The Mathematics Education into the 21st Century Project
The Future of Mathematics Education
Pod Tezniami, Ciechocinek, Poland
June 26th – July 1st, 2004

Word Problems – Can the Model Method Help?

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Abstract : Primary children in Singapore are taught to use the model method to solve word problems. With the model method children are required to interpret information presented in three modes – text, structural and procedural. This paper discusses how 151 eleven year old primary children used the model method to help them solve a series of increasingly complex word problems. These children from five primary schools were asked to use the model method to solve word problems in a one-hour long pencil and paper mathematics test. Children's responses to these items were analysed according to how they used the model method to solve word problems. The findings showed that those children who successfully used the model method to solve word problems had to be competent at five levels – reading, comprehending and transforming, processing and relating. Also these children carried out many mental processes which were not recorded. The paper concludes that the model method did help children solve word problems. Because of its very visual nature, models constructed by children provided evidence into the types of difficulties children had solving word problems. Therefore these models provide teachers with visual clues as to what are some of the conceptual difficulties faced by children in solving word problems and hence offer possibilities how to help children with such difficulties.

Introduction

Primary pupils in Singapore are exposed to a variety of problem solving heuristics. Of those listed in the syllabus (CPDD, 2000), the model method is the most popular. Children can use the model method to represent all the information presented in a given problem as a cohesive whole, and not as distinct parts. This cohesive whole called the model is a structure made up of rectangles. These rectangles which represent unknowns are known locally as units. In formal algebra these units could be replaced by letters. The value of the unknown is solved by solving for the value of one unit. The following example (Figure 1) illustrates how the model method can be used to solve complex word problems which are common in the Singapore Primary mathematics syllabus. The procedures on the right are those recommended by the local teacher's guide.

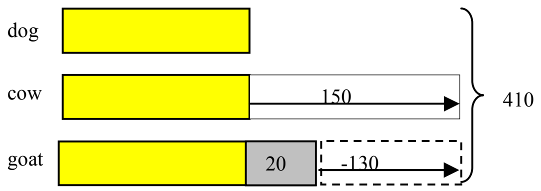
A cow weighs 150 kg more than a dog. A goat weighs 130 kg less than the cow. Altogether the three animals weigh 410 kg. What is the mass of the cow?  	Mass of dog is 1 unit 3 units + 150 + 20 = 410 3 units + 170 = 410 3 units = 240 1 unit = 80 Mass of the cow is 80 + 150 = 230 kg

Figure 1

In the above example the mass of the cow is found by constructing a possible model using the dog as the 'generator' (Berdnarz and Janvier, 1996). The mass of the cow is found by relating the additional mass of 150 kg to the value of one unit. However should the mass of the cow be taken as the 'generator', than the problem can be solved by treating the mass of the cow as one unit. Where the mass of the cow was taken as the generator, one could infer that these children were able to relate that the mass of one unit was the answer they were looking for. Then the following equations are the product of this particular perspective.

Mass of cow as 1 unit. 3 units = 410+ 150 + 130 3 units = 690
1 unit = 230 The mass of the cow is 230 kg.

Through the model method, children are then presented with three modes of representation of a given problem - text mode (T), structural mode (S) and procedural mode (P). To be successful in using the model method to solve word problems, children have to successful in the three modes of presentation –text, structural and procedural (TSP). Also children must necessarily be competent at five different levels and I discuss these below.

Level 1 - Reading the Text

To engage with the problem, children must be able to read the text.

Level 2 - Comprehending the Text

Children have to comprehend what they read. Even if they are competent at Level 1, non-comprehension of the text may mean that children could either construct erroneous models or carry out erroneous procedures.

Level 3 - Translating Text into the Structural mode (TS)

Once they have comprehended the information provided in the text, children then transform the information into the structural mode, a very demanding task. It is important to toggle between T and S to confirm that the S mode accurately depicts the T mode.

