WORKING GROUP B / GROUP DE TRAVAIL B

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"MATHEMATIQUES ET VIVRE ENSEMBLE": POURQUOI, QUOI, COMMENT?

"MATHEMATICS AND LIVING TOGETHER": WHY, HOW, WHAT?

Report Working Group B / Report Group de Travail B

Mathematics and sustainable development / Mathématiques et développement durable

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FRANÇAIS:
Les articles présentés et le dialogue qui a émergé dans le groupe de travail B étaient reliés au sous-thème 1 du document de discussion, "Mathématiques et éducation durable".
Même si le sous-thème suggérait un focus sur trois aspects (développement durable, genre et pédagogies collaboratives), aucun article a incité l'émergence d'une recherche et d'un dialogue sur « mathématiques et genre ». Une considérable présence d'hommes dans le groupe a été un exemple que les inégalités persistent mondialement dans le système de l'éducation mathématique, en contraste avec le fait que les deux co-animatrices et les cinq conférencières étaient des femmes.
Les cinq communications ont mis en lumière l'importance des pédagogies coopératives en soulignant le rôle de l'individu (élève ou future enseignant) dans le collectif à tous les niveaux de l'éducation, au primaire, au secondaire et à l'université. Des différences entre les articles sont apparues en prenant en compte les principales compétences qui, individuellement, en coopération et en collaboration, étaient nécessaires et développées par les élèves et les enseignants pour s'engager activement dans la compréhension et la construction des mathématiques.
Le travail de Panero et Brunero a soutenu l'importance de développer et d'évaluer les compétences argumentatives. Les autrices ont suggéré trois critères pour évaluer un discours argumentatif : aucune erreur mathématique dans la conclusion et dans la justification, la compréhensibilité pour l'enseignant et les pairs et l'exhaustivité du raisonnement à partir des données jusqu'à la conclusion. Ces trois aspects ont été compris par l'auditoire comme une garantie de la construction et de la compréhension des connaissances développées à travers le discours argumentatif. De plus, le dialogue avec l'auditoire a explicité l'importance de clarifier la différence entre la compétence argumentative et les attitudes nécessaires à une communication précise, en termes de justification, de dialogue et de raisonnement.
Le travail de Nazzal et Sodqi a mis en évidence l'importance de développer des compétences de recherche dans les applications et la modélisation avec des étudiants universitaires pour innover dans les cours traditionnels de mathématiques pures. La participation active des étudiants universitaires à ces innovations nécessite des attitudes et d'une motivation renouvelées qui augmentent la confiance en soi et l'estime de soi de la part des étudiants.
En ce qui concerne le développement des compétences, Panero et Brunero ont souligné l'importance de l'argumentation, en particulier, et de la communication, en général, pendant le processus de compréhension des connaissances impliquées dans une tâche particulière ; Nazzal et Sodqi ont considéré l'importance de la communication pour des présentations efficaces des applications et des modèles utilisés par les étudiants pendant un processus non routinier de résolution de problèmes.
Vale, Barbosa et Cabrita ont étendu le processus de résolution de problèmes à la pose de problèmes. Elles ont présenté une approche isomorphe pour la formation des enseignants, car les enseignants pourraient enseigner à leurs élèves du primaire en développant des « trails », des séquences de tâches que les élèves et les enseignants en formation initiale doivent résoudre le long d'un itinéraire préétabli. Ces tâches se concentrent à la fois sur la résolution de problèmes et sur la pose de problèmes dans le but de contribuer à l'acquisition de connaissances mathématiques, mais aussi au

développement d'autres compétences (par exemple, communiquer, raisonner, argumenter, représenter et critiquer).
En complément de cette présentation, l'article de Vale et Barbosa a proposé la méthodologie de la « Gallery Walk » pour identifier et comprendre les stratégies utilisées pour résoudre des problèmes à résolutions multiples, en particulier pour identifier l'utilisation de stratégies visuelles ; analyser/discuter les stratégies des pairs ; et caractériser la réaction à la « Gallery walk » comme une stratégie d'apprentissage et d'enseignement.
Les communications de Vale et Barbosa et Vale, Barbosa et Cabrita ont favorisé la discussion entre enseignants, chercheurs et inspecteurs sur les difficultés liées aux contraintes institutionnelles nationales : temps, programmes et contexte (à l'intérieur et à l'extérieur de l'école). Cependant, pour surmonter ces difficultés, les enseignants et les élèves devraient devenir des apprenants actifs, avec un nouveau rôle clé pour les inspecteurs et les formateurs des enseignants.
Les quatre articles soulignent que le fait de devenir des apprenants actifs, à travers l'acquisition de nouvelles attitudes dans la pose et la résolution de problèmes dans des situations de mathématiques appliquées et de modélisation à l'intérieur et à l'extérieur de l'école, a certainement encouragé l'action et le développement des certaines compétences que nous avons appelées les 4Cs (communication, y compris le raisonnement et l'argumentation, la collaboration, la pensée créative et critique). Même si l'activité de pose et de résolution de problèmes favorise la construction et la compréhension des connaissances de la part des élèves (Vale et Barbosa ; Vale, Barbosa et Cabrita), les attitudes positives envers les mathématiques (Nazzal et Sodqi) ainsi que le développement de compétences méta-cognitives (Panero et Brunero), la promotion de la pensée critique ne suffit pas à fournir aux élèves à la fois des connaissances cognitives et des connaissances méta-cognitives.
Enfin, l'article de Serradó, Romero et Prieto visait à mathématiser les croyances, les pensées et la prise de décision des élèves du secondaire lorsqu'ils sont impliqués dans la résolution de sept problèmes mal posés pour comprendre les conséquences du changement climatique. Les auteurs ont discuté des questions méthodologiques concernant la manière de transformer ces sept problèmes mal posés en une tâche authentique pour une éducation mathématique durable. Ces méthodologies tiennent compte des pédagogies collaboratives pendant le processus actif de résolution de problèmes et de l'individualité concernant les connaissances méta-cognitives déclaratives, procédurales et conditionnelles utilisées par les élèves.

Une approche commune a été adoptée lors de la présentation des cinq communications et des discussions qui ont suivi, ce qui nous a conduit à affirmer que « la didactique des mathématiques devrait être une opportunité sociale pour tous vivre ensemble ». Malgré cette approche intégrante au développement de compétences et attitudes mathématiques, à la garantie de connaissance et à une vue approfondie des possibilités méthodologiques de résolution de problèmes, d'applications et de modélisation à l'intérieur et à l'extérieur de l'école, de futures recherches sont encore nécessaires pour saisir la complexité de mettre en œuvre des pédagogies collaboratives pour une éducation durable des mathématiques.

ENGLISH:
The papers presented and dialogue that emerged in WGB was related with the subtheme 1 of the discussion document, "Mathematics and Sustainable Education". Although the subtheme suggested to focus on three aspects (sustainable development, gender and collaborative pedagogies), none of the papers facilitate the emergence of research and dialogue concerning "mathematics and gender". A major presence of men in the group was an example that the inequalities persist in the mathematical education system worldwide, contrasted by the fact that both co-animators and the five speakers were women.
The five papers enlightened the importance of cooperative pedagogies pointing out the role of the individual (student or pre-service teachers) in the collective in all the stages of education, from

Primary, Secondary and University. Differences between the papers emerged when considering which were the main competences that individually, cooperatively and collaboratively were needed and developed by students and teachers to actively engage in understanding and constructing mathematics.

The work of Panero and Brunero argued about the importance of developing and assessing argumentative competences. Authors suggested three criteria for assessing the argumentative discourse: no mathematical errors in the conclusion and in the justification, understandability for the teacher and the peers and completeness of the reasoning from the data to the conclusion. Those three aspects were understood by the audience as a warrant of the knowledge construction and understanding developed through argumentative discourse. Moreover, dialogue with the audience explicited the importance of clarifying the difference between the argumentative competence and the skills needed for an accurate communication, in terms of justification, dialogue and reasoning.

Nazzal and Sodqi's work highlighted the importance of developing research skills in applications and modelling with University students to innovate traditional courses in pure mathematics. The enjoyable involvement of University students in those innovations needs renewed attitudes and motivation that increases students' self-confidence and self-esteem.

Concerning with the development of competences, Panero and Brunero pointed out the importance of argumentation, in particular, and communication, in general, during the process of understanding the knowledge included in a particular task; Nazzal and Sodqi considered the importance of communication for effective presentations of the applications and models used by the students during non-routine problem-solving process.

Vale, Barbosa and Cabrita extended the process of problem-solving to problem posing. They presented an isomorphic approach for teacher education as the teachers could teach their Primary students through the development of trails, sequences of tasks that students and pre-service teachers have to solve along a pre-planned route. Those tasks focus on both problem posing and solving with the aim of contributing to the acquisition of mathematical knowledge, but also to the development of other skills (e.g. communicating, reasoning, arguing, representing and criticizing).

Complementing this presentation, the paper of Vale and Barbosa proposed the methodology of the Gallery Walk to identify and understand the strategies used when solving problems with multiple resolutions, in particular to identify the use of visual strategies; analyse/discuss peer strategies; and characterize the reaction to the gallery walk as a learning and teaching strategy.

Papers of Vale and Barbosa and Vale, Barbosa and Cabrita propitiated the discussion between teachers, researches and inspectors about the difficulties related to national institutional constrains: time, curriculum and context (inside and outside school). Nevertheless, in order to surpass those difficulties teachers and students should become active learners with a new key role for inspectors and teacher educators.

