The Mathematics Education into the 21st Century Project
The Future of Mathematics Education
Pod Tezniami, Ciechocinek, Poland
June 26th – July 1st, 2004

E-LEARNING IN TEACHING OF MATHEMATICS USING COMPUTER ALGEBRA SYSTEM MAPLE

Jiří Hřebíček[1], Jana Hřebíčková[2], Ivan Mezník[3], Zuzana Chvátalová[4]

ABSTRACT: E-Learning in the teaching of mathematics is supported by the information and communication technologies of the Maplesoft™ company, called an "Academic Enterprise". This is an infrastructure that comprehensively addresses the information and communication technology requirements for education institutions wishing to deploy mathematical courseware for Internet. This paper presents an overview of our proposed methodology of e-Learning in the teaching of mathematics using Maple™, MapleNet™, Maplet™ technologies and Maple T.A.™ of the Academic Enterprise. This methodology was developed with the grant of the Czech ministry of education "Innovation of teaching with computer algebra systems". Also some requirements are mentioned for installation, maintenance, development & publishing of live course content of mathematics, assessment, etc.

1. INTRODUCTION

In the field of computer algebra systems, the Maplesoft company, http://www.maplesoft.com, has developed an *"Academic Enterprise (AE)"* for e-Learning in the teaching of mathematics, see Fig. 1. This AE infrastructure consists of the products Maple™, MapleNet™, and Maple T.A.™ and Maplet™ technology.

Maple™ is the well known mathematical software for teaching, learning, and doing mathematics, which has been developed since the nineties of the last century. Its comprehensive suite of symbolic and numerical mathematical solvers, visualization tools, and document-processing features allows both teachers and students to step into a complete environment where mathematics is easily implemented and explored. In the process of e-Learning teaching in mathematics, Maple worksheets are used to present concepts and provide an environment for exploration. Maple is able to export worksheets to their HTML version. When Maple is available on the student's local computer, the worksheets can be downloaded for study and further exploration of the concepts. The mathematics in the worksheet is *"live"* and therefore can be changed and recomputed to answer the *"what if"* questions that lead to a deeper understanding. When Maple is not available on the student's local computer, HTML versions of the worksheets can be downloaded, so that students who do not have Maple can also access the lesson.

Maplet™ technology is a new information technology included in Maple**.** Maple's Maplets package provides the ability to design standard graphical user interfaces that interact with the mathematical engine of Maple. Within this interface, teachers can perform calculations, plot functions, or activate other windows and dialogs. The teacher can create customized user interfaces that access Maple functionality ordinarily implemented in a worksheet via the standard Maple syntax. These interfaces are included in the e-Learning mathematical course modules, and provide the student with simulations for enhancing intuition, and tools for implementing sophisticated mathematical constructs. Educational tools created using the Maplet technology are called Maplet Learning Objects. If Maple is installed on the student's computer, these Maplet objects can be used via the built-in Maplet viewer. Alternatively, if Maple is not installed on the student's computer and the system MapleNet is available, the Maplet objects can be accessible over the Internet via MapleNet.

[1] Prof. Dr. Jiří Hřebíček, Ph.D., Faculty of Informatics, Masaryk University Brno, Botanická 68a, 602 00 Brno, Czech Republic, e-mail: hrebicek@fi.muni.cz
[2] Ing. Jana Hřebíčková, Faculty of Civil Engineering, Brno University of Technology, Veveří 331/95, 602 00 Brno, Czech Republic, e-mail: hrebickova.j@fce.vutbr.cz
[3] Prof. Dr. Ivan Mezník, PhD., Faculty of Business and Management, Brno University of Technology, Technická 2896/2 , 616 69 Brno, Czech Republic, e-mail: meznik@fbm.vutbr.cz
[4] Dr. Zuzana Chvátalová, Faculty of Business and Management, Brno University of Technology, Technická 2896/2 , 616 69 Brno, Czech Republic, e-mail: chvatalova@fbm.vutbr.cz

MapleNet™ consists of information and communication technologies (ICT), which allow a designated host computer to function as a server that provides Learning Objects to its clients. Students' computers that do not have Maple installed, but which have access to a MapleNet server, can run Maplet objects over the Internet using any Java™-enabled browser. Maplet objects viewed in this fashion are fully interactive in the browser on the student's computer.

Maple T.A.™ is a Web-based system for generating questions and exercises, and automatically assessing a student's responses. The responses can be as simple as a multiple-choice selection, or as complex as the free-form entry of mathematical expressions and formulae in which case Maple intelligently evaluates the response to decide if it is mathematically equivalent to the correct answer.