Level 4 - Processing the Information

Although they may have constructed the correct model children still need to process the information encapsulated in the model into a correct set of procedures. Children who use models to solve a given word problem could be competent at Levels 1 – 3 but may still fail at competency Level 4. This does not mean that such children were not competent up to Level 3. Rather this shows that another level of understanding is needed to transform information captured by the model into the appropriate set of procedures. Also we propose that at this level, children need to toggle (↔) back and forth between the information presented in the structural mode with the translated information in the procedural mode (S↔P) If children fail to do this, although they may have constructed the correct model, the procedures may not be correct as these procedures may not match the information provided in the structural mode (See Figure 6).

Level 5 – Relating the Answer to the Question

Once they have found the solution, children relate the solution to the question in text mode and check if they have answered the question appropriately.

In summary, because they are taught to use the model method to solve word problems, primary children in Singapore are engaged with processing information presented in three different modes. Children who used the model method successfully to solve word problems could be said to have facility in the three modes – text, structural and procedural. However it is also possible that children who could solve word problems using the TP route and offer the correct model for the solution even though to these children, the model may not be necessary. Thus these children are competent in the T, S and P modes although the S mode could be redundant.

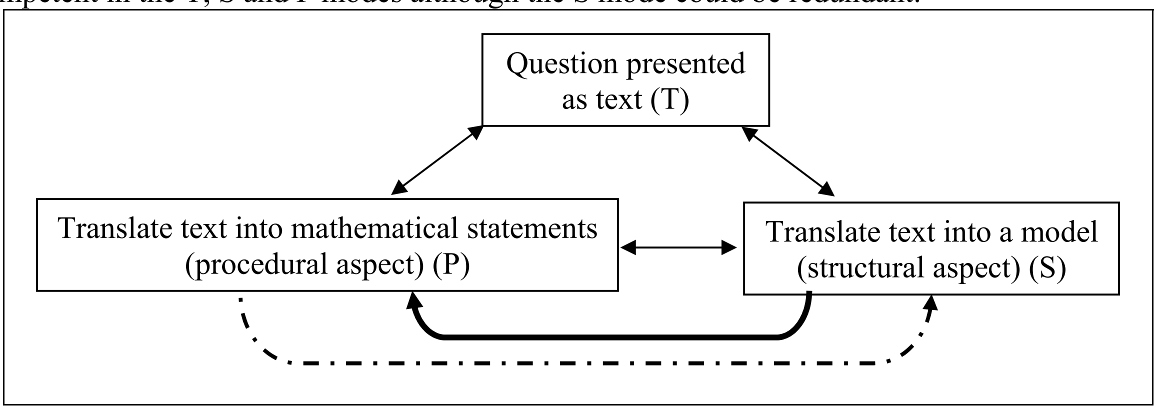


Figure 2: Modes of representation and possible routes to solve word problems when the model method is used.

We offer the following scheme (Figure 2) to explain how children, who are taught to use the model method to solve word problems, engage with the different modes of presenting information.
Because of the visual nature of the structural mode, teachers could identify, through the models offered by children, what were some of the problems these children could have solving word problems. In this paper I will offer evidence to support the above scheme. I will also discuss how the models drawn by children offer insights into their misconceptions. The viability of the above scheme was tested using children's responses to word problems.

The Study
In August 2003, one hundred and fifty-one primary five children (11+) from five different schools took part in a study the aim of which was to ascertain how children use the model method to solve word problems. These children were given one hour to answer ten questions and they were specifically asked to use the model method to solve the problems.
Because of limited space I will discuss children's response to five of the ten test items. Table 1 lists the five items used in the test and the corresponding success rate for each. These questions test concepts of whole numbers and fractions which children were taught from primary three (9+) to early primary five (11+).
Table 1 – Five Test Items