The four papers stick out that becoming active learners, through the acquisition of new attitudes in problem posing and solving in situations of applications and modelling inside and outside the school, surely encouraged the action and development of 4Cs skills (communication including reasoning and argumentation, collaboration, creative and critical thinking). Even though the activity of problem posing and solving promotes students' knowledge construction and understanding (Vale and Barbosa; Vale, Barbosa and Cabrita), positive attitudes towards mathematics (Nazzal and Sodqi) as well as the development of meta-cognitive skills (Panero and Brunero), fostering critical thinking is not enough for providing students with both cognitive and meta-cognitive knowledge.

Finally, the paper of Serradó, Romero and Prieto aimed to mathematize the beliefs, thoughts and decision-making of Secondary School students when involved in solving seven ill-defined problems to understand the consequences of climate change. The authors discussed about the methodological issues concerning how to transform these seven ill-defined problems into an authentic task for sustainable mathematics education. Such methodologies consider the collaborative pedagogies during the active process of solving the problems and the individuality concerning the declarative, procedural and conditional meta-cognitive knowledge used by the individual students.

A common approach was taken during the presentation of the five papers and the successive discussions, which led us to claim that "didactics of mathematics should be a social opportunity for all living together". Despite this integrative approach to the development of mathematical competence and attitudes, to the warrant of knowledge and to an extensive view of the methodological opportunities of problem solving, applications and modelling inside and outside of school, further research is still needed to understand the complexity of undertaking collaborative pedagogies for sustainable mathematics education.

Argumentation in mathematics for learning to live together - Argumentation en mathématiques pour apprendre à vivre ensemble

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Abstract. Through argumentation in mathematics, students can learn to live together. If students are constantly stimulated to verbalise what they did, to explain how they reasoned and why, mathematics can largely contribute to develop the capability to communicate and discuss, to argue in a correct way, to understand other people's points of view and arguments. This paper presents a didactic method to foster argumentation, based on formative assessment strategies and feedback provided at different levels. It has been implemented all along the current school year in eleven grade 5 classrooms in Turin (Italy). The study is part of a wider project aimed at designing and experimenting formative uses of national standardized tests (INVALSI) in mathematics. Our preliminary results show effects on the evolution of students' argumentations and on students' meta-cognitive skills, developed through peer assessment and interactive constitution of what counts as an acceptable argumentation.

Résumé. Grâce à l'argumentation en mathématiques, les élèves peuvent apprendre à vivre ensemble. Si les élèves sont constamment encouragés à verbaliser ce qu'ils ont fait, à expliquer comment ils raisonnent et pourquoi, les mathématiques peuvent largement contribuer à développer la capacité de communiquer et de discuter, de raisonner correctement, de comprendre les points de vue et les arguments des autres. Cet article présente une méthode didactique pour favoriser l'argumentation, fondée sur des stratégies d'évaluation formative et des rétroactions fournies à différents niveaux. Elle a été mise en œuvre dans onze classes en dernière année de l'école primaire à Turin (Italie). L'étude fait partie d'un projet plus large visant à concevoir et expérimenter des utilisations formatives des tests standardisés nationaux (INVALSI) en mathématiques. Nos résultats préliminaires montrent des effets sur l'évolution des argumentations des élèves et sur leurs compétences métacognitives, développées par l'évaluation entre pairs et la constitution interactive de ce qui compte comme une argumentation acceptable.

Introduction

The development of argumentative competencies is a transversal learning goal to be pursued since the early years of education. Mathematical activity can largely contribute to this goal, if students are constantly stimulated to explain what they did and why they did so. In this way, the student can verbalise the reasoning that led her to solve a problem, for example, and benefit from this individual activity helping her clarify and organise her thinking - a sort of Vygotskian inner speech. Moreover, once uttered, an argumentation is no longer an inner product, but it is socialised and serves as an act of communication. Through argumentation in mathematics, students can learn to live together. In

this perspective, the Italian National Guidelines (MIUR, 2012), since the primary school, stress that mathematics "contributes to develop the capability to communicate and discuss, to argue in a correct way, to understand other people's points of view and arguments". At the end of primary education, a student should be able to "construct reasoning formulating hypothesis, supporting her own ideas and comparing her point of view with the others". Therefore, argumentation is a competence to be developed, a learning goal. Furthermore, as Morselli (2014) underlines, if the teacher is able not only to grasp the occasions to discuss about "how" and "why", but also to analyse students' explanation, making emerge what is implicit for them, argumentation can become an efficient didactic method focused on the construction of mathematical meanings. The curriculum points out a fruitful context in which the teacher can create occasions for argumentation: problem solving activities. Indeed, among the competencies to be developed by the end of the primary school, the student should be able to "solve simple problems in every content domain, by managing both the resolution process and the results" and to "describe the developed process and recognise solution strategies which are different from her own strategy". Hence, argumentation is intertwined with problem solving: to gain insight into the resolution of a problem we need to have information on the activated processes and strategies ("explain how") and on the justification of the undertaken choices as well ("explain why") (Di Martino, 2017).
The methods and the example discussed in this paper attempt to foster argumentation within problem solving activities throughout a sequence of interventions in eleven grade 5 classrooms in Turin (Italy). The study is part of a wider project aimed at designing and experimenting formative uses of Italian national standardized tests (INVALSI)[1] in mathematics. This means that questions taken from these tests or their results are used to elicit evidence about student understanding, which then has to be "interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black & Wiliam, 2009, p.9).

Theoretical elements

The wider project in which this study is framed is driven by formative assessment principles (Wiliam & Thompson 2007, Black & Wiliam 2009). Key strategies of assessment *for* learning, centred on feedback, are implemented all along the school year, namely:

- Clarifying and sharing learning intentions and criteria for success;
- Engineering effective classroom discussions that elicit evidence of student understanding;
- Providing feedback that moves learners forward;
- Activating students as instructional resources for one another;
- Activating students as the owners of their own learning.

These strategies, adopted in our case mainly by the researchers playing the role of teachers, allow the different actors of the process (the teacher, the individual students and their peers) to understand where the students are in their learning, how far they are from the learning goals and how to help them bridge the (possible) gap (Wiliam & Thompson, 2007). As stressed before, feedback plays a central role and its effectiveness depends on the level at which it is provided: product level (right or wrong), process level, self-regulation level, and personal level (Hattie & Timperley, 2007).
Strategies involving sharing, discussion and peer assessment are important to create the good conditions for living together, and argumentation is at the core of such strategies. An argumentation is a discourse that, drawing on data leads to a conclusion (a claim). What bridges data and conclusion is a warrant (an explanation), which in turn relies on a backing (Toulmin, 1975). In the classroom dynamics, an argumentation can be seen as an interactive constitution of the participants (Krummheuer, 1995), composed by several statements situated, not predetermined but negotiated by the participants as they interact. Similarly, what counts as an acceptable mathematical

[1] In Italy, students have to take a national standardized test in mathematics, administered by INVALSI (National Institute for the Evaluation of the School System), in grade 2, 5, 8, 10.

explanation and justification can be interactively constituted as a "sociomathematical norm" (Yackel & Cobb, 1996) which is elaborated as taken-as-shared basis for communication through classroom discussions guided by the teacher. To assure a good communication with the teacher and with their peers, students have to learn to formulate their argumentations in a *correct*, *clear* and *complete* way (called the 3Cs, in the following). An argumentative discourse is: correct, when there are no mathematical errors in the conclusion and in the warrant; clear, if it is understandable for the teacher and the peers; complete, if all the steps in the reasoning leading from the data to the conclusion are explicit (Cusi et al., 2017). Nevertheless, in the light of formative assessment, the 3Cs criteria for successfully producing an argumentation can be better internalised if students construct such criteria by themselves and interactively constituted them with the teachers, as sociomathematical norms of the classroom. In the methodology section, we will show how the formative assessment strategies and the levels of feedback are combined to build a didactic method centred on argumentation.

Research methodology

The main project involved 254 students and 11 teachers in three different schools in Turin. We, as researchers and authors of this paper, followed the students all along the school year 2017/2018. This means that we met the teachers and the students as regularly as possible, about once every 5-6 weeks. Classroom teachers and researchers met prior to each classroom intervention in order to decide the theme and the modalities of work (e.g. work in groups, in pairs, collective discussion). Then, the materials were prepared and the intervention was conducted by the researchers. Every lesson lasted averagely two hours and a half with this structure: feedback about the previous meeting; mathematics laboratory activity[2] on a specific mathematical theme; three individual questions-problems taken from 5th grade INVALSI tests, related to the theme of the day, and re-adapted as explained here below.

During classroom discussion, students were always asked to justify what they were claiming. The same happened in the individual written test: questions, which were generally proposed in an open form, were always accompanied by the request "Explain your reasoning" or "Explain why your answer is correct". A few questions, however, were proposed in the original multiple-choice form, but followed by the request: "Justify your choice". Such requests are additional with respect to the original INVALSI test, that for different reasons (e.g. time constraints, objective assessment criteria, …), does rarely ask for an explanation of the given answer. Anyway, this possibility is generally left to teachers in the classroom, along with questions or problems to initiate interesting discussions. Grasping this opportunity for argumentation, in this project, researchers collected data focusing both on the answer (product) and on the explanation (process). In parallel with Toulmin's model of argumentation, the answer is the claim, while the explanation is the warrant allowing the student to derive that claim from the given data.

Students' written productions were collected and analysed by the researchers, engaging also the classroom teachers. In this way, together we elicited and interpreted evidence of student achievement about the mathematical content and competencies at stake. Afterwards, to exploit such data, the researchers engineered a discussion about students' answers at the beginning of the following meeting. In doing so, feedback was provided to students at different levels which evolved over time.

