

The Mathematics Education into the 21st Century Project
 The Future of Mathematics Education
 Pod Tezniami, Ciechocinek, Poland
 June 26th – July 1st, 2004

On symbolic calculation with the ClassPad300
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Abstract:

Let us consider the ClassPad300 (with the operating system OS 01.24.0000) and discuss on some new commands e.g. sequence(list)-command, sumSeq(list)-command, solve-, dSolve-, rSolve -command, ... to see the possibilities using the new tool in the learning process of our students.

By the help of several examples the interactive work with the ClassPad300 is considered. The student can solve difficult exercises of practical applications step by step using the symbolic calculation of the calculator. Sometimes several fields of mathematics are combined to solve a problem, e.g. the technique of partial fractions to write a quotient of two polynomials as a sum of simpler terms.

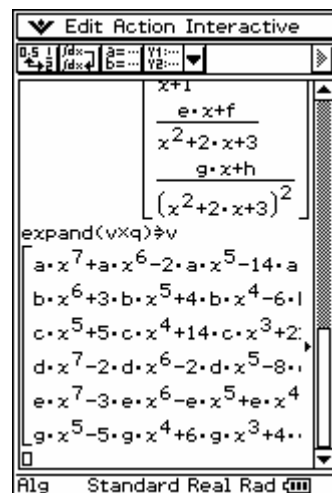
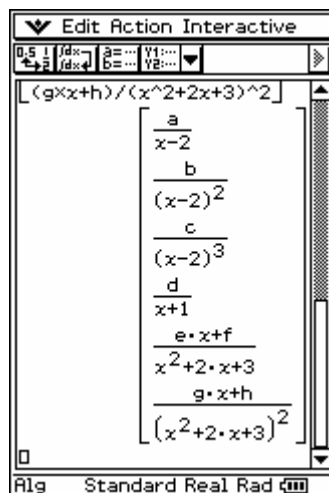
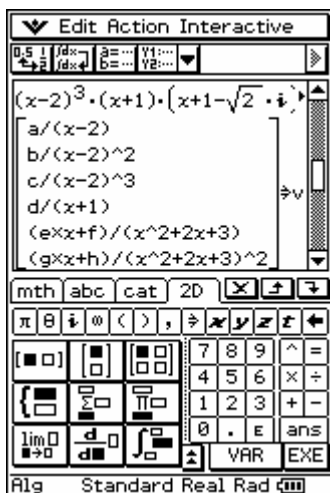
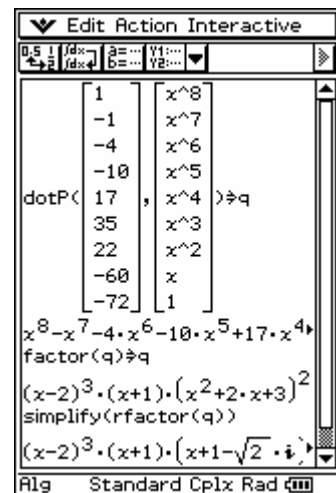
1. Example on partial fractions:

Consider the quotient $p_1(x)/q_8(x)$ of two polynomials $p_1(x)$ and $q_8(x)$ respectively, where

