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COMMON - SENSE LOGIC FOR MANAGING UNCERTAINTY

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Abstract: The aim of the paper is to give a view on basic tools for managing uncertainty. In more details method of common sense logic is mentioned and case study - a qualitative model of a social system is presented
Key words : uncertainty, uncertain knowledge, vagueness, expert system, fuzzy set and logic, common-sense logic, qualitative model

1.UNCERTAINTY AND HANDLING UNCERTAINTY

UNCERTAINTY AS A REAL PHENOMENON

A human life is realized as a finite sequence (individually long and preferably as long as possible) of miscelaneous decisions as a result of brain activity. Practically all forms of human behavior involve decision making under the supervision of a mysterious guide - *uncertainty.* In this sense uncertainty is considered an objectively existing real phenomenon. Presumably, uncertainty is a complicated and elusive topic. Its presence tends to destabilize things and any decision made under uncertainty, which effects future events, possess some risk to it. Altough risk, ambivalence, insecurity and vagueness are present in nearly all aspects of everyday life, the most surprising things in the human condition is the ability to behave in a changing world in which nothing is certain. How to "define" the concept of uncertainty ? Despite the complexity of this phenomenon (some authors paraphrase that uncertainty has an uncertain meaning), the following verbal specification is generally accepted :

Uncertainty *is a phenomenon, whose characteristic features are given by elements of vagueness, insecurity, imcompleteness, inexactness and randomness.*

It is obvious, that the issue has impact on personal and organizational decision making, executive behavior, managerial attitudes and information systems. Many profitable businesses exist in part to the presence of uncertainty. Consequently, the subject of managing uncertainty is of general interest. In this point there is also one natural query - would it be possible to take the way of eliminating (or at least of reducing) uncertainty than to go in for methods of handling uncertainty ? It is apparent, that getting for instance more exact information, uncertainty may be diminished, but there are theoretical aspects of accuracy itself leading eventually to contradictory outcome. It concerns the relation between relevancy and accuracy of information, which was formulated by Zadeh(1973) as *The principle of incompability:* Roughly speaking, if we want to describe any reality, we must decide between relevancy of information, that will be less accurate and accuracy of information, that (as from a certain boundary) will be less relevant.

Example : The teacher´s command at driving lecture " brake slightly" is relevant, but not accurate. The command "brake by the force 10N " is accurate, but not relevant.

Increasing the accuracy, we reach a point, when accuracy and relevancy become mutually excluding characteristics.

Example : To give a real picture of the run of the university, we need several sentences, describing its hierarchy, study programmes, the number of teachers and students and some more . In order to provide more and more accurate information, we must gradually add more and more materials (detailed lists, history,…) to the extend that is useless as a relevant information. We must politely return to some relevant information expressed by means of natural language.

The mentioned examples point to a crucial role of a natural language as the best tool to express and carry relevant and uncertain information.

For making decisions maximum accessible information, say *knowledge* in the sequel, about a real situation must be utilized. Such integrated knowledge consists of data of two types. First, the data possessing an exact representation (laws of nature, exactly defined quantities, properties described by equations, given numerical characteristics,…) - they form an exact (or a certain) part, which is reffered to as *certain knowledge.* Second, the data not possessing(from many reasons) an exact representation - they form an inexact (or an uncertain) part , which is reffered to as *uncertain knowledge*.

KINDS OF UNCERTAINTY AND THEIR MODELING

There are many kinds of uncertainty arising in real -world problems. A clasical kind of uncertainty is that of *randomness,*as exemplified by the uncertainty of the outcome of some experiment. Randomness is typically modeled using probability theory, theory of stochastic processes or statistics. *Vagueness* is the type of uncertainty that arises when the boundaries of objects (or information) are not sharply defined or are given by some measure of possibility.

Example : (a) In the declaration " if the driver has a long practice and the car is slow then the risk of an accident is low"- the objects (variables) LONG, SLOW and LOW contain information that does not have clear-cut boundaries.

(b) In the declaration " if a man is young (0.8) and his income goes up (0.9) then his index of atractivity goes up (0.7) - the measures of 0.8, 0.9, 0.7 are numerical representations of uncertainty (vagueness).

A number of mathematical theories crystalized to model vagueness- fuzzy sets theory (fuzzy modeling, fuzzy logic), possibility theory, Dempster- Shafer theory of evidence, common sense logic, rough sets theory. Each theory has its associated calculus.