Didactic method

In the first meeting, students got two individual coloured strips: they received one red point for each correct answer (product) and one blue point for each valid explanation (process). The classroom strips were constructed as the sum of all students' points. This was a feedback about the task.

[2] It is not to be intended as a physical place out of the classroom, but rather as a structured set of activities aimed at constructing the meaning of the involved mathematical objects.

In the second meeting, students analysed in pairs some argumentations selected by the researchers, after the analysis carried out with the teachers. Students were supposed to detect criteria for success, namely the three Cs: correctness of both the answer and the explanation, clarity and completeness. This feedback was both about the process and at the self-regulation level, aiming at helping students identify key-questions they have to consider while writing an explanation (e.g. *Have I written all the steps that I made?*).

Always in a perspective of self-regulation, in the third meeting, the researchers introduced half points for argumentations that were clear and correct but not complete, or clear and complete but not completely correct, or complete and correct but not that clear. Thus, students working in pairs became evaluators and tried to assign points (0, ½ or 1) to some argumentations appositely selected by the researchers.

In the fourth and following meetings, researchers created homogeneous pairs or groups of students according to their individual tests so that their answers and explanations were somehow "complementary". For example, if student A answered correctly only to questions 1 and 2, and gave valid explanation only to question 1, she was paired with student B who answered correctly only to question 3 and gave valid explanation to questions 2 and 3. Indeed, the correct answers and valid explanations were not revealed to students, but they had to find them out by comparing what they wrote and providing additional oral argumentation. They finally had to agree on the assigned points to their respective productions and received their individual strips for a comparison. In this case, the feedback was given by peers and after by the teacher at the level of both the process and the product.

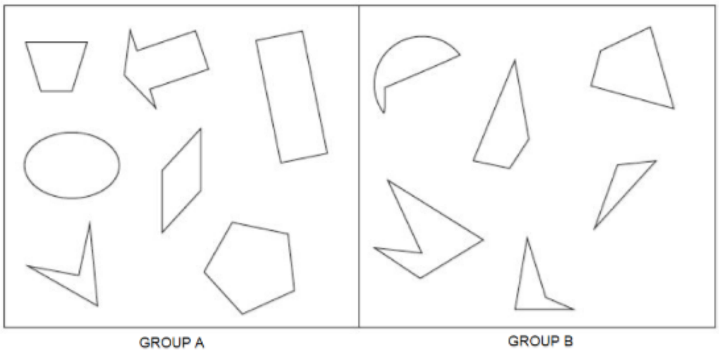
Analysis of an example

In this section, we present a short excerpt taken from the beginning of the fourth meeting in one of the classrooms. At the end of the previous meeting, students have individually answered three questions about the geometrical properties of plane figures, which was the topic of the lesson. Now, working in groups of three, students have to compare their answers and explanations and to evaluate them. The three Cs have already been established as criteria for success by the students themselves, drawing on some examples in the previous meetings. Now, every student has to assign herself points, comparing her answers with those written by the other components of her group. The researchers specify the modalities of work: "While comparing your answers, if you find that some answers are different, maybe one of them is right. So, try to discuss and understand which answers among yours is the right answer. If it's wrong, you can correct it with the red pen".

Guido, Elena and Luca work together. To one specific question about symmetry (Fig. 1), all of them have correctly answered B and correctly drew the axes of symmetry for figures in group A. Nevertheless, Luca's explanation is not that clear: "Mario put those figures in group A because he made sure that he could divided them in the right way". Instead, Guido and Elena have written good explanations (having all the 3Cs): "Figures of group A may have an axis of symmetry while those of group B cannot have it" (Guido); "For me Mario put in group A symmetric figures because if you divide them by half with the symmetric line the figures of group A you see that are symmetric while those of group B aren't" (Elena).

Discussing with Guido and Elena, Luca decides to assign ½ points to his explanation and shares it in the collective discussion which follows: "I think that, well, verifying each other's explanation, I think that I didn't express it so well". In Luca's utterance, we can recognise a deeper awareness of what he did and, mostly, how he did it. Through this action of self-assessment, he takes the responsibility for his learning (*activating students as the owners of their own learning*). This occurs thanks to the comparison and discussion with his peers or, by using Luca's words, "verifying each other's explanation" (*activating students as instructional resources for one another*).

Mario divided the following figures in two groups using a criterion.



Which criterion did he use?
A. Mario put polygons in group A and non-polygons in group B
B. Mario put symmetric figures in group A and non-symmetric figures in group B
C. Mario put quadrilaterals in group A and non-quadrilaterals in group B

Justify your choice (you can also write on the figure): ………………...

Fig. 1: Adapted from Q6 of the grade 5 INVALSI test administered in 2015.

This brief example shows effects on the students' meta-cognitive skills, developed through peer assessment and interactive constitution of what counts as an acceptable argumentation. Following Zan (2002), we refer to meta-cognitive skills as the management of cognitive resources: "This management consists of two moments: the awareness of one's own resources; the regulation of one's own behaviour according to such resources (that is the activation of control processes)" (ibid., p. 36).

Conclusion

In this paper, we presented, and illustrated with a brief example, a didactic method grounded on argumentation and formative assessment strategies that allowed children to work together and learn to live together. Students built criteria to successfully express their ideas within the classroom community, learnt to listen to other people's argumentations and, as Luca did, were even ready to modify their explanations to better communicate with the others.

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The Effect of Exploring Applications on Pure Mathematics Material on Mathematics Students

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Abstract. This study was conducted to examine the effects of research projects on applications of pure mathematics courses on students' attitudes towards pure mathematics, variables considered are self-confidence, value, enjoyment, motivation, understanding, and knowledge and skills. A pretest-posttest with control group quasi-experimental design was used in the study. The questionnaire used in this study consists of a modified version of Martha Tapia Attitude toward Mathematics scale which measures the studied six variables. The sample consists of two groups of participant, a total of 30 junior and senior students. The experimental group consists of 15 students and the control group consists of 15 students. Results using ANCOVA revealed that exploring applications of the pure material has statistically significant positive effects on attitudes towards pure mathematics in all studied variables except on enjoyment. This research indicates that incorporating applications of the pure mathematics could improve mathematics education in Palestinian universities.

Résumé : Cette étude vise à examiner les effets des projets de recherche sur les applications des cours de mathématiques pures aux attitudes des élèves à l'égard des mathématiques pures; les variables considérées sont la confiance en soi, la valeur, le plaisir, la motivation, la compréhension ainsi que les connaissances et les compétences. Un pré-test post-test avec une conception quasi-expérimentale du groupe de contrôle a été utilisé dans l'étude. Le questionnaire utilisé dans cette étude consiste en une version modifiée de l'échelle de l'attitude envers les mathématiques de Martha Tapia, qui mesure les six variables étudiées. L'échantillon comprend deux groupes de participants, soit 30 étudiants juniors et seniors. Le groupe expérimental comprend 15 étudiants et le groupe témoin, 15 étudiants. Les résultats obtenus avec ANCOVA ont révélé que l'exploration d'applications du matériau pur avait des effets positifs statistiquement significatifs sur les attitudes à l'égard des mathématiques pures pour toutes les variables étudiées, à l'exception du plaisir. Cette recherche indique que l'intégration d'applications des mathématiques pures pourrait améliorer l'enseignement des mathématiques dans les universités palestiniennes.

Introduction. Most of our teaching of pure mathematics courses has been teaching pure mathematics for its own sake. Unfortunately, this attitude isolates pure mathematics from the real world and introduces pure mathematics as a boring complicated material which has no feasible use. It is natural then for students to ask: "Why should we learn pure mathematics?" I have been particularly influenced by this question. Trying to figure out reasonable answers to my students' question, I realized that we should also teach mathematics so that students learn to value the importance of pure mathematics, one way is to let them see mathematics in their lives and as an inevitable tool in the development of other sciences and technology. I'll let students explore their passion for pure mathematics through projects involving applications on the course material. So, each student is going to study one application on the material and then, give a talk discussing her/his project. At this point, students will also learn some basic research skills and they will also acquire skills on how to deliver effective presentation. The purpose of the study is to determine the effects of research projects on applications of pure mathematics courses on students' attitudes towards pure mathematics, the considered variables are self-confidence, value, enjoyment, motivation, understanding, and knowledge and skills.