$$p_1(x) = 2x + 3 \quad \text{and}$$

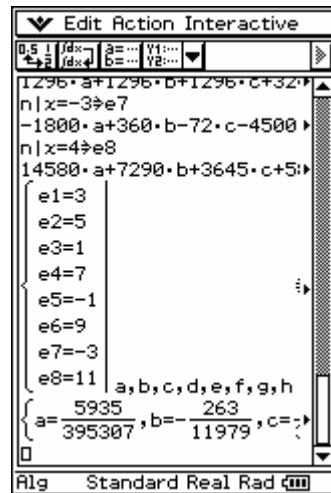
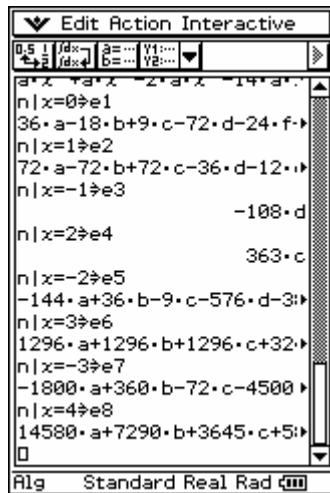
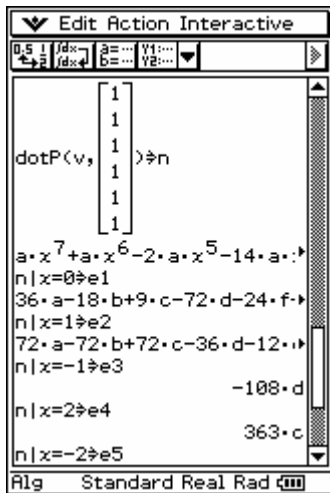
$$q_8(x) = x^8 - x^7 - 4x^6 - 10x^5 + 17x^4 + 35x^3 + 22x^2 - 60x - 72$$

The technique used depends on the factorization of $q_8(x)$. Here we have the real **factor**-command, cp. screenshot. To input the polynomial we use the 2D-input of the virtual keyboard. The polynomial we get by the help of a dot product (scalar product) – so we can see all terms of the polynomial.



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In order to determine the constants a, b, \dots, g, h , multiply the partial fractions by the denominator $q_8(x)$ so that there are no denominators remaining, and substitute any 8 values for x , yielding 8 equations for the constants a, b, \dots, g, h .



The result of the considered linear equation system is the list-variable, say s .

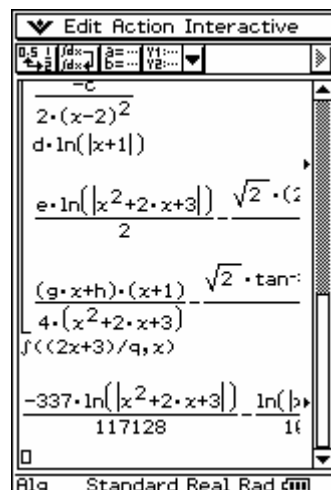
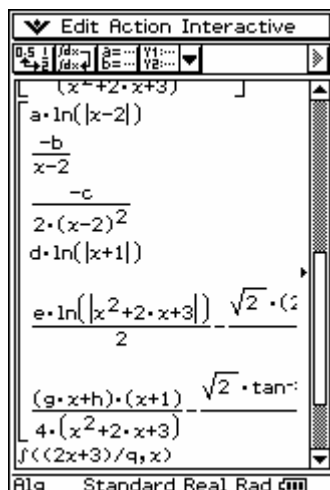
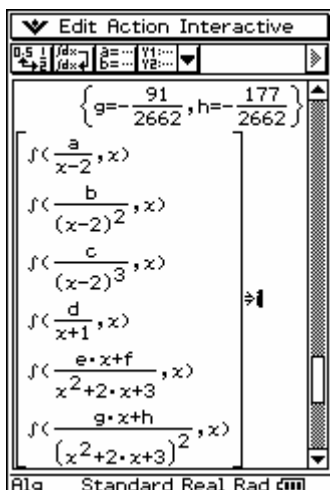
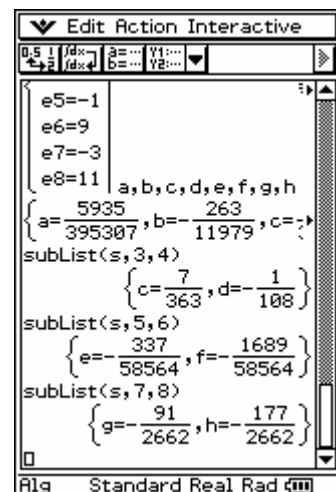
We can observe the elements of the list by the help of the subList-command.

So we see, the ClassPad300 is a useful tool to compute the constants in the partial fractions according the theoretical knowledge of the students.

Here we used elements of the vector-calculation and of the work with linear equations.

Finally we can integrate each of the partial fractions and we can compare our result with the complete integral.