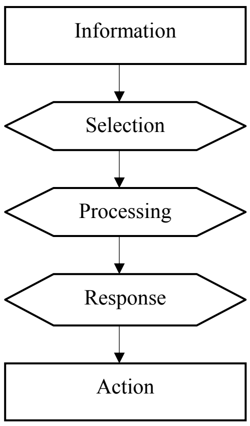
In complicated real-world cases, several kinds of uncertainty may coexist. For instance, a population of humans can be chosen at random and one might be interested in its average IQ, political spirit, the quality of mutual relations and so on. In a given situation, one theory may be more advantageous to use than another and one may even use several in combination. Dealing with uncertainty is a complicated problem and the practitioner has to be creative and use understanding in order to choose the right approach.

Viability of the methods and theories indicated above demonstrate realized applications(among others) :

- control parts of household products (cameras, washing machines, vacuum cleaners)
- automatic regulators (ABS systems and gear units for cars, thermal processes-kilns)
- pattern recognition (symbols, writings, pictures, shapes)
- logical parts of expert systems (banking and capital operations)
- evaluation of credibility of enterprises
- modeling of social and environmental systems

UNCERTAINTY MANAGEMENT

Managing uncertainty is closely connected with the problems of *artificial inteligence*, that deals with the modeling of human reasoning. For this purpose knowledge systems called *expert systems* are designed. An **expert system** is characterized by a data base of facts and a logical engine for exploiting the facts in a purposeful manner (Forsyth(1984)). Practical expert systems for problem solving should také into consideration two distinqushed components of the uncertainty, namely the uncertainty of knowledge itself and the uncertainty of the reasoning process. The mechanism is required for dealing effectively with both components. Such a mechanism must have theoretical base in logical tools. It must also reflect the key fact, that people typicall handle uncertainty by relying on how the information is stored in their memories, rather that on the laws of uncertainty (Katzan(1992)). From this viewpoint expert systems involving procedures utilizing laws of uncertainty may be more safe in outcomes than human reasoning. The cognitive process for succesful handling uncertainty should consists of three steps : selection, processing and response. *Selection* is the choosing the applicable part of information, ie. information that is relevant to a particular situation. *Processing* is the performing of methods for handling uncertainty. *Response* is the task following the computed results realized as an action (see figure below).



There are three components of knowledge required to handle uncertainty. The first component is the collection of facts, states, or values, called *indicators* , concerning the particular situation. For instance prices go up, moral is improving, or temperature is up. The second component is the belief in what the indicators mean. The implication stemming from indicators, belief and meaning is referred to as *evidence*. The third component is a collection of so called *inference rules* for determining new facts, states, or values and answering queries.

Now we will present some concrete examples employing different theories of handling uncertainty and their "calculi" without specifically going into their mathematics. In the next section we will concentrate on common-sense logic approach.

EXAMPLE 1(Fuzzy sets).

The word "great" has not a precise meaning, there can not exist a generally accepted criterion to decide uniquely of what is "great" or not. Therefore the collection of great (real)numbers can not be treated as a set. To handle with such collections of objects , the notion of so called *membership function* assigning to each object of the given collection a number from the interval < 0,1 > is introduced. The collection of objects together with the associated membership function is referred to as a *fuzzy set*. The membership function has natural interpretation - the closer is the number assigned to a particular object to number 1, the more belongs the object to the fuzzy set and similarly, the closer is the number to 0, the less belongs the object to the fuzzy set. The representative values of membership function of our fuzzy set A of great numbers may be given in many ways (they reflect how a person subjectively comprehends the meaning of "great number"), for instance :

A={…, (-100|0),…, (-10|0),…, (0|0), (1|0),…, (100|0,001),…, (110|0,01),…, (120|0,1),…, (130|0,2),…,(150|0,5),…, (180|0,8),…, (190|0,9),…, (200|1),…, (210|1),… }.

Apparently, in our case the meaning of "great number" may be connected with the tallness of adult men, so our fuzzy set A could be renamed the fuzzy set of tall men. Now, consider further natural constructions. For instance, if we strengthen the meaning of "great" to "very great", the membership function of such a new fuzzy set B of very tall men must be in its values less(or possibly equal) than the membership function for A. Such relation between vague objects ("great" and "very great") is modeled by the concept of a (fuzzy)subset, in our case B is a subset of A. There is another , elegant construction, how to model the process of strenghtening the meaning, using the concept of a power A^2 of fuzzy set A; simply, the memberhip function of A^2 is squared. Then it holds

A^2 = {…, (-100|0),…, (0|0),…,…, (100|0,00001),…, (150|0,25),…, (190|0,81),…, (200|1),… }.