Methodology. This part presents study's design, data collection, the instrument of the study, sample and population, and test of validity & reliability. Research design of this study is quasi-experimental with plan of pretest-posttest and a control group. Statistical population includes all mathematics students taking number theory course at Palestine technical university (PTUK) in Palestine, spring semester 2017-2018. The place is chosen due to its accessibility to the researchers. Besides, due to similarity in the curriculum of pure mathematics courses in all Palestinian universities, this sample of (PTUK) students is representative. Two groups of participant, 30 junior and senior students, are randomly assigned to experimental and control groups responded to the questionnaire including six types of attitudes towards pure mathematics.
The scale of attitudes towards pure mathematics was applied before and after the intervention of the study. The mean and standard deviations were calculated and their results showed the absence of statistically significant differences between the groups at level of significance (α=0.05) which indicates the two groups are equivalent.
Scale of attitudes towards pure mathematics was used in this study as an instrument to measure research variables. This scale includes six types of attitudes towards pure mathematics are considered: self-confidence, value, enjoyment, motivation, understanding, and knowledge and skills. The scale contains thirty six likert type statements. For each statement, a five levels-scale is assigned, starting with a score of one up to five; where 1= strongly disagree, 2=disagree, 3=neutral, 4=agree, and 5=strongly agree. There is no instrument to measure attitudes towards pure mathematics versus applied mathematics. However, the researchers modified Tapia & Marsh (2004) instrument to build the scale. To check reliability of this scale, it was applied on an exploratory sample out of the population of the study. The internal consistency was calculated using Chronbach Alpha, this was obtained to be 84.1. To check validity, it was presented to a group of arbitrators made up of university professors, whose views were taken into account. Scale of attitudes towards pure mathematics contains 36 items. The distribution of questions in the questionnaire according to their orientations type is as follows: Self-confidence: 1-10, Value: 11-17, Enjoyment: 18-22, Motivation: 23-26, Understanding: 27-30, and Knowledge and skills: 26-30

Study Hypotheses and Results. This research examines seven hypotheses. In this section, we state each hypothesis and check its validity depending on the collected data. First, we will investigate the general attitude of mathematics undergraduates towards pure mathematics.
i) Attitude towards Pure Mathematics. One definition of attitude is students´ general emotionally toned disposition towards mathematics Haladyna et. al (1983). Aiken (1974) defines attitude as a learned predisposition, or a tendency to respond negatively or positively to mathematics. The first hypothesis states that there is no significant difference at (α=0.05) between the mean response of Mathematics students at scale of orientation towards pure mathematics which could be attributed to the study's intervention. To examine the hypothesis, the difference between pretest and posttest response is calculated and analysis of covariance (ANCOVA) is used.

Source of variation	Sum of squares	Degrees of freedom	Mean of squares	F	P-VALUE	Partial eta squared
pretest	2.226	1	2.226	172.67	*0.0001	
Group	2.805	1	2.805	213.68	*0.0001	0.89
Error	0.354	27	0.013			

Table 1.

Observe that the F value given in Table 1 is 213.68, which shows that there is significant main effect of exploring applications on mean response of students at scale of orientation towards pure mathematics at (α=0.05). When the eta squared value is investigated, the intervention explains for the 89% variation in the post test response in the scale of orientation towards pure mathematics, independent of the pre-test scores. This shows that the experimental group benefited from the intervention on the overall attitudes towards pure mathematics. The studies conducted by Tapia (1996), Karjanto (2017), National Research Council (1979), Blum (2002), and Hannula (2002) show that modeling affected attitudes towards pure mathematics. They also indicated that attitude affects achievement.

Now, we examine each of the six orientation types separately[3].

ii. Self-confidence. Perceived self-efficacy is defined as 'it is not a measure of the skills one has, but a belief about what one can do under different sets of conditions with whatever skills one possesses' Bandura (1997). It is clear that *self-confidence* and *self-efficacy* are broadly similar. To decide if the differences between the two means (for the control and experimental groups) are significant at the level (α = 0.05), ANCOVA is used, the computed F value is 35.43, indicating statistically significant difference at level of significance (0.0001) between means of the two groups' performance in the scale of students' performance dealing with self-confidence, and so the null hypothesis is rejected. The eta squared value indicates that the study's intervention explains for the 56.8 % variation in the post test response in the scale of self-confidence, independent of the pre-test scores. Parsons et al. (2009) found out that there is a statistically significant relationship between self-confidence and students' achievement in engineering mathematics at university. Literature shows that students who have low levels of mathematics self-efficacy are at a high risk of underperforming in mathematics, despite their abilities Bandura (1997) and Schunk and Pajares (2009). The study conducted by Durant and Jacobs (2014) shows that mathematical modeling positively affected mathematics students' self-confidence.

iii. Value. Bishop (1999) defines value as the deep affective qualities which education fosters through the school subjects of mathematics. To examine the third hypothesis which states that there is no significant difference at (α = 0.05) between mean response of mathematics students scale orientation-value which could be attributed by the intervention, ANCOVA is used. The F value is calculated to be 62.74, indicating statistically significant difference at level of significance (0.0001) between means of the two groups performance in the scale of value of pure mathematics, and so the null hypothesis is rejected. The eta squared value indicates that the study intervention explains for the 69.9 % variation in the post test response in the scale of value, independent of the pre-test scores. Sullivan and McDonough (2007) showed that students consider mathematics as a

[3] Tables (like table 1) are obtained using SPSS for the analysis of covariance ANCOVA. However, due to the limited size of the paper in this act, we will not include them. For the same reason we will not include any of the questions in the questionnaire. Also, we restricted the theoretical frame to the literature included in the analysis.

worthwhile subject when they see that mathematics helps with problem solving in other areas and that mathematics is important in everyday life. Papageorgiou (2009) found that there are no significant differences between the groups which adopted modeling in education and the control group. However, there is evidence supporting the assumption that such activities can have positive impact on various aspects of the teaching and learning in the mathematics classroom. In general, students value something more when they are able to make connections with it Elliott, et al. (2001) and Witten (2005).

iv) **Enjoyment.** Enjoyment is defined as a positive activating emotion that can affect whether students will engage and reengage with the enjoyable content Goetz, et.al (2008). Another definition of enjoyment was found to be related to effort and performance Schukajlow and Krug (2014)**.** The forth hypothesis states that there is no significant difference at ($\alpha = 0.05$) between mean response of mathematics students scale orientation-enjoyment which could be attributed to the intervention. The F value is 1.347 which means there is no significant difference on students' enjoyment due to the intervention. Statistics shows an increase in the number of students who responded favorably to some of the questions. However, the proportion of increase is not statistically significant to show evidence of a change in their overall enjoyment. Wallace (2000) found out that by doing mathematics in interdisciplinary projects, students begin to believe that mathematics is useful, important and even interesting. This increased interest may be more important than their perceived math ability in determining whether they study more mathematics. Dickinson and Butt (1989) suggest that students tend to enjoy a task more when they have a level of success in it.

v) **Motivation.** Motivation may be defined as the drive behind an individual's actions towards a certain situation Middleton & Spanias (1999). Hannula (2006) defines motivation as "a potential to direct behavior that is built into the system that controls emotion. This potential may be manifested in cognition, emotion, and/or behavior". Ryan and Deci (2002) define motivation as reasons that students engage in different school activities**.**

The fifth hypothesis, states that there is no significant difference at ($\alpha = 0.05$) between mean response of mathematics students scale orientation- motivation which could be attributed to the study's intervention. To determine whether these differences between the two groups are significant at the level ($\alpha = 0.05$), ANCOVA is used. The F value is 53.08, indicating statistically significant difference at level of significance (0.0001) between means of the two groups in the scale of motivation of pure mathematics and so the null hypothesis is rejected. The eta squared value is shows that the intervention explains for the 66.3 % variation in the post test response in the scale of motivation, independent of the pre test scores.

Enhancing students' motivation in the mathematics classroom is one main goal for educators since motivation is strongly related to students' behavior and achievement. Real-life situations taken from students' areas of preference have the potential of helping students engage and find their motivation for engaging in mathematics Kacerja (2012). The obtained results are also consistent with many other studies; many veteran educators recommend using real-world applications of abstract math concepts as a motivational tool, see Blum (2002) and National Research Council (1979).

vi) **Understanding.** Understanding can be thought of as an actual or potential mental experience Sierpinska (1994). Michener (1978) defines understanding mathematics as a process that can be understood and to some extent taught, a good part of the process is concerned with building and enriching a knowledge base. This includes creating associations of many kinds as well as items. It also involves differentiating between various kinds of items according to their function in acquiring knowledge, familiarity, and expertise. To examine the sixth hypothesis which states that there is no significant difference at ($\alpha = 0.05$) between mean response of mathematics students scale orientation-understanding which could be attributed to the study's intervention. The F value is calculated to be 45.37, indicating statistically significant difference at level of significance (0.0001) between means of the two groups' performance in the scale of students' response in the scale of understanding of pure mathematics and so the null hypothesis is rejected. The eta squared value indicates that the study’s intervention explains for the 64% variation in the post test response in the scale of understanding pure mathematics, independent of the pre test scores. This had also been long observed through several studies on the effect of modeling on students’ attitudes and performance; for example, National Research Council (1979) assures that if mathematics is taught to stress understanding, the subjects should be treated to emphasize the relations to scientific problems. Immersing students in situations which can be related to their own direct experience allows students to deepen and broaden their understanding of the scope, Nourallah and Farzad (2012). Jeffes, et.al (2013) found out that students are beginning to acquire a deeper understanding of mathematics and how it can be applied.

vii) Knowledge and Skills. Wagner (2008) defines skills and knowledge as the abilities and techniques that prepare students to be successful outside of an educational setting. The seventh and last hypothesis states that there is no significant difference at ($\alpha = 0.05$) between mean response of mathematics students scale orientation-knowledge and skills which could be attributed to the study’s intervention. ANCOVA is used; we found the F value to be 205.2 indicating statistically significant difference at level of significance (0.0001) between means of the two groups’ performance in the scale of students’ response in the scale of understanding of pure mathematics and so the null hypothesis is rejected. When the eta squared value is investigated, the intervention explains for the 88.4% variation in the post test response in the scale of understanding pure mathematics, independent of the pre test scores. Blum (2002) emphasized that to place the students in an environment in which their mathematical skills and point of view can be applied, experience in the application of mathematics, combined with the interaction with workers in other disciplines, will greatly strengthen the students' education. Özdener and Özçoban (2004) found out that research projects are concurred with an increase in student’s skills both personally and as collaborative groups. Newell (2003) claims that using project, helps students develop better comprehension, problem solving skills, and teaches students to learn. Prahmana (2017) indicated that using research enhance students’ research and academic writing skills.