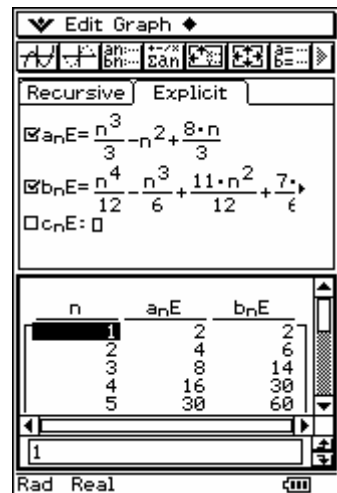
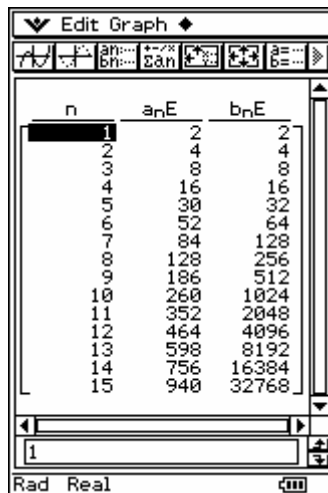
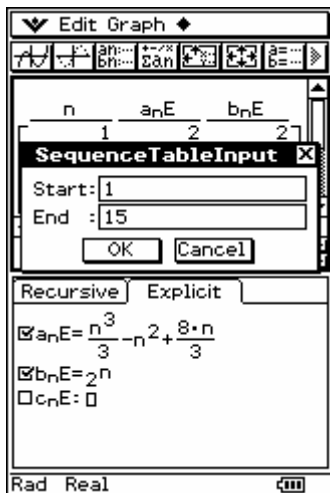
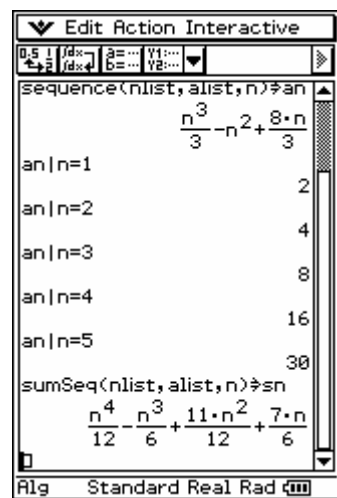
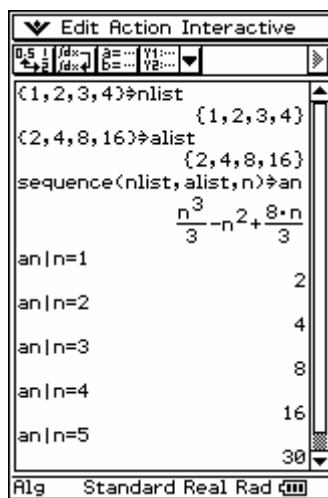
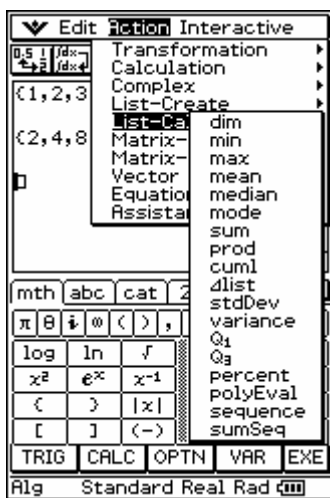
Thus the symbolic calculation is useful for better understanding the mathematics and solving difficult problems.



2. Examples on sequences of numbers and partial sums:

Consider following the sequence of numbers $a_1 = 2, a_2 = 4, a_3 = 8, a_4 = 16, \dots$.
 What do you think is the next number a_5 of the sequence? Is this a geometric sequence?
 If you know the kind of sequence you can build the general law for the number a_n .

If we assume the general law of the considered sequence is a polynomial, then the ClassPad300 can compute the general term of a_n . The first three screenshot are from the Main-menu, the next are from the sequence-menu. Here b_n is a geometric sequence.



Now we can compute the partial sums by the help of the **sumSeq**-command in the Main-menu or by the help of $\Sigma a_n E$ in the Sequence-menu.