	Item	Success
1	Dunearn Primary school has 280 pupils. Sunshine Primary school has 89 pupils more than Dunearn Primary. Excellent Primary has 62 pupils more than Dunearn Primary. How many pupils are there altogether?	95 (62.9%)
2	At a sale, Mrs Tan spent $530 on a table, a chair and an iron. The chair cost $60 more than the iron. The table cost $80 more than the chair. How much did the chair cost?	67 (44.4%)
3	A cow weighs 150 kg more than a dog. A goat weighs 130 kg less than the cow. Altogether the three animals weigh 410 kg. What is the mass of the cow?	56 (37.1%)
4	A tank of water with 171 litres of water is divided into three containers, A, B and C. Container B has three times as much water as container A. Container C has $\frac{1}{4}$ as much water as container B. How much water is there in container B?	30 (19.9%)
5	A school bought some mathematics books and four times as many science books. The cost of a mathematics book was $12 while a science book cost $8. Altogether the school spent $528.How many science books did the school buy?	22 (14.6%)

Findings
Although children were instructed to use only the model method to solve the test items, children's responses suggested otherwise. Analysis of their responses showed that children chose the methods that were most likely to help them solve the given problems and this was useful as their responses provided information as to what children knew. One child wrote that he did not know how to use the model method and used the TP route for all his solutions, which unfortunately were wrong. In the following section I provide examples of children's work to support the scheme. Because of limited space only selected examples are possible

T → S → P route

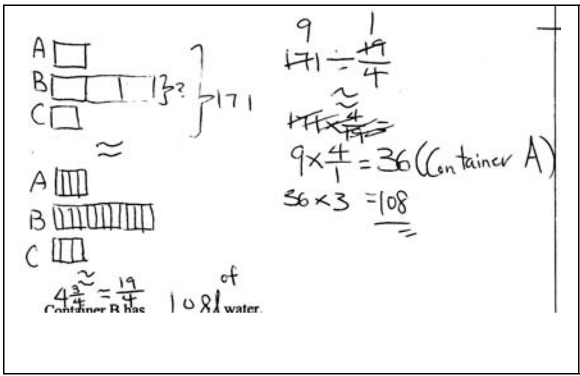


Figure 4 – Item 4

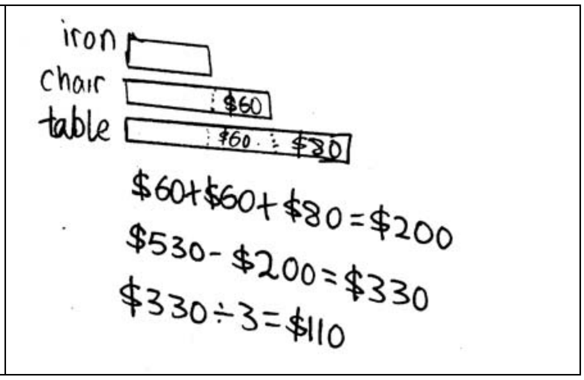


Figure 5 – Item 2

Figure 4 shows the TSP route of the child. However the fact this child drew two sets of models for the same problem, one raw and one refined, showed that many mental processes were carried out and these were not recorded on paper. Figure 5 shows the work of a child who had the correct model and procedures but did not relate the solution to the question and found the cost of the iron instead of the chair.

The example in Figure 6 shows that it is possible that children (12.5% for Item 3) carried out T → S correctly but translated incorrectly the relationships represented by the model, thus committing a S → P translation error. It is not surprising for children to have a structural-procedural gap as the children had to make sense and translate the visual representation of the problem into the appropriate mathematical procedures which in this case was demanding on children as they had to remember that the dotted boxes represent information to be removed and this could be more difficult than having information that had to be added on instead.

