The Future of Mathematics Education
Pod Tezniami, Ciechocinek, Poland
June 26th – July 1st, 2004

Workshop Summaries

Helena Bereková, Faculty of Mathematics, Physics and Informatics, Comenius University, Bratislava, Slovakia
Graphs and systems of equations with a real parameter solved by a graphic calculator

In the course of workshop, I wish to present a solution of equation and inequation containing a real parameter. The goal is to show how the graphic calculator can help to solve the tasks in the case when the numerical and also graphical solution is not entirely successful.

Mathematical tasks: Task 1. Equation $x^2 = a^x$, (1) can have in dependence on the parameter $a \in (1, \infty)$

    a) two, three or four solutions
    b) two or three solutions
    c) one, two or three solutions
    d) two, three, four or five solutions
    e) non, two or four solutions

The task 1 can be formulated also in the following way.

    How many intersections can have the graphs of functions *f: y = x²* and *g: y = aˣ*, where *a* is the real parameter $a \in (1, \infty)$?

    1) Let us solve the Task 1 in as many ways as you can.
    2) Let us find such value of parameter a, for which the equation (1) has just one solution.
    3) Let us perform experiments and describe your findings.

Task 2. For which value of the real parameter *a,* the solution of inequation $\log_a(x^2 + 2) > 1$ (2) is the set of all real numbers? Second interpretation of the Task 2: **For Which value of the real parameter *a* "lies" entire graph of the function *f:* $y = \log_a(x^2 + 2)$ above the graph of the function *g:* y = 1?**

Let us try solving the Task 2 numerically or graphically. Record your findings anonymously on the enclosed sheet.

Using the calculator Graphs of the functions described in the previous can be drawn in the GRAPH mode. We shall choose concrete values of the real parameter and look for all intersections of graphs of given functions. When simultaneously displaying graphs of several functions, it can be helpful to use a different color for each. The color can be selected by highlighting a desired function and then selecting F4. We shall compare results of numerical or graphical solutions with results of solutions, at which the graphic calculator has been used.

George Booker, Griffith University, Brisbane, Australia
Using games to develop meaning and proficiency with fraction concepts and processes

While whole number understanding builds on everyday experiences with natural numbers, an understanding of fraction ideas is not so strongly rooted in the real world. Students often focus on the size of the parts, rather than the relationship among parts or numbers. This requires a significant change in thinking, and has led to fractions being described as the first artificial numbers that students will meet as they build ever more complex number systems. This workshop will provide a framework and a range of games that will provide authentic settings for the development of meaningful initial concepts, recording and renaming among the various forms of fractions (improper and mixed number, equivalent common fractions) and equivalence among decimal fraction, per cent and common fraction forms.

RNDr. Monika Dillingerová
Faculty of Mathematics, Physics and Informatics , Comenius University, Bratislava, Slovak Republic
How to solve geometrical problems with CLASS PAD

If we are saying "Mathematical Olympiad", so a big mathematical problem occurs in each of us. To solve such a problem is very difficult, time consuming and demanding. In a lot of cases we need to use no traditional methods, but set some hypothesis, which's truth we than have to show. To solve this art of problems the new Casio calculator – CLASS PAD – has some famous software, especially the geometry software built there in. I will show you how this software can help to solve problems from

our national Mathematic Olympiad. There are problems with finding the set of all possible points fulfilling some rules, finding a point with maximizing an area etc.

Gunnar Gjone

1) **Investigating geometry on ClassPad 300. Introductory workshop**.
We will present elementary constructions of geometrical figures. The participants will use ClassPad to solve ordinary geometry problems, by investigating properties of figures.
2) **Number theory on ClassPad 300**.
ClassPad contains several number theoretic functions. We will present some of these functions and show how to use ClassPad for investigating number theoretic relations.
3) **Using the ClassPad 300 spreadsheet**. As an additional feature ClassPad 300 contains a spreadsheet, with ordinary spreadsheet functions. We will look at how this spreadsheet can be used in mathematics education to investigate number relations.

Iris DeLoach Johnson, Miami University, Teacher Education Department, Oxford, OH USA
Using Technology to Support Inquiry Learning in Mathematics

This workshop will provide a hands-on exploration of activities designed to help students discover such mathematical concepts as divisibility rules, multiplication/factoring of binomials, finding derivatives. We'll use manipulatives and technology and discuss sample work of students who've engaged in these activities. As we look toward "The Future of Mathematics Education" we will discuss how technology might be used to support students' mathematical proficiency in the future.

Lilla Korenova, Faculty of Mathematics, Physics and Informatics Comenius University, Bratislava, The Slovak Republic
Some models of using ClassPad in teaching mathematics for secondary students

Mathematics is about ideas. One of the most important is the idea of function. It is functions which bring algebra to life, but it is hard to describe exactly what they are. However we can see functions at work when we make tables of values, draw mapping diagrams, solve equations and draw graphs.