So, A^2 may model the fuzzy set of very tall men. Of course, the process may be repeated, getting fuzzy sets of extremely very tall men,… and so on.

The concept of a fuzzy set enables effective handling vague objects and it proved to provide extremely useful tool in a number of applications (see Novak(2000), Katzan(1992) among others).

EXAMPLE 2(Fuzzy logic).

For the regulation process the following rules (based on experience of an expert) are given:

Rule 1: **if** deviation is great **and** the change of deviation is small
then the change of action operation is very big

Rule 2: **if** deviation is small **and** the change of deviation is more great
then the change of action operation is small

Exploiting these rules, the following inference process employing fuzzy logic rules may proceed :

Observation : deviation is roughly great and the change of deviation is small

Conclusion : (allowing Rule 2) the change of action operation is big.

Unlike (mathematical) logic, that works with statements (either true or false sentences), fuzzy logic works with sentences that are not statements. For instance "deviation is great" is not a statement because of vagueness of "great". With a view to the concept of a fuzzy set (Example 1) it may be spoken about *a fuzzy statement* (this concept is not domestic so far).

EXAMPLE 3(Possibility theory).

The folowing rules are given :

Rule 1 : **if** A: the patient has permanently high temperature (0,7)
or B: the patient´s blood diagnosis is very bad (0,5)
then E: there is a strong evidence (0,9) that the patient should go to the hospital for the treatment

Rule 2 : **if** C: the patient has a high blood pressure(0,6)
and D: the patient is very busy(0,3)
then E: there is evidence (0,5) that the patient should go to the hospital for the treatment

Each rule is associated with a certainty factor, which reflects *the degree of belief* in the validity of that rule. Hence, an "inexact" reasoning systém is based on a construct, reffered to as *a production rule* of the form :

if condition **then** action (to a degree a)

There is frequently also uncertainty associated with the antecedent logical expression-condition, because of measurement errors and inherent incorrectness in a test or measurement. In this example (Rule 1), the evidence for a high temperature is uncertain (0,7) because of the vagueness of the word "high", and similarly with the evidence for very bad blood diagnosis (0,5). The certainty associated with the rule is (0,9). For the inference process, the calculus on rules and the corresponding degrees of belief is employed as follows. In the antecedent part of a rule evidence is combined in accordance with the sense of logical operators :

(1) p **and** q = **min**(p,q), p **or** q = **max**(p,q), **not** p = 1-p .

Rules are evaluated according to (1), (2) :

(2) The belief for the conclusion produced by a rule is given by the multiplication of the belief for the antecedent and the belief of this rule

(3) The belief for the fact produced as the conclusion of one or more rules is given as the maximum of the beliefs of all the rules that yield that conclusion.

By the following computation we get a belief of 0,45 for E (BEL_X denotes belief of X) :

max(BEL_A, BEL_B) = **max** (0,7;0,5) = 0,7 by (1)
$BEL_{Rule\ 1}$= 0,5x0,9 = 0,45 by (2)
min(BEL_C, BEL_D) = **min**(0,6;0,3) = 0,3 by (1)
BEL_{Rule2} = 0,3x0,5 = 0,15 by (2)
max($BEL_{Rule\ 1}$, BEL_{Rule2}) = O,45 by (3)

2. COMMON-SENSE LOGIC AND UNCERTAINTY

Common-sense logic technique has been developed to analyse complex systems in a realistic environment (Tabucanon (1989), Davis(1990), Dohnal(1991) among others). The bulk of data forming the knowledge of such systems is of uncertain character. The calculus for common-sense logic is *qualitative modeling* . Qualitative modeling provides an effective tool to handle with both, certain and uncertain knowledge. For practical reasons it is supposed, that certain knowledge will be expressed in terms of an equation (in the context of qualitative modeling it is usually called *equational knowledge*), uncertain knowledge is mostly expressed by a natural language (alternatively *nonequational knowledge*).