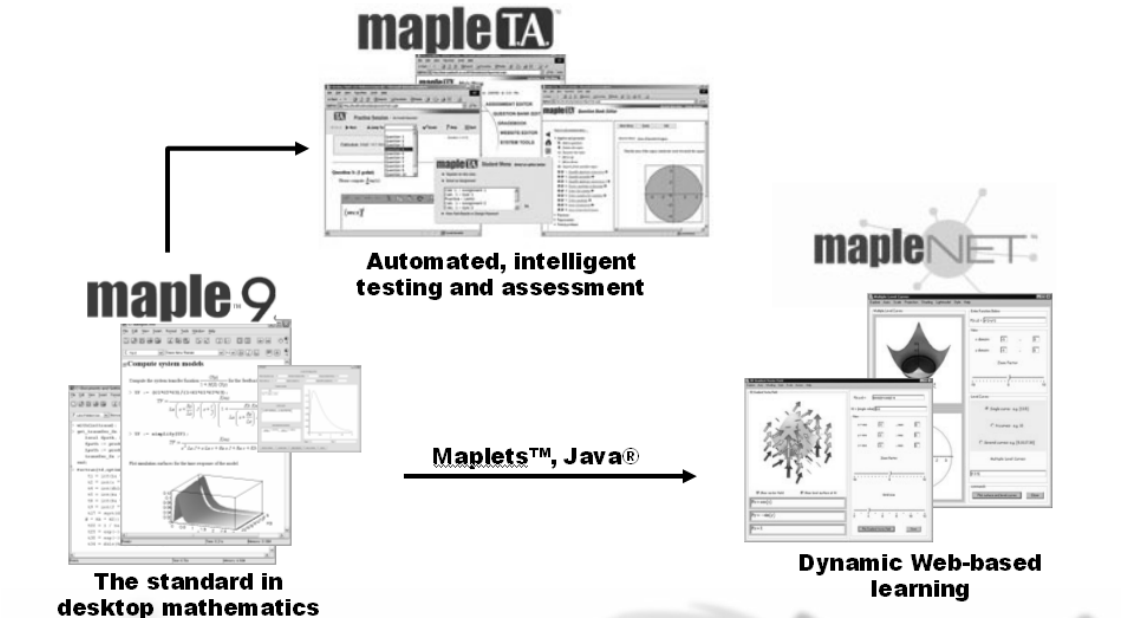

Figure 1: The Maplesoft's Academic Enterprise for e-Learning

2. DEVELOPMENT OF E-LEARNING COURSES USING ACADEMIC ENTERPRISE

In the prepared e-Learning mathematics and econometrics courses, students are encountered with problems in one of these forms: *practice, assignment*, or *test*.

A practice session is one in which no records are kept, and the student can repeat algorithmically-generated versions of questions.

Assignments and *tests* are drawn from the same question banks that provide the practice, and the results of both will be stored by the Maple T.A. system.

Students receive feedback on all three forms of activities. For practice sessions, the feedback appears immediately. For assignments and tests, the feedback can be delayed due to assignment date. Assignments and tests do not differ structurally, but only in how the teacher uses them. Students need to be registered in the Maple T.A. class in order to complete the graded assignments and tests.

The Maplesoft's AE offers a complete ICT to enhance mathematics and related courses over the Internet. It is ideally suited to meet the emerging challenges of distance, continuing, professional, and many other non-traditional forms of education.

In some sense, the AE possess software tools for all subjects participating in e-Learning mathematics:

- **Teachers:** The AE offers innovative ICT to make it easy for teachers to create and distribute electronic supplements and enhancements to mathematical courses.

- **Students:** The AE provides students with a rich interactive environment to explore difficult concepts in mathematics. MapleNet applications and mathematical lesson components can be easily launched from any browser and do not require any advanced computer knowledge to run. Students can spend their time learning mathematics rather than operating software.
- **E-Learning system administrators:** Components Maple, MapleT.A. and MapleNet of the AE are easy to install and maintain. They provide an easy-to-use administration model of e-Learning system that makes it easy to set up student access, manage user IDs, and monitor and adjust processes.
- **Educational administrators:** In many ways, the AE is about helping any educational institution meet its academic and educational goals. As important as curriculum enhancement is, today's educational process is forcing educational administrators to develop new education programs offering to expand the revenue base, continue to improve the quality of the educational experience, and to offer high levels of innovation to attract the majority of students.

The advantage of using Maplesoft's AE has always been the integration of all of the necessary components for comprehensive applications of on-line mathematical education and mathematical modelling, see Fig. 2.

