

The Mathematics Education into the 21st Century Project
The Future of Mathematics Education
Pod Tezniemi, Ciechocinek, Poland
June 26th – July 1st, 2004

On Improving School Maths Curriculum through Fashionable Maths

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Abstract: The last few decades have shown a big revolution in Maths that surpasses all past revolutions. Whereby what is known to be fashionable Maths emerged. This Maths is characterized by its vast applications, its fundamental role in developing recent theories (such as Chaos Theory, Non-linear Dynamics theory or rather complexities), and in developing computer graphics and virtual objects.....

Some of currently fashionable Maths reflects mathematical art and wonders as in Non-commutative Geometry (of Cones) and Fractal Geometry (of Mandelbrot) which have roots in Modern Maths and illuminated by the artistic ancient Egyptian's and Arab's decorations and structures.

To improve school Maths curriculum by making it more alive, more realistic, more accessible and up-to-date in my opinion is to shape it by fashionable Maths. So the aim of this paper is to present my preliminary tentative efforts in that respect... i.e. in introducing some ideas and features of fashionable Maths - mainly Fractal geometry - in school Maths.

Introduction Getting closer to nature inspires mathematicians (and scientists) to discover and invent theories. Newton's contemplations of dropped apple lead to his laws of gravity and motion, which are suitable for heavy bodies, pendulum motions....etc. Getting closer to nature enabled Einstein - who was fond of forests - to invent his theory that dealt with minute objects with light speed; a discovery that Newton's law could not cover.

Getting closer to nature inspired Mandelbrot to invent fractal geometry - while contemplating sea shore and clouds - that models natural phenomena with its complexities, chaos and formless shapes. In fact Euclid's geometry failed to deal with such phenomena.

Combining fractal geometry with dynamical systems lead to non-linear dynamical systems, which can help in solving problems - in our current technological era- that Einstein's theory was unable to solve.

Fractal geometry is considered fashionable maths, humanistic and quasi pragmatic. Besides, its connection with nature, art, science, engineering, the marvels of computer and IT development; fractal geometry has also a special nature, beauty, and a particular way of thinking, that can be felt in the mind and heart.

Therefore, one can improve school maths by introducing fractal geometry fundamentals - that reflect its wonders, features and connections- or by simply updating some topics by related ideas in fractal geometry. Moreover its intrinsic nature that makes it more alive, more realistic, more informative, more accessible, more up-to-date can help in shaping school maths in that respect. This paper is limited to show how to benefit from fractal geometry being more alive and realistic in shaping school maths in that way i.e. in making school maths more alive and realistic.

1- Why fractal geometry is more alive and realistic?

The reasons can be easily clarified from its connection to nature, life, art and its dynamic features, its humanistic characteristics as follows:

(i) Fractal geometry is more closer to nature and life around us

This is obvious from many examples of self similar objects (fractals) around us, e.g. cauliflower head (where the head is repeated on smaller scales as its branches into smaller units similar to the head), clouds, tree branches and tops, sea shores, rivers branches, tops of mountains plant roots, blood vessels.... Rough surfaces of metal, ferns, also, in Pollack's painting, in modern non-Euclidean wire antenna based on Von Koch snow flakes and Seirpinski's gasket or carpet curves (fractals).

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N.B. you may be reminded by: (a) any object composed of smaller models of itself is self similar, (b) a fractal is a rough geometric shape that can be divided in parts, each of which (approximately) a small version at the whole⁽⁵⁾, -i.e. it is self similar object.

(ii) Fractal geometry is more alive for its dynamics.

Fractal dynamics is a sort of personification, that can be noticed from: (a) the continuous chastic behaviors of points moving as if they are dancing around the roots of functions in complex plane and near the bounding of its basins, (b) creating fractals by IFS that simulate natural objects (e.g. ferns), creating realistic imaginary landscape, (c) simulating nature phenomena, characterizing the shape and texture of complex surfaces..., (d) the brownian motion of points when creating strange attractors such as the most beautiful Mandelbrot and Julia sets.

(iii) Fractal geometry is more alive as it is more humanistic

For Robin Hursh fractal geometry (being fashionable maths) is humanistic for: (a) it is invented by people. (b) it is social as a group of mathematicians contributed in its development, (c) it is political as it is a function of space and time, i.e. the product of our technological age & the advances of computer science & IT.

It is also sociable as it is based on the work of Julia, Cantor (and their sets), Hausdorff and his curve filling and box dimension, Lorenz and his strange attractor, Smale and his horse shoe geometry....

Fractal geometry is appealing and attractive, it touches the feelings, emotions, and mind. Its connections with nature and life make it familiar, as if you know it for long time. Simplicity of producing famous fractals by generators is fascinating and exciting. Producing exact (or statistical) fractals by IFS based on simple ideas of (affine) transformations, iteration makes it familiar.

Incredible beauty of fractals such as Mandelbrot set (fig1), Julia set (fig2) that the computer - with its innovations in graphics, colouring, animation - is able to show their beauty, which is felt in heart and soul. & activate imagination to wonder about their constituents: "Are they dynamic creatures and flowers full of life?!". "Are they glittering diamonds?!". "Are they artistic paintings of impressionists or abstractists?!".