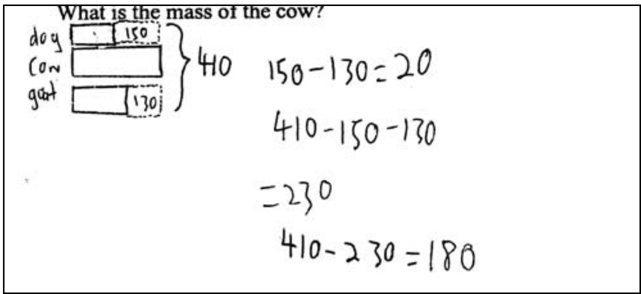


Figure 6 – Item 3 Structural-Procedural Gap

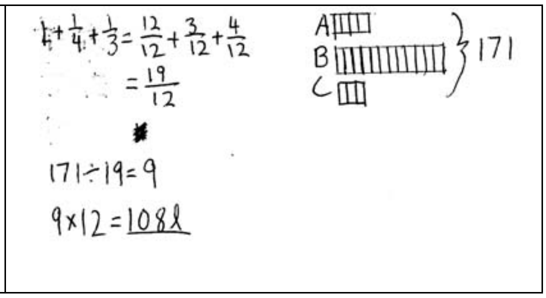


Figure 7 – Item 4

T → P →S→P route

The response in Figure 7 shows that some children could toggle successfully between T, and P and S modes. The child's superior knowledge of fractions allowed him to translate information in the text into a set of procedures which then allowed him to translate those procedures into the model. He then used the information contained in the model and the procedures to solve the problem very elegantly. The TPSP is an additional route to the proposed scheme.

Nature of Errors

Lack of attention to relationships

Because of its visual nature, wrong models would also suggest what some of the misconceptions held by children. Wrong models offered by children suggest that children did not pay attention to the relationships stated in the questions. Table 2 shows the proportion of children who drew models which showed that they misconstrued the relationship between the different items in the question. For example, 25.8% of the children used the iron as the generator for both the chair and the table.

Table 2: Generator type errors

Item 1	Item 2	Item 3	Item 4	Item 5
19 (12.6%) – used Sunshine for Excellent Primary	39 (25.8%) – based cost of table on iron's.	23 (15.2%) – based mass of goat on dog.	Nil	Nil

Difficulties with relational terms

Children in this study showed that they had difficulties understanding the meaning of relational phrases such as 'more than', 'less than', 'as much as' and 'as many as'. The following example shows how these children (9.9% for Item 1) had constructed a model using the number and ignoring the context. This child (figure 8) did not comprehend what she had read. Also children's 'reversal error' (MacGregor, 1991) to Items 4 and 5 showed that children (1.3% and 11.3% respectively) had difficulties making sense of the comparative phrase 'as much as' and 'as many as'.

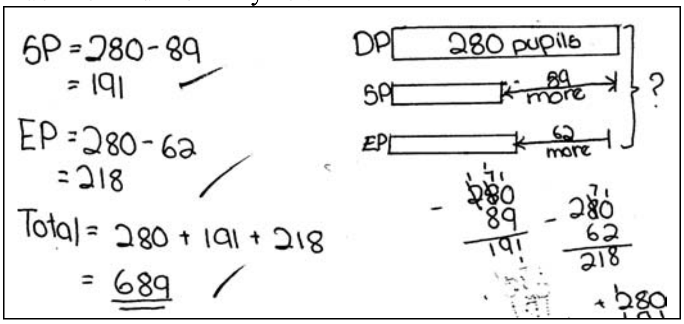


Figure 8 – Item 1

Difficulties with meanings of the rectangles

Children's correct responses to Item 5 showed that while they were successful in using the model method to solve the problem, their understanding was erroneous. In Item 5, each rectangle no longer represents the unknown value; rather the rectangles represent the costs of the books and the number of rectangles, the proportional relationship between the two sets of books. This representation is not the same for Items 1 – 4 where the rectangles are of the same size and represent unknown values. However children's models for Item 5 showed that they drew rectangles of the same size but indicated the cost of the books in the appropriate rectangles. They then solved the problem by asking how many groups of $44 there were in $528. However it could be argued that it did not matter to these children that their understanding was erroneous because the method works for all problems of similar ilk.

Conclusion

The proposed scheme with the additional route of TPSP helps map how children use the model method. This study shows that the model method offers children a way to tease out the relationships presented in the text mode. Even when the method fails, their models offer insight as to where their difficulties lie. Teachers can use the scheme and the five levels of competency to help identify children's limitations in word problem solving.

References

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