PRINCIPLES OF QUALITATIVE MODELING

In qualitative modeling, only four "qualitative" values are considered :

positive(+), negative(-), zero(0) and irrelevant(*)

and for them an algebra with two operations - *the sum,*denoted by +, and *the multiplication*, denoted by . (mostly omitted), reflecting usual algebra of numbers is defined. The value(or the result) (*) is interpreted in such a way, that it may be (+) or (-) or (0) from the reason that qualitative values can not be distinguished with respect to their size. The operations are given in Tables 1 and 2 :

+	(+)	(-)	(0)	(*)
(+)	(+)	(*)	(+)	(*)
(-)	(*)	(-)	(-)	(*)
(0)	(+)	(-)	(0)	(*)
(*)	(*)	(*)	(*)	(*)

Table 1

.	(+)	(-)	(0)	(*)
(+)	(+)	(-)	(0)	(*)
(-)	(-)	(+)	(0)	(*)
(0)	(0)	(0)	(0)	(0)
(*)	(*)	(*)	(0)	(*)

Table 2

Certain knowledge may contain quantities(variables) quantitative or qualitative nature. Quantities of quantitative nature are transformed to qualitative ones by the process called *degradation* . For instance,

0,02x - 3y + 10z = 0 is degraded to X - Y + Z = 0.

Uncertain knowledge does not require any degradation, because it is qualitative itself. For instance, knowledge "if technical conditions of a car deteriorate then the probability of an accident goes up" is uncertain knowledge (of qualitatitative nature).

Our aim is to introduce now minimal tools to show how seemingly elusive real-life situations full of uncertain knowledge may be modeled and even some decisions for them may be made. For this purpose we need to describe the behavior of some variable (quantity) for a variable usually characterizes the behavior(character of changes) of some phenomena during given time period.Let x=x(t) be a function of time t describing the trajectory of variable X. A *qualitative behavior of variable* X is defined as a triplet

(X, DX, DDX),

where DX is the first and DDX is the second qualitative derivative of x(t), ie. DX is the qualitative degradation of dx/dt and DDX is the qualitative degradation of d^2x/dt^2.

<u>Interpretation :</u> (brackets are omitted for qualitative values)

(X, DX, DDX)	Verbal description as time increases, then X
+ + +	increases, rate of increase grows(convexity)
+ + 0	increases, rate of increase is zero(linearity)
+ + -	increases, rate of increase slows(concavity)
+ - +	decreases, rate of decrease grows(convexity)
.	.

At the beginning of any qualitative analysis the second derivatives are ignored until the first derivatives are known. Sometimes, the second derivatives are difficult to determine. To describe more complicated kind of behavior of variables, particularly relations in their behavior, the concept of a qualitative relation is useful. *A qualitative relation(between variables Y_i , Y_j*) is of the form

$$Y_j = F_{ij} (Y_i) .$$

In a similar way to describe their qualitative behavior, the first (and possibly higher) derivative of Y_j with respect to Y_i is used, denoted by D_{ji} . When considering only the first derivative, then we obtain the following interpretation :

D_{ji} = + means if Y_i increases, then Y_j increase
D_{ji} = - means if Y_i increases, then Y_j decreases
D_{ji} = 0 means if Y_i increases, then Y_j is constant.

Qualitative relations are usualy represented by macroinstructions together with some additional clauses, when necessary. After creating a qualitative model in terms of qualitative behavior of variables and their qualitative relations, the set of so called scenarios (solutions to the model) is found. Suppose that the model M is described in terms of variables $X_1,\ldots, X_n$. Consider (we limit only to first derivatives) n-tuple S,

$$S = ((X_1, DX_1),\ldots, (X_a, DX_a)).$$

S is said to be *a scenario (for M)* when substituing S into the expressions describing M (or posing query S to M) no contradiction is reached; the set of all scenarios for M is denoted by S(M). After all scenarios are calculated, the problems of optimal solution, ie. finding a scenario satisfying given optimization criteria, may be solved.