The largest and most difficult part of the syllabus for the secondary mathematics is formed by the functions. Teachers of mathematics, using classical methods, are limited significantly to present only several basic types of functions. However, new teaching methods using graphic scientific calculators in the classroom, such as Cassio`s ClassPad, enable students to make limitless experiments with functions, to draw their graphs and achieve some knowledge of higher quality even from advanced, non-traditional functions.

Exercises based on methods of optimum choice are taught only in the last year of secondary education in Slovakia when students have already learnt finding the turning points (maxima, minima) of the functions by the differential calculations. Many fascinating exercises may be solved by methods of optimum choice which are much more useful from a didactic point of view when tasks are solved by mathematical modeling rather than just by the mechanical differential calculations.

Graphic scientific calculators draw the graphs quickly and find the maxima and the minima of the functions easily allowing students to solve interesting exercises which otherwise may not be solved at the secondary level.

In my workshop I would like to present how students can be motivated to make experiments in graphing and finding relationships, values of functions and solving exercises based on methods of optimum choice.

Ludwig Paditz, University of Applied Sciences Dresden (FH), Germany

1. **Several aspects of 2D-graphic with the ClassPad300** (functions y=f(x) or parametric x=x(t), y=y(t) or polar r=r(theta) and nonlinear regression in a scatter-plot and random functions and finally using the new log-log-scaling view-window):

2. **Several aspects of 3D-graphic with the ClassPad300** (functions z=f(x,y), how to get the best view of the surface?, discussion on several examples):

Felix Rieper & Dr Andreas Filler
Analytic Geometry with 3D Computer Graphics in Maths Education

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What do have computer games, movies or today's weather forecast in common? They are the result of mathematical calculations visualised with 3D computer graphics. Maybe you are fascinated by the realistic and, at the same time, virtual results of the computer graphic artists. It is even more fascinating to make computer pictures yourself. This workshop intends to give participants an introduction to the computer graphics software POVRay and its effective usage in mathematics classes.

POVRay is on the one hand a computer language for describing arrangements of geometrical objects (scenes) by means of analytical geometry. Pictures in your mind have to be analysed in terms of basic shapes and then set into a 3D coordinate system. At times this can be strenuous – but the results are the more amazing since on the other hand POVRay is a powerful (and free) 3D rendering software. It transforms the geometrical scene via ray tracing into a photo realistic picture: Every pixel is the result of an intricate back-tracing of light rays from the objects to the light sources. Unlike educational maths software for analytical geometry POVRay produces beautiful pictures, which can be very motivating for students.

The workshop starts with a talk introducing the major features of POVRay and demonstrating the use of the software in different projects in maths education. The presentation continues with some students' results of projects undertaken by Dr. Filler and Mr Rieper.

The main part of the workshop is a hands-on-practical to get a first impression of the work with POVRay. Participants can work in pairs with a self study booklet conceived especially for the workshop. A number of templates facilitate the first steps with the software and the two teachers organising the workshop will give the participants a helping hand if problems occur.

All example images in the workshop booklet are on a CD ROM, in form of jpg-files along with the POVRay code files. For all problems posed in the booklet participants can find solution proposals on this CD ROM.

It would be nice if participants owning a portable PC, could bring it to the workshop. The (free) software will be distributed in the beginning but can also be downloaded from www.povray.org. Please download the version 3.5 to avoid compatibility problems with the templates.

Anna Rybak, Bialystok, Poland
Presentation of the educational software – program VUStat for learning and teaching of statistics and probability.

Authors of VUStat are Piet van Blokland and Carel van de Giessen from Netherlands. They are teachers and programmers with big experience in math teaching. Program VUStat may be used in schools of different types and levels: from middle school until university. The main menu of VUStat contains the following options (in the brackets I follow sub-options of each option):

STATISTICS - handling data and data analysis,

DATAPLOT – a small environment using frequency tables and graphical representations of data,

PROBABILITY (Tree diagram, Urn, Galton's grid, Frequency grid, Call and put options),

SIMULATIONS (Coins, Dice, Smoking, Reservations, Reaction speed, Fast food shop, Monopoly, Roulette, One armed bandit, Banks, Regression lines, Randomized response, Artificial intelligence, Random numbers, Sampling),

DISTRIBUTIONS (Binomial distribution, Normal distribution, Hypergeometric distribution, Poisson distribution, Plots of distributions, Central Limit Theorem),
TESTING HYPOTHESES (Binomial test, t-test, z-test, Poisson test, Confidence intervals, Power of the binomial test).