4. Conclusion. Exploring the applications of the pure material has positive significant effect on attitudes towards pure mathematics on six out of seven scales. This study suggests incorporating applications in the curriculum of pure mathematics courses.

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Mathematics outside the classroom: examples with pre-service teachers

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Abstract : The classroom is only one of the "homes" where education takes place. The use of non-formal teaching contexts, such as the surrounding environment, constitutes an educational context that can promote positive attitudes among students and an additional motivation for the study of mathematics. Teaching should be enriched with challenging tasks, aimed at developing cognitive abilities, such as problem posing and solving, and also encourage creative thinking. Thus, arise the trails, which consist of a sequence of tasks that the students have to solve, along a preplanned route. In this process, teacher education has a fundamental role, providing (future) teachers with the same experiences they are expected to offer their own students. The trails have great potential for all the students who experience them. Thus, we will discuss some of that potential developed in the context of pre-service teacher training.

Résumé : La classe n'est qu'un des "foyers" où l'éducation a lieu. L'utilisation de contextes d'enseignement non formels, tels que l'environnement environnant, constitue un contexte éducatif susceptible de promouvoir des attitudes positives chez les étudiants et une motivation supplémentaire pour l'étude des mathématiques. L'enseignement devrait être enrichi de tâches stimulantes visant à développer les capacités cognitives, telles que la résolution de problèmes, et à encourager la pensée créatrice. Ainsi, surgissent les sentiers, qui consistent en une séquence de tâches que les élèves doivent résoudre, le long d'un itinéraire planifié. Dans ce processus, la formation des enseignants joue un rôle fondamental: elle offre aux futurs enseignants les mêmes expériences qu'ils sont censés offrir à leurs propres étudiants. Les sentiers offrent un grand potentiel pour tous les élèves qui les vivent. Nous aborderons donc une partie de ce potentiel développé dans le cadre de la formation initiale des enseignants.

Introduction

A large part of the mathematical failures has their origin in the affective environment that is created and that can compromise students' initial expectations and motivations (e.g. Hannula, 2004). In an attempt to reverse the situation, and since teachers play a key role in what happens in the classroom, teacher training must allows future teachers to experience new approaches that they are expected to use with their own students. Mathematical learning should include more than routine tasks - it should be enriched with challenging tasks such as problem posing and problem solving, contributing to the development of creative thinking. Within this perspective, as it contemplates all the previously mentioned features, comes up the learning outside the classroom, such as the environment surrounding schools. In this context, we privilege the mathematical trails. It is also important to state that nowadays young people have sedentary habits of life and our students spend long hours sitting inside the classroom, with all the implications it brings, in particular, at the level of attention. So, it is appropriate to give them opportunities to leave the formal classroom space, to get involved and to experiment the mathematics around them by relating it to real phenomena, aspects they can also take into the classroom to be discussed and deepened. Simultaneously, the opportunity to know the historical, architectural, cultural and natural heritage of the neighborhood where the school is located also arises. Thus, after a brief theoretical contextualization, an exploratory study is presented. It's based on a larger project in development, with future teachers of elementary education (3-12 years old), that attended a course of Didactics of Mathematics. We intended to understand the impact, knowledge and attitudes towards mathematics during a math trail outside the classroom.

Theoretical framework

Today, people are no longer rewarded only for what they know, but for what they can do with what they know. So, in order to promote an effective teaching, students should have a meaningful learning, through individual and collaborative experiences, that promote their ability to make sense of mathematical ideas. They should be engaged in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies (NCTM, 2014). Thus, since teachers are the main agents of change in students learning, it is important that they develop some type of (creative) abilities: exhibiting a deep knowledge of mathematics, presenting it as a coherent and connected enterprise; having a sound pedagogical knowledge; selecting, constructing appropriate and worthwhile tasks, since different tasks influence student's learn and provide different learning opportunities; promoting mathematical discussions; or reflecting about the teaching practice (e.g. Ball, Lubienski & Mewborn, 2001; Smith & Stein, 2011).

On the other hand, while creating, selecting and/or adapting tasks, teachers should incorporate elements related to contexts, culture and language. In fact, the involvement of students in the solution process will be more connected with their sense of identity, leading to increased engagement and motivation. Challenging tasks are valued because they arouse curiosity, require imagination and appeal to creativity, becoming interesting and pleasant to solve. But they only makes sense in an exploratory teaching, where the teacher is the orchestrator of the activity in the classroom (Smith & Stein, 2011). Therefore, special attention should be given to the training of teachers, providing experiences that allow them to acquire a deep knowledge of the mathematics to teach and how to teach it. Only this way they can establish connections between themes, highlighting the conceptual understanding and considering problem solving as a central aspect in mathematics teaching. So, it is fundamental that (future) teachers have opportunities to experience didactic situations in the same way that they will conduct them with their own students.

Tasks that focus on problem posing and problem solving may contribute to the acquisition of mathematical knowledge, but also to the development of other skills (e.g. communicating, reasoning, arguing, representing, criticizing). By learning with problem solving, students have numerous opportunities to connect mathematical ideas and develop their conceptual understanding (Barbosa & Vale, 2016). In essence, authors almost always refer to the same ideas concerning the process of creation (invention, formulation) of problems - problem posing implies generating new problems or reformulating a certain problem, based on the knowledge and mathematical experience and the personal interpretations of situations (e.g. Silver, 1997). Brown and Walter (2005) propose two problem posing strategies that students can use: *accepting the give strategy* - students start from a static situation (e.g. expression, table, image, sentence, calculation, dataset) from which they ask questions in order to have a problem; and *what-if-not strategy* - consists of extending a given task by changing what is given. On the other hand, teachers have a crucial role to play in developing students' creative potential, providing them with appropriate learning experiences, such as problem posing and solving, which is not only developed within the classroom, but can be complemented by other educational environments, such as the contexts outside the classroom (e.g. Silver, 1997; Barbosa & Vale, 2016).

Learning to solve real-life problems has proved to be a more difficult task than solving the traditional class-type problems in textbooks. Effective outside classroom learning mobilizes problem-solving, collaborative, cooperative, and interpersonal communication skills, all of which are essential skills for today's young people. Is intended to contribute to the success of students in mathematics through practices that favor the use of contexts outside the classroom and, on the other hand, helps students not to spend too much time sitting. Thus, each student should experience the world beyond the classroom as an essential part of personal learning and development, regardless of their age, ability or circumstances, experiencing meaningful learning opportunities, because the classroom should only be one of the homes where education takes place (e.g., Kenderov, Rejali, Bartolini, Bussi et al., 2009). The attitudes, conceptions, feelings that students create about mathematics, can seriously compromise their relation with this subject throughout their academic course and beyond, since those influence the whole process of teaching and learning (e.g. Hannula, 2004). Learning outside the classroom can promote positive attitudes in students and an additional motivation for the study of mathematics because it allows them to understand its applicability, but also to develop mathematical skills and knowledge associated with all subjects of the curriculum. It allows the establishment of connections between various mathematical themes and other disciplinary areas in an atmosphere of adventure and exploration. We privilege the mathematical trails, considered as a sequence of tasks along a preplanned route (with a beginning and an end), composed of a set of stations in which students solve mathematical tasks in the environment that surrounds them (Cross, 1997; Barbosa & Vale, 2016).

The study

Taking into account the aim of this study, we chose an exploratory approach of qualitative and interpretative nature. It was conducted with 60 future teachers of elementary education (3-12 years old) that attended a Didactics of Mathematics unit course, during which they had to develop a math trail outside of the classroom. The data collection uses the productions of the future teachers (tasks/trails), classroom observations, questionnaires and photo records. The questionnaires were carried out at the end of the semester to collect students' impressions about the whole experience (e.g. difficulties, positive aspects, potentialities, impact). The inductive data analysis was carried out by the two teachers of the curricular unit, according to some criteria such as diversity of the tasks and mathematical contents involved, rigor/accuracy of the mathematical contents, creativity of the proposals, and reactions of the future teachers.

To carry out the conception of the Math Trail the students experienced a sequence of steps. First, during the classes of this unit course, students were exposed to teaching modules about problem solving/posing, communication, reasoning, mathematical connections and creativity in mathematics. Then, in small groups, they went to the city surrounding the school and selected an artery in which they defined a route. After, students walked through that route and photographed elements of the local environment that would allow mathematical exploration (e.g. monuments, windows, gardens, pavements). To design the math trail, they had to pose problems inspired by the chosen elements, adequate to basic education. To refine this process, they had the opportunity to share their proposals in class, with the teachers and the peers, and get feedback. During this phase, the groups of students went back to the real context as many times as they needed. Finally, they sequenced the tasks in the form of a trail and constructed support mathematical kits to aid in solving the tasks (e.g. writing material, measuring tape, calculator, map).

Some preliminary results

As expected, the tasks formulated by the students were inspired by the elements of the local environment. This experience allowed them to have a direct contact with the surroundings of the school (the city of Viana do Castelo), using a mathematical lens.

To organize the trail, each group had to choose a number of stops along a route of their choice, collecting photographs of elements of the city. Considering that they were asked to formulate problems based on those choices and that they started from a static situation, the photographs, posing questions without changing what was given, they mainly used *Accepting the data* as a problem posing strategy (Brown & Walter, 2005). Many of them were questions focused on the knowledge of specific facts (e.g. "discover the polygons you identify in the window", “count the axis of symmetry in the tile”), or routine tasks (e.g. involving perimeters, areas, ...). The solvers have to collect the necessary data in the real context to answer the questions. Privileging exercises and problems can be explained by the lack of experience of the participants with problem posing and the choice for tasks that were most familiar to them or had more expression in Portuguese textbooks. The contents involved were mostly of geometric nature, fact that can be explained due to the visual nature of the elements involved in the trail. The design of the tasks was recognized by these future teachers as one of the main difficulties in the development of the math trail, due to the need to diversify the level of complexity and the contents involved, to maintain the interest of the solver. However, as shown in Figure 1, some groups made an effort to include tasks in their trail that approached contents from algebra, probability, number and operations and measurement. The presentation of the trails was left to the criteria of each group, which lead to quite different formats that varied from maps to flyers, complemented with support material organized in kits.

