Working Group 1: Powerful learning environments for mathematics problem solving

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Position Paper

Major conclusions of the Working Group at the Palermo conference

Presentations and discussions of group 1 show that, in dealing with arithmetical word-problems, language understanding is not the only pre-request we need to develop in students, but as well mathematical concepts understanding. The understanding of mathematical concepts and propositions are the main pre-request to solve algebraic and geometric problems. The lack of geometric background of students is an obstacle to deal with other mathematical problems like integral calculus problems.

In order to make connections in students' mathematical problem solving, the use of concrete, realistic, and authentic problem situations is recommended, especially for young children. However, it is more important that problems are meaningful for the learner. It has to be remarked that an abstract problem is realistic, once it is personally meaningful for the problem solvers. Moreover, we should keep in mind the dual nature of mathematics: on the one hand, mathematics offers tools for modeling and solving problems in everyday life, but on the other hand, mathematics is a culture of formal and abstract structure. From this perspective it is important to stimulate reciprocal interactions between the concrete and the abstract levels. As a result, one should also be aware of taking into account individual differences among learners. Disparate problem situations should be used. Using a contextual approach and relying on visualization we can develop young students understanding of algebraic concepts. Young students have to learn algebraic relations and solve algebraic problems through generalizing number properties.

In addition besides problem solving, problem posing or problem generating by the learners themselves should be stimulated more than is hitherto the case.

The teaching of problem solving should be more process- and strategy-oriented than product-oriented. In view of stimulating in students' constructive and progressively more self-regulated learning, a change in the role of the teacher is essential. Instead of being the sole source of knowledge and solutions, the teacher should create a classroom climate and culture that encourages and facilitates students' own initiatives and stimulates interactive and collaborative problem solving. The teacher's input is thus becoming more essential than before as the architect of the learning process. Problem solving in an inquiry-based classroom activity helps students to create, modify, and promote their cognitive constructions. In solving problems, teacher should appreciate discussions and arguments of students. Well-planned dialogue led by teacher is an appropriate way to facilitate the reveal of different solutions for a particular mathematical problem.

An important issue, however, is how such approaches to learning and teaching mathematics can be disseminated. Teachers' and students' beliefs can be an obstacle in improving the teaching of problem solving. In that perspective, substantial efforts are necessary to develop instructional materials that are in line with the so-called 'new conception of teaching and learning mathematics', and to introduce this conception in initial and in-service teacher education and training. To be successful it is also essential to change teachers' beliefs about mathematics and mathematics education. Finally, it is useful in view of disseminating the new conception of teaching and learning mathematics problem solving to identify and share what might be called "success stories", i.e. cases that illustrate in a convincing way how the new approach can be implemented appropriately and effectively.

Initial discussions at the Palermo conference on "The importance of mathematics-related beliefs"

Over the past decade many researchers have been studying students' beliefs aiming, on the one hand, at identifying the different kinds of students' beliefs that influence mathematical learning and problem solving, and on the other hand, at understanding the processes through which they develop and determine learning and problem solving. This work has revealed how different categories of students' beliefs shape their cognitive as well as conative and affective processes in the classroom. First, several studies have demonstrated how *beliefs about the nature of mathematics and mathematical learning* and problem solving determine how one chooses to approach a problem and which techniques and cognitive strategies are used. Research on the relevance of subject-specific manifestations of *epistemological beliefs* for mathematical learning and problem solving further supports these findings. Apart from the research on these two categories of beliefs that deal with the way students' cognitive processes are influenced by their beliefs, other scholars have investigated the motivational and volitional relevance of students' beliefs. More specifically, studies on students' *value and/or expectancy beliefs* in the context of mathematical learning and problem solving clearly show how these beliefs relate to students' motivation and the way they engage in mathematical learning and problem solving; these investigations also substantiate their influence on achievement. Finally, *students' beliefs about teaching and the practices characterizing their specific classroom context* have been found important factors to be taken into account if we want to understand fully the academic behavior in the mathematics classroom. More than students' beliefs about the specific classroom context as such, it appears to be the closeness of fit between students' more general beliefs about mathematics teaching, learning, and the self, on the one hand, and the perceived practices typical for their classroom, on the other hand, that enables us to explain some of the motivational and emotional reactions of students. At present, little is known about this relation between students' beliefs and their emotional processes in the classroom. Particularly, the few studies that investigated the relation between beliefs and emotions indicated that indeed students' beliefs about mathematics education provide an important part of the context within which emotional responses to mathematics develop. Aside from a general agreement among researchers that students' beliefs have an important influence on mathematical learning and problem solving from a conceptual as well as from an empirical viewpoint, there is still a lack of clarity. Indeed, so far research on this topic has not yet resulted in a comprehensive model of, or theory on students' mathematics-related beliefs. As a matter of fact, most of the studies are situated in, respectively, cognitive, motivational or affective research traditions and in many cases operate in relative isolation from each other. The isolated study of specific categories of beliefs within these distinct research traditions in many ways has prevented the study of different students' beliefs in relation to each other, i.e. the analysis of students' belief *systems* related to mathematics learning and problem solving. It is precisely the study of students' mathematics-related belief systems, more than the study of isolated beliefs that can push the field forward. Indeed, such study may present a unifying framework for research on students' mathematics-related beliefs, resulting in more systematic research efforts and leading to a more comprehensive understanding of how beliefs influence mathematics learning and problem solving. During the sessions of the working group initial research in that perspective was reported.

A new perspective for discussion: "Intervention research: A tool for bridging the theory – practice gap in mathematics education?"

Recent research on learning and instruction in mathematics has substantially advanced our understanding of the processes of knowledge and skill acquisition. However, school practices have not been innovated and improved in ways that reflect this progress in the development of theory and research.

To be successful in making psychological theory and research applicable to mathematics education one should develop a strategy that combines the following basic characteristics:

- good communication with practitioners which means that the relevant outcomes are translated in such a way that they become palatable, accessible, and usable for the teachers;
- an orientation toward a fundamental change of teachers' belief systems about the goals of mathematics education and about good teaching and productive learning;
- a holistic (as opposed to a partial) approach to the teaching-learning environment, i.e. all relevant components of the learning environment should be addressed.

Taking this into account a promising strategy for bridging the theory-practice gap consists in carrying out intervention research involving the creation and evaluation in real classrooms of complex instructional interventions that embody our present understanding of effective learning processes and powerful learning environments. In order to make a reasonable chance to be successful, such attempts at fundamentally changing the classroom environment and culture should be undertaken in partnership between researchers and reflective practitioners. Such partnership is essential to promote mutual good communication, but also in view of modifying and reshaping teachers' beliefs about education, learning, and teaching. This intervention approach has a twofold goal: it intends to advance theory building, while at the same time contributing to the optimization of classroom practices.