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n	a _n E	b _n E
1	2	2
2	4	6
3	8	14
4	16	30
5	30	60
6	52	112
7	84	196
8	128	324
9	186	510
10	260	770
11	352	1122
12	464	1586
13	598	2184
14	756	2940
15	940	3880

n	a _n E	Σa _n E
1	2	2
2	4	6
3	8	14
4	16	30
5	30	60
6	52	112
7	84	196
8	128	324
9	186	510
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11	352	1122
12	464	1586
13	598	2184
14	756	2940
15	940	3880

Finally we consider well-known special cases of power-series (powers of integers) and the arithmetic series.

```

seq(n,n,1,5)⇒nlist
  {1,2,3,4,5}
seq(n^2,n,1,5)⇒alist
  {1,4,9,16,25}
sumSeq(nlist,alist,n)
  n^3/3 + n^2/2 + n/6
factor(n^3/3 + n^2/2 + n/6)
  (2·n+1)·n·(n+1)
  / 6
factor(∑_{k=1}^n (k^2))
  (2·n+1)·n·(n+1)
  / 2·3
  
```

```

seq(n,n,1,5)⇒nlist
  {1,2,3,4,5}
seq(n^3,n,1,5)⇒alist
  {1,8,27,64,125}
sumSeq(nlist,alist,n)
  n^4/4 + n^3/2 + n^2/4
factor(ans)
  n^2·(n+1)^2
  / 4
factor(∑_{k=1}^n (k^3))
  n^2·(n+1)^2
  / 2^2
  
```

```

{1,2,3,4}⇒nlist
  {1,2,3,4}
(a+d,a+2d,a+3d,a+4d)⇒alist
  {a+d,a+2·d,a+3·d,a+4·d}
"arithmetic sequence"
"arithmetic sequence"
sequence(nlist,alist,n)
  a+d·n
"arithmetic series"
"arithmetic series"
sumSeq(nlist,alist,n)
  d·n^2
  / 2 + n·(a+d)
  / 2
  
```

Next we study the **rSolve**-command to get from a recursion formula the general term of a sequence. Here we consider the Fibonacci numbers $a_1=1, a_2=1, a_{n+2} = a_{n+1} + a_n$

```

rSolve(an+2=an+1+an, a1=1, a2=1)
  {
  an = (sqrt(5) * ((1+sqrt(5))/2)^n - ((-1+sqrt(5))/2)^n) / 5
  }
rSolve(an+2=an-an+1, a1=1)
  {
  an = 4 * (3 * (1/2)^n - 3 * (-1)^n) / 9
  }
  
```

```

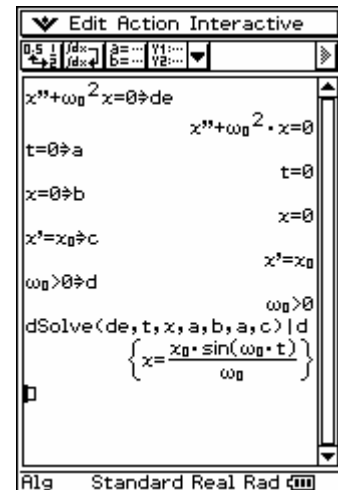
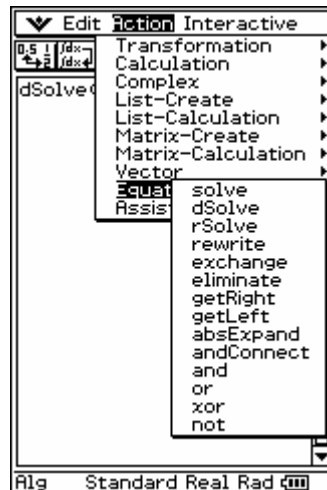
rSolve(an+2=an+1+an, a1=1, a2=1)
  {
  an = (sqrt(5) * ((1+sqrt(5))/2)^n - ((-1+sqrt(5))/2)^n) / 5
  }
rSolve(an+2=an-an+1, a1=1, a2=0)
  {
  an = 4 * (3 * (1/2)^n - 3 * (-1)^n) / 9
  }
  