Example 1

You can find this flowerbed in the *Marginal Garden*.

How did the gardener build it? Explain the process.

Example 2

Can you find a store named *Opipaua*? Find as many geometric shapes as you can in the iron railings above the store.

Example 3

Look around and find the sign with the name of the street where you stand. Take a picture of it. If you put the letters in a bag, which one will you most likely draw? And least likely?

Example 4

Go to *Praça da República*. Once there you will find a fountain.

1. Knowing that the girl at the edge of the fountain measures 1,55m, estimate the height of the fountain.

2. How could you measure the perimeter of the fountain?

Example 5

This is the skylight of *Palacete Abrunhosa*.

1. Which geometric shapes and solids can you identify?

2. If you were to paint all the glasses of the skylight knowing that adjacent glasses needed to have different colors, what is the minimum number of colors you would need?

Figure 1. Examples of tasks formulated by the future teachers

From the questionnaires, we can see that the construction of mathematical trails allowed future teachers to perspective Mathematics in a more dynamic and motivating sense in relation to their own experiences as students, compelling them to think about mathematics in a less formal and more creative way. They also recognized the difficulty of organizing a trail, assuming the role of the teacher: formulating the tasks (correctness, clear language, diversity of the type and the contents); sequencing a balanced trail (e.g. distance, number of stops, time of exploration); ellecting more natural themes (e.g. figures, area, perimeter, patterns). However, they also valued the potential of this type of work to promote a positive image of mathematics, highlighting the opportunity to experience its applicability.

Some of these participants had the opportunity to conduct the experience of a math trail with elementary school students in the school surroundings. Figure 2 illustrates an example of a task created by the future teachers, (e.g. Barbosa & Vale, 2015) and its' implementation. The engagement and excitement of the students was noticeable. They were able to give meaning to mathematical concepts, applying them to real situations, in a context that was familiar to them, work collaboratively when solving problems and making decisions.

The school rubbish bins are quite degraded, and the school board decided to paint all the red iron in new blue.

1.Calculate an approximate value of the area of iron to be painted.

2. We have three cans of blue paint enough to paint 3 square meters of iron and the school has twelve rubbish bins

Figure 2. Example of a task and students doing a trail

These experiences were also significative to the pre-service teachers involved, that confirmed the effectiveness and the potential of this approach.

Final considerations

Organizing and executing the math trails helped our future teachers to have a more positive attitude towards mathematics and gain a broader view of the possible connections that can be established between mathematics and the world around us. The trails created, inside the school spaces or in the city surroundings, allowed them to analyze it through a "mathematical eye", but also to know a little more about its history and architecture (e.g. Barbosa, Vale & Tomás-Ferreira, 2015; Barbosa & Vale, 2016). The design of the tasks was not an easy process, namely from the point of view of the mathematical knowledge involved, the degree of challenge and requirement, as well as the diversity of the nature of the tasks. The experience has shown potentialities for future teachers, in particular, related to problem posing. They reflect on different types of mathematical tasks, and develop their creative skills, both when posing and solving the tasks. It contributed to promote a positive attitude towards mathematics and see its applicability, as well as improve their competences as future teachers. The participants understood the importance of having a deep knowledge of mathematics, of having a sound pedagogical knowledge, of selecting appropriate and worthwhile tasks, and reflecting about the choices made and its' implications (e.g. Ball, Lubienski & Mewborn, 2001; Smith & Stein, 2011).

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Mathematizing the consequences of climate change: beliefs, metacognition and decision-making

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Abstract : Transforming mathematic curriculum into a mathematic sustainable one is one of the current challenges of education. This transformative approach aims to question what activities should be put in place to enable the teaching of mathematics to be a sustainable teaching. In this regard, this paper presents an "authentic" task that aims to enhance forty-eight Spanish Secondary School students (ages 12 and 13) in mathematizing the consequences of climate change. The task is composed by seven ill-defined complex problems connected by open-ended self-reflective metacognitive questions. We have analysed the declarative, procedural and conditional self-reflective metacognitive knowledge that students had used to justify beliefs, thoughts and decisions when answering these questions. Moreover, we have analysed if involving students in answering these kind of questions helps to transform the seven ill-defined complex problems into an authentic task for sustainable mathematics education.

Résumé : Transformer le programme d'études mathématiques dans un programme mathématique durable est un des défis actuels de l'éducation. Cette approche transformative vise à s'interroger sur les activités à mettre en place pour que l'enseignement des mathématiques soit un enseignement durable. À cet égard, cet article présente une tâche « authentique » qui vise à améliorer quarante-huit élèves (âges 12 et 13) des écoles secondaires espagnoles en mathématisant les conséquences du changement climatique. La tâche est composée de sept problèmes complexes mal définis reliés par des questions métacognitives autoréflexives ouvertes. Nous avons analysé les connaissances métacognitives autoréflexives déclaratives, procédurales et conditionnelles que les élèves avaient utilisées pour justifier leurs croyances, leurs pensées et leurs décisions lorsqu'ils ont répondu à ces questions. De plus, nous avons analysé si impliquer les élèves dans la réponse à ce genre de questions aide à transformer les sept problèmes complexes mal définis en une tâche authentique pour l'éducation durable aux mathématiques.

Introduction

Climate change is one of the greatest challenges of our time and its adverse impacts determine the ability of all countries to achieve sustainable development. Increases in global temperature, sea level rise, ocean acidification and other climate change impacts are seriously affecting coastal areas and low-lying coastal countries (United Nations, 2015). United Nations for 2030 aims to reduce the adverse impacts of climate change, challenging national educational systems to ensure that all learners acquire the knowledge and skills to promote sustainable development and increase awareness-raising and human and institutional capacity on climate change mitigation, adaptation, impact reduction and early warning.

Aligned with these principles, the CIEAEM70 Discussion Document proposes to question about: what activities should be put in place to enable the teaching of mathematics to be sustainable teaching? In this regard, we present a task that aims grade 7 (ages 12 and 13) Secondary School Students to answer the question: *It is possible that the Trocadero Island disappears due to climate change?* (Romero, Serradó and Prieto, 2017).

In this paper, we analyse if the didactic proposal of challenging students' development of metacognitive knowledge about their beliefs and thoughts on how the consequences of climate change could affect the Trocadero Island help them to develop awareness-raising on climate change mitigation. Furthermore, we retrospectively analyse if this didactical proposal helps to transform an everyday problem contextualized in the Trocadero Island into an "authentic" task for sustainable mathematics education.

A mathematic sustainable curriculum for decision-making about climate change

Sustainable mathematics education is the project of reorienting mathematics towards environmentally-conscious thinking and sustainable practices (Renert, 2011). For Renert (2011), these sustainable practices emerge from an epistemological reorientation of the traditional deterministic approach of data analysis to a stochastic one. This stochastic epistemological approach should help to understand the dialogue between the local and global. The relations stablished between the local and global when analysing climate change aims to avoid the certainties that are locally true to feeling the connections and possibilities that the analysis of large series of numbers would provide to develop an environmentally-conscious thinking about the global.

To ensure this epistemological approach, we have designed a task, contextualized in the city where students live, Puerto Real (Spain). The task wants to answer the question: *"Is it possible that the Trocadero Island disappears due to climate change?"*. The question was reformulated to surpass its impersonal nature and capture the initial opinions and beliefs of students. This first question was: *"which do you think is the probability that the Trocadero Island disappears?"* (problem a). The colloquial use of different levels of possibility of occurrence could be consequence of the students' opinions (beliefs) about the consequences of climate change. The beliefs about the global effect of climate change would emerge in Science and Geography subject. The knowledge acquired in this subjects, Science and Geography would provide students with a social and scientific understanding of the local situation. It also could be a first approach to a conscious understanding of the uncertain and random nature of the causes and consequences of climate change, scientifically related with the "Theory of Chaos".

The transformation of these opinions in an environmentally-conscious thinking was designed through a horizontal and vertical mathematization process (Treffers, 1987). The horizontal mathematization process engaged students in conjecturing, experimenting, inductively reasoning and organizing the information obtained with the aim of modelling or applying the mathematic knowledge needed to answer the questions: which do you think is the predictable setback of the beaches in Spain? (problem b), where should a coast guard live to be equally distant of the three coasts with higher probability of disappearing? (problem c), which is the surface of the Trocadero Island? (problem e), which should be the length of a dam to keep the Island undamaged (problem f). The vertical mathematization process consisted in the resolution of these questions, its generalization and formalization. Students constructed and used mathematic knowledge: statistic data analysis, synthetic geometry of the loci of the triangle (barycentre (centroid), incentre, circumcentre and orthocentre), shapes, perimeters and surfaces of polygons as an approximation to the fractal geometry of the Trocadero Island.

Two extra questions were posed: could the island be portioned or even flooded due to climate change (problem d) and do you think that this dam could help to save from disappearing the Island? (problem g). These questions would help students to integrate numeric, stochastic and geometric knowledge to cognitively enhance students in answering the main question (*"Is it possible that the Trocadero Island disappears due to climate change?"*).