CASE STUDY - QUALITATIVE MODEL OF A SIMPLE SOCIAL SYSTEM

We will create qualitative model M of a social system , where the following variables are taken into account :

X_1 … standard of living
X_2 … quality of human relations
X_3 … solidarity
X_4 … satisfaction of people
X_5 … social differences
X_6 … unemployment

First step to set up qualitative model is to define qualitative relations between variables. We must respect existing laws (sociology, macroeconomy,…) and to utilize experience and also intuition (expert knowledge). Some relations between variables may be irrelevant (in this case the corresponding qualitative relation does not exist), some may seem to be questionable (in this case we are aware of personal resposibility). The most suitable way to describe qualitative relations is to fill the following table(matrix). The symbol ↑ in the row of X_i and the column of X_j is to be understood as " if X_i increases then X_j increases " and similarly the symbol ↓ "if X_i increases then X_j decreases". With a wiew to the nature of social laws and in order to simplify the text it is agreed that the statement "if X_i increases then X_j increases" implies " if X_i decreases then X_j decreases" and "if X_i increases then X_j decreases" implies " if X_i decreases then X_j increases". Irrelevant qualitative relation between variables corresponds with the empty place in the table.

	X_1	X_2	X_3	X_4	X_5	X_6
X_1	♥	↑	↑	↑		
X_2		♥	↑	↑		
X_3		↑	♥	↑		
X_4		↑	↑	♥		
X_5		↓	↓		♥	

X_6		↓		↓	↑	♥

Now, we can describe qualitative model M by means of macroinstructions having the form :

M_+ X_i X_j : D_{ji} = + (if X_i increases then X_j increases)

M_- X_i X_j : D_{ji} = - (if X_i increases then X_j decreases).

Using data of the above table, the following macroinstructions describe M :
1. M_+ X_1 X_2 2. M_+ X_1 X_3 3. M_+ X_1 X_4 4. M_+ X_2 X_3 5. M_+ X_2 X_4 6. M_+ X_3 X_2 7. M_+ X_3 X_4
8. M_+ X_4 X_2 9. M_+ X_4 X_3 10. M_- X_5 X_2 11. M_- X_5 X_3 12. M_- X_6 X_2 13. M_- X_6 X_4 14. M_+ X_6 X_5
The next step is to find all scenarios S(M) for our model. We will pose queries S - sixtuples of couples

$$S = ((X_1, DX_1), (X_2 , DX_2), (X_3, DX_3), (X_4, DX_4), (X_5, DX_5), (X_6, DX_6))$$

to M finding all that do not conradict M . Since X_i = + for all i = 1, 2, 3, 4, 5, 6 , we wil pose only sixtuples (DX_1, DX_2, DX_3, DX_4, DX_5, DX_6).Then we get the following set S(M) = { S_1, S_2, …, S_{13} } of scenarios for M :

	DX_1	DX_2	DX_3	DX_4	DX_5	DX_6
S_1	+	+	+	+	-	-
S_2	+	+	+	+	-	0
S_3	+	+	+	+	0	0
S_4	-	-	-	-	+	+
S_5	-	-	-	-	+	0
S_6	-	-	-	-	0	0
S_7	0	+	+	+	-	-
S_8	0	+	+	+	-	0
S_9	0	+	+	+	0	0
S_{10}	0	-	-	-	+	+
S_{11}	0	-	-	-	+	0
S_{12}	0	-	-	-	0	0
S_{13}	0	0	0	0	0	0

For example sixtuple (+, +, +, +, +, +) is not a scenario (X_5 increases, X_2 increases, but if X_5 increases then X_2 must decrease, which is a contradiction with G).

References

Davis, E.(1990) : *Representation of Common-Sense Knowledge*. Morgan Kaufmann, San Mateo
Dohnal, M.(1991) : *Methodology for Common-Sense Model Development*. Computers in Industry 16, 141-158
Forsyth, R.(1984) (ed.) : *Expert systems : Principles and Case Studies.* Chapman and Hall, London
Jura, P.(2003) : *Principles of Fuzzy Logic for Controling and Modeling* (In Czech original- *Zaklady fuzzy logiky pro rizeni a modelovani*). VUTIUM, Brno
Katzan, H.(1992) : *Managing Uncertainty*. Van Nostrand, New York
Nguyen, H.T., Walker, E.A.(2000) : *A First Course in Fuzzy Logic*. Chapman and Hall/CRC, New York
Novák, V.(2000) : *Principles of Fuzzy Modelling* (In Czech original-*Zaklady fuzzy modelovani*). BEN, Prague
Tabucanon, M.T.(1989) : *Multiple Criteria Decision Making in Industry*. Elsevier, Amsterdam
Zadeh, L.A.(1965) : *Fuzzy Sets*. Information and Control 8, 338-353
Zadeh, L.A.(1973) : *Outline of a New Approach to the Analysis of Complex Systems and Decision Processes.* IEEE Trans.Syst.Man and Cybern 1, 28-44