```

3. Examples on ordinary differential equations (odf):

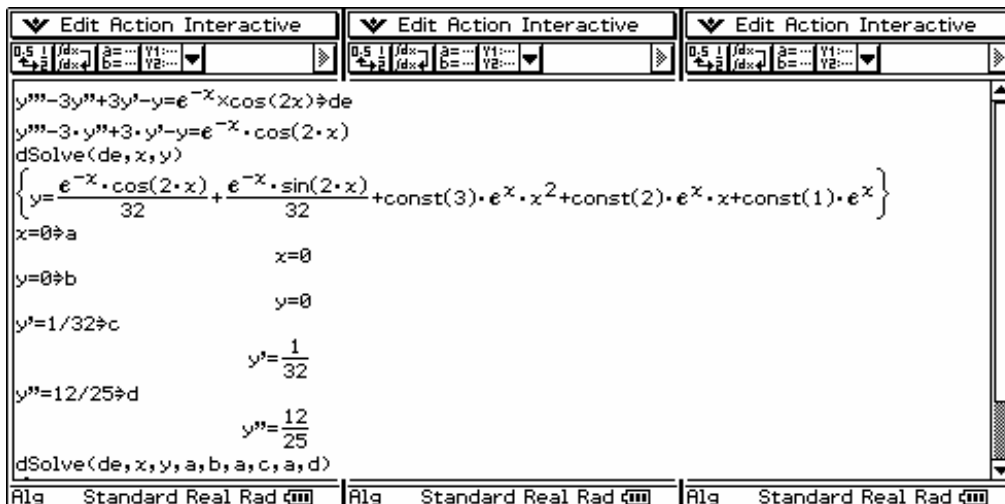
We consider a simple second order constant coefficient equation using the virtual keyboard to input greek symbols or indices.

Additionally we have two starting conditions and a condition on ω_0 .

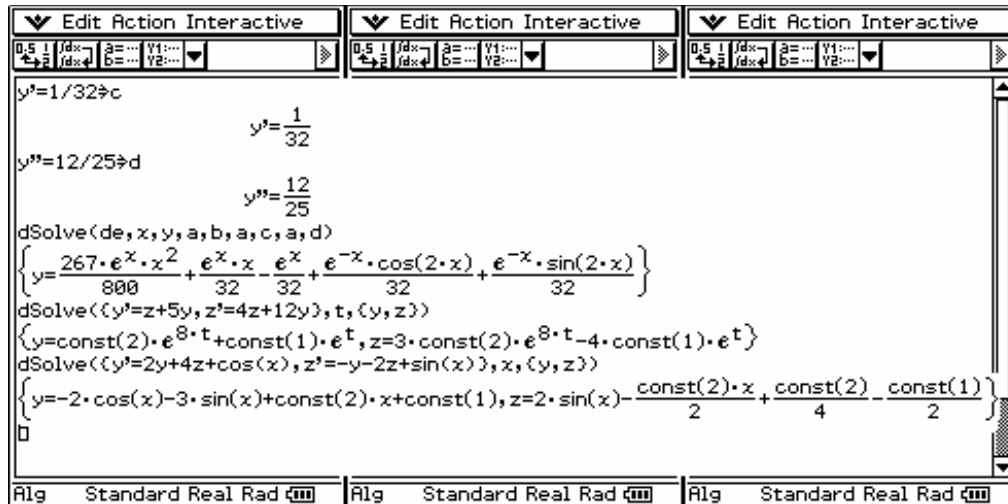
Here you see the syntax of the **dSolve**-command.



Next we consider an ordinary differential equation of 3rd order and a system of two odf:



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During the lecture the virtual keyboard with the 2D-input-windows and the possibility of Drag&Drop are used to show how convenient it is to work on the touch-screen.

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References:

- [1] CASIO: **ClassPad300 User's Guide** (670 p.)
cp. <http://www.classpad.de/euro/files/manualeng.pdf>
- [2] Paditz, L.: **Mathematische Modelle und wissenschaftlich-technische Anwendungen** (100 p.), Hrsg. CASIO Europe GmbH Norderstedt 2004
- [3] Paditz, L.: **Rechnen und Darstellen mit dem neuen CASIO-ClassPad300** (2003)
cp. <http://www.classpad.org/pdfs/Videoclips.pdf>