Wrongly, this transformative approach to sustainable mathematics education could be conceived as an agenda for improving students' exclusive acquisition of knowledge through the action of problem solving. Nevertheless, the United Nations (2005) agenda for sustainable development education aims to ensure that all learners acquire knowledge and skills, such as: critical thinking, collaboration and cooperation, decision-making and problem solving. In this agenda, decision-making and problem solving are considered disconnected skills. Despite this initial disconnection, there exists the possibility of embracing the theoretical approach of "problem solving-in-action" (Schoenfeld, 2006), a function of the complex interaction of students and teachers' goals, beliefs, knowledge and decision-making procedures.

The collaboration and cooperation between students with the mediation of the teachers could help to develop students' cognitive knowledge to achieve the goals of a sustainable mathematics education. Nevertheless, the quality of decision-making that students could perform is much more a function of the individual's metacognitive skills, beliefs and affect (Schoenfeld, 2006; 2015).

The extension of the role of education to the development of individual's metacognitive skills is not a neutral option. This extension needs of a theoretical framework that analyse the relevance and efficacy of metacognition as theory-based instructional principle. Baten, Praet and Desoete (2017) propose a theoretical framework related with the development of three kind of metacognitive knowledge: declarative, procedural and conditional (or strategic). Declarative metacognitive knowledge can be defined as the "what" knowledge or the knowledge of the strengths and weaknesses of one's own processing ability as a learner and the knowledge about cognitive strategies. Procedural metacognitive knowledge can be described as knowing "how" to successfully employ particular cognitive strategies in order to achieve learning objectives. Conditional metacognitive knowledge can be used for "when" and "why" knowledge, referring to knowledge of the appropriateness of particular cognitive strategies when taking into account external learning conditions, including awareness of the underlying reasons for cognitive strategies effectiveness. These three metacognitive knowledges could emerge and develop during the self-reflective process of deciding how to solve the realistic problem on hand.

Nevertheless, research on decision-making in the context of stochastics informs about the existence of difficulties when making decisions under uncertainty or decisions of risk (Serradó, 2018). These difficulties are related with assigning a deterministic nature to the data to be analysed. Teachers and students could face some difficulties related with relinquishing the deterministic predictability and embrace the contingency and stochastic probabilities for a sustainable mathematic education (Renert, 2011). Renert (2011) goes further and extends these requirements, for transforming the traditional mathematic education into a sustainable one, with turning mathematics from a collection of objects, or a series of competences, into an open-ended state of observing the world. Furthermore, he suggests engaging students and teachers in ethical actions for healing the world.

Methodology

We have adopted a collaborative design based research strategy as a form of professional development (Voogt et al., 2015) for designing and retrospectively analyse the "authenticity of the task" (Romero, Prieto and Serradó, 2017). The task was implemented to 48 grade 7 students (ages 12 and 13 years old from a deprived coastal town in the South of Spain. Students, divided in two classrooms (A and B), were collaboratively organized in six-collaborative teams of four or five students. Students were asked to solve collaboratively the seven ill-defined complex problems (a, b, c, d, e, f and g). After solving the problems (a), (b), (e) and (g), students answered individually four self-regulative metacognitive questions, which were adaptively designed in function of the cooperative knowledge used when solving the seven ill-defined problems. Examples of these questions are: *"What strategy have you used to determine the total area of the Island?"* or *"Have you changed your opinion over these weeks? Why?"*.

These open-ended questions were designed with the aim that students justify their beliefs, thoughts and decisions using self-reflective metacognitive knowledge (declarative, procedural and conditional). In this paper, we want to retrospectively analyse if this metacognitive knowledge presents an environmental-conscious thinking for the sustainability of the world. The retrospective analysis consisted in a case study of the answers of these self-reflective metacognitive questions. For the case study, we selected 12 students (one of each team of the classroom) and we analysed the cooperative information included in the group portfolio, the visual and verbal information video-taped during the collaborative discussion of the answers of the seven ill-defined problems and the individual written data related with the answer of the self-reflective questions.

This individual written data was codified twice to pursuit a multidimensional analysis of the beliefs, thoughts and decisions of the students. Firstly, we discriminated the kind of scientific, social or mathematic metacognitive knowledge (declarative, procedural or conditional) used by the students to justify their beliefs, thoughts and decisions when answering the self-reflective questions. The information obtained allowed to analyse the possible evolution in the use of these metacognitive knowledges during the resolution of the seven ill-defined problems. Then, a second codification took place analysing which elements of the transformative approach to mathematics sustainable education described by Renert (2011) were used by the students to justify their beliefs, opinions and thought about the consequences of climate change.

Results and discussion

Figure 1 summarises the evolution of the metacognitive knowledge used by the students to justify their beliefs, thoughts and decisions in problems (a), (b), (e) and (g). The category of what science, what math and what social refers to the students use of scientific, mathematic and social declarative knowledge, respectively. The how category corresponds to the use of procedural metacognitive knowledge, and the "when" category indicates the use of the conditional metacognitive knowledge.

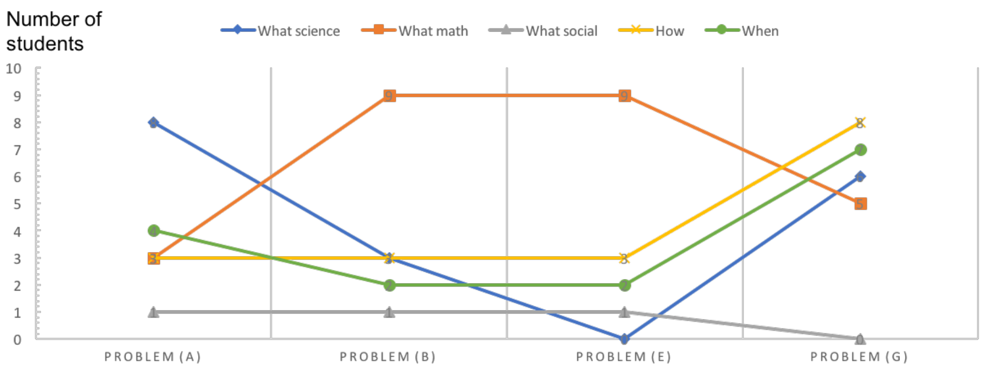

Figure 1. Evolution of the students' metacognitive knowledge

Mostly all the students (8 of 12) initially used the scientific declarative metacognitive knowledge learned in science subject and refreshed during the solving of the problem a. Students were questioned to individually reflect if the consequences of the climate change in the Pacific Island could be transferred to the Trocadero Island. The student 3 group B argues: *"Yes, because, in the future, the sea level rise will affect the Trocadero Island"*. This scientific declarative metacognitive knowledge strengths the student processing ability to transfer the scientific knowledge about climate change in the Pacific Islands to the Spanish Islands. The use of this scientific declarative metacognitive knowledge decreased during the solving of problems b and e. During process of solving the problem b, students used stochastic knowledge to predict the setback of the beaches. The problems (e) and (f) gave students the declarative and procedural metacognitive knowledge to analyse the shape, surface and perimeter of the Trocadero Island. This cognitive and metacognitive knowledge helped them to justify the possibility that the Trocadero Island portioned in two, or more parts, or even completely flooded.

The scientific declarative metacognitive knowledge of the student 3 group B emerged again in the last problem g. She argued: *"Yes, it is probable, because if we do not change the effects of climate change, the sea level will continue increasing, so in the future it will end up disappearing".* Moreover, the student uses the mathematic metacognitive knowledge acquired in problems b and e to conjecture: *"It is probable that this would have happened in the disappearance of the Pacific Island".*

Her justification expresses her concern about the relationship between the consequences of climate change in the Trocadero Island and the Pacific Island. The integration of the declarative stochastic, geometric and scientific knowledge would provide students with metacognitive procedures and conditional strategies to understand the connections between the local and the global consequences of climate change. In this sense, the analysis of the data about the stochastic setback of the beaches in Spain and the fractal geometry of the island is a point of depart to avoid the certainties that are locally true and to feel the connections that exist worldwide and develop and environmental-conscious thinking about the global situation. Therefore, providing students with open-ended problems, which help to develop cognitive knowledge, and self-reflective questions, which aim to integrate metacognitive scientific and mathematic knowledge, could be a path for transforming a traditional problem solving in action in a sustainable mathematic task (Renert, 2011).

Furthermore, students have been able to integrate mathematic declarative, procedural and conditional metacognitive knowledge. For example, the student 2 of group A answered the self-reflective metacognitive question related with problem e: "in which unit do you think that will be better to express the final result in cm^2 or m^2? Why?". The student answered: *"In m^2. We have transformed the cm^2 to m^2 to compute the surface. And, make it easier to finally express it".* The declarative metacognitive knowledge used in the first part of the answer informs about the strengths of the student' processing ability to discriminate "what" measure is more useful to compute the total surface of an island. The second part describes how successfully employ the knowledge about the measures, due to the fact that students computed the surface using a map scaled of the Island in cm^2. Then, he transformed the total surface expressed in cm^2 to m^2. This part of the sentence provided information about how the student used the procedural metacognitive knowledge. Finally, the third part of the sentence is an attempt to describe the appropriateness of the measure selected to use the cognitive strategy of transforming cm^2 to m^2 and obtain numbers with an easier expression. Using in this case, conditional metacognitive knowledge answering why the strategies used were appropriate.