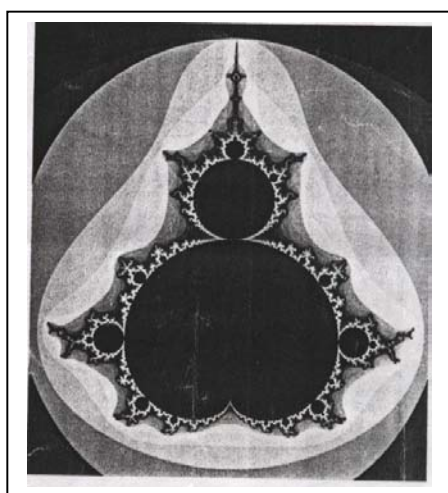


Fig. (1) Parts of Mandelbrot set



Fig. (2) Julia set

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Pollack's painting as fractals reflects his feeling of the nature's rhythm, is very appealing. Also artists use fractal software to produce highly artistic paintings.

Producing well known fractals such as Koch's snow flakes, and Seirpinsky's gasket, using generator and iteration, makes it a thrilling activity while getting more details every iteration. One feels that it develops gradually as if it is a flower blossoming (fig3).

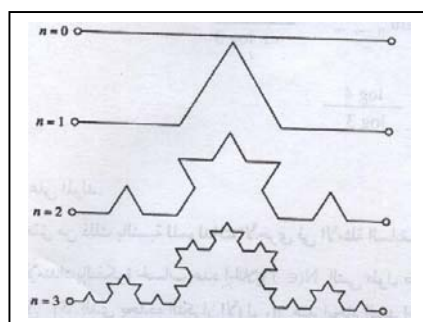


Fig. (3)

2-Implications in making school maths more alive.

The above features that make fractal geometry more alive (& real) can be a guide for school maths to be also more alive; by

i) Showing its connections with nature through:

- Giving the learner opportunities to be close to nature as if he is a part of it, to help logging in data from the real world, where order and disorder are related, rough and formless shapes are dominant, nonlinear paths of creatures are apparent, and different scales of the same figure are found. This will shape his thinking to be nearer to the core of fractal geometry
- Connecting ideas of maths with nature :e.g. *relating one* by the one moon, one sun, heart, nose, unicorn; *relating quadratic function graph* with dolphin's jump path, water from fountain, jaw with teeth; *relating big number*: (millions) by number of seconds in a year or distances of sun & earth and other planets; *relating growth or extinction graph* with logistic map.

ii) Developing the feeling of its dynamics:

Feeling school maths (figures and ideas) as dynamic animated objects can be stimulated and developed through:

- The motion of mathematical figures by geometric transformation; drawing loci and geometric constructions; dissections and combinations, either by sketchpad or computer graphics.
- Offering learning opportunities for the learner to live and feel the motion of pencil trace along a ruler at drawing a line segment, as if he is moving linear points in sequence.
- Watching the funfair wheel (or a model of it) when it is stopped then when it is moving slowly then quickly to show the learner the circle as moving points. Then let him feel when drawing a circle using a compass, the circular moving points. Also, when drawing the circle as an envelope of line segments using moving ruler touching a point (the center).
- Feeling the same impression of moving points when drawing any graph functions.
- Activities using paper folding, making solids from their nets.
- Using Java sketch pad or other computer software to show the dynamics of geometric models in construction.

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-When teaching complex equations such as $Z^5 - 1 = 0$ (or the zero function of $F:Z \rightarrow Z^5 - 1$), draw attention of the dynamic chaotic behavior of points near the solution and at the boundary of its basins forming the related Newton's fractals, using geometric sketch pad (or fractal software), fig (4), then let the students reflect about its beauty and personifications.

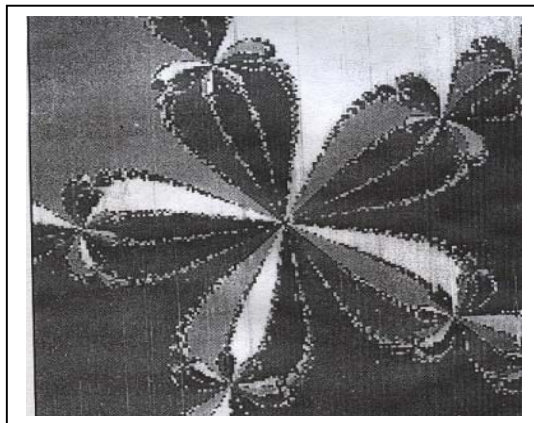


Fig. (4)

iii) viewing it as humanistic

Making school maths more humanistic utilizing the humanistic features of fractal geometry can be through:

- Providing learning opportunities for the learner to do maths by himself or by the help of others.
- Using discovery, inquiry, laboratory and cooperative learning methods which contribute in viewing, maths is sociable, as one or many do it.
- Making school maths learning more interesting, exciting and appealing to feeling, imagination and mind; by showing its outer (apparent) beauty resembled in geometric and number patterns, and its inner beauty embodied in its reasoning, logic, proofs, solving problems, and rules.
- Simplifying maths to help learners succeed in achieving it with love

- Developing appreciation and love of maths by: (a) mathematicians' contribution in developing a subject and stories about its creation and not just mentioning historical notes. (b) connections with art (music, paintings, sculptures, ancient and modern decorations, (c) thrilling activities and puzzles, (d) Let the learner search for related ideas, life applications, mathematicians wonders, then make his own collections.

-Finally, in the same way one can benefit from fractal geometry as more informative, up to date, more access in developing school moaths in that respect.

The ideas presented above are extracted from my book " On fractal geometry and developing creative maths teaching"

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