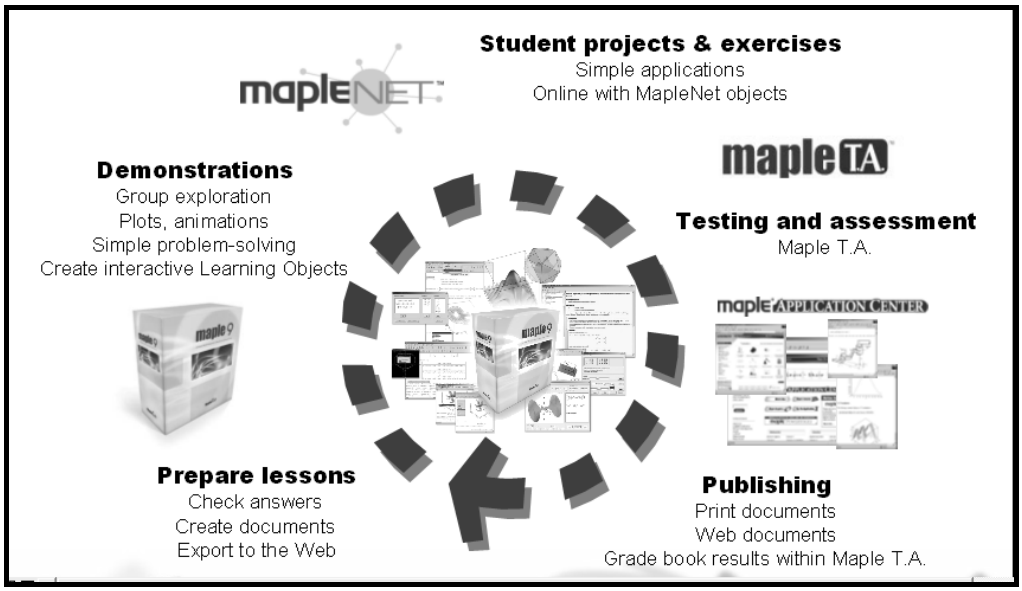

Figure 2: Full circle of e-Learning solution with AE

The full circle of e-Learning teaching in mathematics by using the AE consists of:

- using Maple for the preparation of lessons and publishing documents by using prepared educational worksheets and documents from the Maple applications centre, http://www.mapleappps.com,
- further using MapleNet for the demonstration of *"live mathematics"* as plots, animations and simple problem solving by using Maplets technology and
- finally using Maple T.A. for specifying student projects and exercises, their testing and assessment.

In order to simplify the overall design of mathematical problems for all components of the AE, we concluded that a sensible approach was to break the e-Learning education system issues for mathematics into four categories:

- **Content creation and publishing**: These issues include the consideration of different types of content in mathematics education (e.g., document-orientation vs. interactive

applets for concept exploration), migration of legacy information, and integration into the new generation of mathematical courseware management systems.

- **Server components and maintenance**: These issues include efficient runtime management of mathematical computation services, system maintenance, scalability, non-mathematical services, and possibly even alternate mathematical engines.
- **Client activities**: These issues include the teachers' and students' experience, authentication, adaptability to various didacties and accessibility.
- **Complementary functionality and connectivity**: These issues include the connectivity to educational databases, assessment modules, and, in general, any functionality that is generally better managed by existing or external components.

As the main result of the grant of the Czech ministry of education "Innovation of teaching with computer algebra systems" at the Faculty of Informatics of the Masaryk University Brno, a new methodology of on-line mathematical courses was develop in cooperation with the Faculty of Civil Engineering and the Faculty of Business and Management of the Brno University of Technology. This methodology has been issued from the following framework. For each lesson, there are up to five educational stages – *Overview of theory, Maplet presentations, Maple T.A. exercises, Homework exercises,* and *Knowledge tests*:

- **Overview of theory.** Every lesson is linked to a Maple worksheet that provides an outline of the lesson. The worksheet contains a discussion of the mathematics relevant to the lesson, and includes examples, graphs, and animations. The Maple worksheet can be viewed *"live"* by students who have access to Maple. Hence, the Maple inputs can be manipulated and altered, allowing students to experiment with *"what if*" questions. The lesson must have hyperlinks to this worksheet. Clicking on this link launches the worksheet if Maple is installed on the local computer. Students without access to Maple can read these fully executed worksheets as HTML documents in any appropriate browser by clicking the HTML version. The HTML version contains all the discussion, mathematics, and graphics available in the Maple worksheet. However, the mathematics is not *"live"*. If Maple is available, the Maple worksheets can be used *"live"* as the basis for classroom lectures, or they can be assigned to the student for study outside the regular lectures. Used "*live*", they can be projected onto a screen, with additional mathematical notes being written on a chalk or marker board. Generally, the mathematical presentations in the worksheets are sufficiently comprehensive to serve as a complete lecture. The graphs and examples are *"live*", so a student's questions can generally be explored and answered within the worksheet.
- **Maplet presentations**. Every lesson is linked to the Maplet Learning Object that launches on the local machine if Maple is available. These learning objects include simulations such as animations showing, for example, how rectangles approximate the area under a curve, or more sophisticated applications such as the implementation of the rules of differentiation or integration in calculus lessons, etc. The Maplet Learning Objects can also be executed over the Internet via the MapleNet technology by clicking on the "*MapleNet*" link.
- **Maple T.A. exercises**. Every lesson contains links to exercises in Maple T.A. For most lessons, there are practice problem sets focusing on specific topics. For all lessons, there is a set of general practice problems covering the entire lesson. For practice problems, no records of a student's work are maintained, but students receive feedback on individual questions when they are graded.
- **Homework exercises.** Every lesson contains a graded assignment (called homework). Homework exercises could be drawn from the same set of algorithmically-generated questions as the practice problems. Records of a student's homework performance have to be maintained. The student receives feedback on each question in the homework. In

addition, a selection of exercises taken from the course textbook is included in the prescribed homework assignment.

- **Knowledge tests.** Chosen lessons contain a link to Maple T.A. tests. The results of these tests are recorded, and after the due date, the students can obtain feedback on their work.

Together, these five elements combine to form a significant enhancement to the standard lecture course in the instructing of mathematics. The interactive elements in the worksheets and accompanying Maplet objects provide the instantaneous feedback and intuition that can not be gained using current textbooks. The practice problems presented and graded by Maple T.A. deepen a student's skills and clarify just what is understood and what is not. The graded assignments and tests record the progress of the student and allow the student to modify their learning strategies accordingly.

Among the complementary components, the most significant piece is arguably an assessment component. One of the most powerful aspects of a symbolic-based mathematical computation engine is its ability to resolve complex mathematical *"dialog"* such as free-form answers to test questions – for example, a symbolic system can easily discern that a + b is the same as b + a. This offers greater sophistication beyond the current norm of multiple choice or true / false.

Several examples of developed courses will be presented at the conference.

3. CONCLUSION

Within the core Maple community in the Czech Republic, there are several examples of innovative applications of the AE already utilizing this capability. In addition, the Maplesoft's AE can also be integrated into commercial assessment tools such as EDU from Brownstone or courseware management systems, such as WebCT, eCollege, Desire2Learn and Blackboard®.

On the client side of the AE, the current philosophy of the teaching of mathematics is that simpler and smaller is better for most applications. Some of the technical issues that are currently being worked on include an option for local (client-side) computation within the AE web infrastructure (for speed or in case users do not have access to a broadband connection).

Within all of the above MathML language plays an extremely important role. From the beginning we concluded that although Maple was a critical component, it was not going to be the only component in the solution. The real AE advantage or goal has always been the integration of all of the necessary components for comprehensive applications of online mathematical technology.

The most essential aspect of the Maplesoft's Academic Enterprise design is the recognition of the fact that every educational institution is unique.

Correspondingly, the AE design and architecture offers the flexibility needed to address many educational or infrastructure challenges. The first step in the preparation of online courses in e-Learning teaching in mathematics is to reconfigure the powerful AE system to drive online applications connected with the topic of the developed course. Then, through partners and standards, enrich the maths engine with the integration of complementary components while considering the individual needs and concerns of educational institutions wishing to deploy online mathematics content. Combined with a unique customer-centric product management process, the AE stands as the only truly integrated solution for comprehensive online distance education initiatives.

4. References

Dalík, J. at al. (2001) *Using Maple in teaching mathematics and econometrics*. (in Czech) In Procedings Informační technologie pro praxi. Ostrava, Czech Republic, p. 77-82.

Dufek J. and Hřebíček J. (2000) *Paradigm of Solving Problems in Econometrics*. Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis. Vol. XLVIII, No. 2, 33-36.

Gander W., Hřebíček J., (2002) *Solving Problems in Scientific Computing Using Maple and Matlab*. Third, expanded and revised ed. Springer Verlag, Berlin.

Hřebíček, J., Ráček, J. (2001) *Mathematics in pictures*. Manual for e-Learning (in Czech) Brno: Faculty of informatics, Masaryk University Brno, 24 pp.

Mezník, I. (1999) *Econometrics for graduate programs* (in Czech). PC-DIR Real, Brno