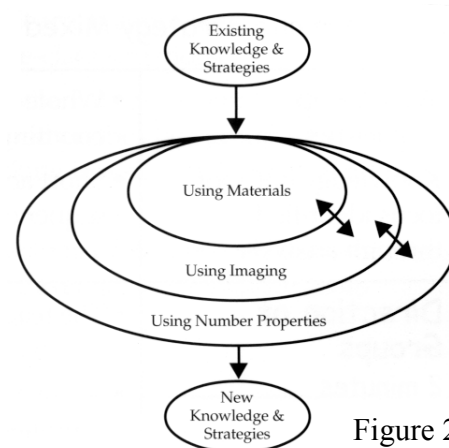


Figure 2

#### Outcomes

To date over 300,000 primary school children have participated in the Numeracy Development Project and by the end of 2004, 14,225 teachers will have undergone intensive one to one training. Each year comprehensive evaluation reports have been undertaken in which the detailed analysis of children's achievement data have been presented. For example, by the end of year 2, 40% are using advanced counting strategies with 15% able to use part/whole strategies to solve addition and subtraction problems; while the evaluation for years 4-6 shows children's improvement across the six aspects of number (Higgins, 2002; Thomas & Ward, 2002). On the basis of results and recommendations arising from these data, MOE has committed additional annual funding to facilitate the participation over time of nearly all primary teachers and students.

The Numeracy Development Project has made and can continue to make a significant difference to the achievement of New Zealand children. Evidence reported by the evaluators show the Numeracy

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Development Project is having a significant impact on children's achievements, teacher confidence, skills and knowledge (Thomas, Tagg & Ward, 2003; Higgins, 2003; Irwin, 2003; Christensen, 2003). One of the finest outcomes, not seen on paper but heard in the teaching community is mathematics being discussed positively with joy and passion.

Conclusion

In this large-scale comprehensive national approach to professional development in numeracy, in which improving teacher capability is fundamental to raising achievement of children, the complexity of the challenge of this change process is now only becoming evident. Close examination of what constitutes quality implementation and the size and nature of the resource, continue to be the focus of sustainability studies. The evolution of the Numeracy Development Project has involved the synthesising of extensive research, implementing and evaluating exploratory studies and pilots, ongoing consultation with the education sector, building capability of key specialist personnel, resource development, consistent, specific and rigorous contract requirements, and constant review and refinement based on research and evaluation evidence. The overview is positive but as a nation we cannot afford to become complacent.

There are aspects of the numeracy project that need to be analysed or explored. The teaching strategy model has yet to be rigorously researched for effectiveness, the stages of place value and basic facts need reviewing and raising achievement of Maori and Pasifika children remains an issue, to name a few.

Teachers can and have made a difference to children's achievement in number through improved capability. At the heart of the Numeracy Development Project is teachers' response to children's needs through effective identification of their number understandings. Teachers can make informed decisions about learning programmes when children's thinking is out in the open. The healthy future of mathematics in New Zealand depends on the ongoing recognition that change process takes time, teachers must be supported and sustained in their practice for continued improvement of the profession, and nurturing collaborative, knowledgeable school communities are crucial for continued success.

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