In contrast with the majority of the students that use of scientific and mathematic metacognitive knowledge when answering the self-metacognitive questions of (a), (b) and (e), the student 4 team B used social declarative knowledge to justify that the human action was the cause of the flooding of the Trocadero Island. After cooperatively solving problem a, she argues: *"I think that it is unlikely that [the Trocadero Island] disappear, because the human activity is produced mainly in foreign countries".* When solving problem b, which engaged students in the stochastic analysis of a diagram about the predicable setback of the Spanish coast in 2050, the team of the student used a direct proportional model to predict the total flood of the Island. Meanwhile, the student used exclusively social metacognitive declarative knowledge to individually justify her beliefs. She argues: *"I continue believing the same, because if we continue polluting so much [...] the poles are* *going to melt and it is going to increase the sea level"*. Finally, in the last problem, she integrated declarative and procedural mathematical metacognitive knowledge to argue again about the probability of disappearing the Island: *"Probably yes; but, in about 500 years, [the Island] will be divided in two. But, unlikely that [it would] disappear"*. The sentence informs about the strength and weaknesses of students use of stochastic metacognitive declarative knowledge to affirm the probability of disappearing the Island. This answer contrast with exact value proportioned (500 years), that she has cooperatively determined using direct proportional models.

The use of this deterministic model was not exclusive of the student 4 team B. Five other students used this model during the answer of the self-reflective metacognitive question of problem b, and it was adopted by mostly all the students in their conclusions. The direct proportional model used proportioned a deterministic view of the risk and uncertainty of the climate change consequences. The use of this deterministic directly proportional model constringed the stochastic nature of the prediction of climate change situations of risk (Serradó, 2018). Surely, the use of this direct proportional model to deterministically predict the total flood of the Island was a difficulty to embrace the contingency and stochastic probabilities.

The problems a and b were designed and implemented with the aim that the students use exclusively stochastic knowledge to justify their beliefs about the total flooding of the Island and predictions about the setback of the beaches. Nevertheless, the students used deterministic directly proportional models. In coherence with the theoretical framework of Renert (2011), the retrospective analysis lets us conclude that the use of these deterministic direct proportional models is a constrain to transform the task into a sustainable mathematic task to understand climate change consequences. Problem g gave students the opportunity to cooperatively discuss about the economic and environmental cost of constructing a dam to keep the Island undamaged. In words of Renert (2011), students were engaged in ethical action for healing the world. When the students used the declarative, procedural and conditional metacognitive knowledge to reflect about the pros and cons of constructing the dam, they trade the linear metaphors about the certainties and complexities that will provide this construction.

Closing up this section, we present an excerpt of one student recapitulating how the task helped him to understand the consequences of climate change: *"Climate change is our fault. I have not changed my opinion. Nevertheless, [I have changed] how I think. Thank you"*. When writing this comment, the student integrates opinions (beliefs), thoughts and his affect about the metacognitive knowledge acquired. We think that involving the students in the action of solving the seven problems and the self-reflective metacognition of the problems (a), (b), (e) and (g) helped students to be aware of the risk decisions they make and provoke the climate change. This decision-making has been a function of the students' individual metacognition knowledge and skills, their beliefs and affect (Schoenfeld, 2006, 2015). In coherence with Baten et al. (2017), the design of mathematic sustainable tasks needs of questions that help students express their affect when engaged in self-regulative metacognitive process. Such is the case of the task presented; students' aware-raising about the local and global consequences of climate change has emerged.

Conclusion

In contrast with the results of previous research that used quantitative methodologies that helped to discriminate a unique metacognitive knowledge (declarative, procedural and conditional) used by the students (Baten, Praet and Desoete, 2017), the students have combined the use of these three metacognitive knowledges when justifying their beliefs, thoughts and decisions about the consequences of climate change.

The seven ill-defined real problems helped students conjecture about their beliefs and engage them in a horizontal and vertical mathematization process to transform the real problems into mathematical ones. The students in this process of mathematization modelled and applied numeric, stochastic and geometric knowledge. In particular, the process of problem solving-in-action provided students with the cognitive stochastic knowledge about the uncertainty and risk of the setback of the beaches and the geometry of the Island.

Although, these problems have provided students the knowledge and skills to answer the questions, these are not enough to transform a mathematic curriculum to be considered a sustainable one. A didactic approach to a sustainable mathematics practices needs of an environmentally-conscious reflection about the consequences of climate change. This conscious reflection has been provided through the inclusion of the self-reflective metacognitive questions to answer after solving the problems. When students have answered these questions, they have integrated scientific and mathematic metacognitive knowledge to understand the connections between the global and local consequences of climate change. Moreover, they have integrated mathematic declarative and conditional knowledge to conjecture the probability of disappearing the island.

Nevertheless, this stochastic approach to understand the consequences of climate change were constrained by the deterministic directly proportional model used to conclude about the total flooding of the Island. Despite this constriction, the justifications of the students showed a commitment to feeling the local and global situation of the consequences of climate change, and making connections between these two realities. Finally, they were engaged in the ethical action of assuming how the human actions affect the climate change.

Summing up, most of the requests stablished by Renert (2011) for considering a transformative approach to a mathematic sustainable education were identified in the self-reflective metacognitions of the students. In conclusion, the task that aims to answer the question *"Is it possible that the Trocadero Island disappears due to climate change?",* integrated by the seven ill-defined complex problems and the metacognitive questions accomplish the conditions for being a task for sustainable mathematics education about climate change consequences.

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Gallery walk a collaborative strategy to discuss problem solving

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Abstract: Mathematics should provide an environment that allows students to conjecture, to prove, to generalize, to question, to discuss, to collaborate, to explain and to communicate their way of thinking by engaging and creating a sense of community. So, tasks have a great influence on students' learning, in particular those that elicit visual resolutions, as well as the way they are explored by the teacher. Moreover, today we have children sitting for a long time in the classroom and the gallery walk, being a strategy that requires students to move around the room, aspect that can be especially attractive for younger students, encourages them to challenge and share ideas. In this context, a study is being carried out with elementary preservice teachers where we analyze the work developed by these students, future teachers, in a teaching-learning environment based on a gallery walk when they solve some problems with multiple resolutions.

Introduction

School mathematics requires effective teaching that engages students in meaningful learning through individual and collaborative experiences, giving them opportunities to communicate, reason, be creative, think critically, solve problems, make decisions, and make sense of mathematical ideas (NCTM, 2014). Assuming that the tasks used in the classroom are the starting point of all students' learning, teachers should orchestrate productive discussions emerging from tasks that allow multiple and varied (re)solution strategies and provide the use of different representations (e.g. NCTM, 2014). In this context, the gallery walk emerges as a strategy to contemplate in classroom practices, which allows students to share ideas and receive feedback from their work, requiring them also to move around the room.

This paper presents an experience that is part of a larger study about the potential of visual strategies in problem solving in pre-service teacher education. It results from a teaching experience carried out with students in a curricular unit of Didactics of Mathematics, where a gallery walk was implemented as a teaching strategy to solve problems with multiple resolutions, of visual and analytical nature. The aim was to identify and understand their contribution as a promoter of productive discussions, as well as the reaction of the students to this experience. We adopted a qualitative methodology of exploratory nature, collecting data through observation and written productions regarding the proposed tasks and written comments on the experience. The results allowed to identify the strategies used by the students, that appeal to visual resolutions, and to verify the potential of the gallery walk strategy for a more effective teaching of mathematics for the future teachers.

Theoretical framework

The purpose of school mathematics is to have students who are flexible mathematical thinkers and who are able to solve problems efficiently and creatively, inside and outside the classroom. Assuming that the learning of mathematics greatly depends on the teacher and the tasks proposed (e.g. NCTM, 2014; Vale, Pimentel & Barbosa, 2018), teaching should provide the use of multiple teaching strategies and task resolution using different representations and approaches. In this way our goal as trainers is to provide these students, future teachers, with these skills so that they can develop them with their own students.

As Krutetskii (1976) points out, one of the characteristics of mathematically competent students is being able to look for a clear, simple, short, and therefore elegant solution to a problem. Research increasingly emphasizes the need for good visuospatial students, so teachers must organize a set of tasks to help them in their practice in order to show the advantages of this approach. The potentialities and limitations of visuospatial reasoning are recognized as part of the classroom mathematical culture (e.g. Presmeg, 2014), however it is not a common practice. Some problems may be complicated for those individuals who are analytical in the solutions they adopt, or, at least can be more laborious due to the number of numeric/algebraic manipulations they require. However, after the discovery of the visual relations involved, with some intuition or an *aha!* experience to begin the solving process, these problems become much simpler and more evident, hence accessible to more students.

Visualization plays a fundamental role as a component of mathematical reasoning (e.g. Jones, 2001; Vale et al., 2018) with strong connections to geometry, so it is necessary to develop, in students, skills like intuition and spatial perception (Jones, 2001). However, these abilities are not always developed or highlighted in the mathematics classroom, and also there are students who are not predisposed to use it. This situation is also reflected in future teachers. We argue that, for this purpose, mathematical learning should include practices that lead students to think visually and to develop this ability through experiences that require such thinking. We consider that visual resolutions include the use of different visual representations (e.g. figures, drawings, diagrams, graphics) as an essential part of the solution process. Non-visual resolutions no longer depend on visual representations as an essential part to reach the solution, appealing to others, such as numerical, algebraic and verbal representations (e.g. Presmeg, 2014; Vale et al., 2018).

To effectively learn mathematics, students should be actively involved. The experiences and interactions that emerge from intellectual, social and physical involvement are of great importance (e.g. Edwards, Kemp, & Page, 2014; Nesin, 2012). According to Hannaford (2005), thinking and learning don't happen only in our mind, on the contrary, the body plays a decisive role in the whole intellectual process, throughout our lives. Students who have the opportunity to move during classes (e.g. mathematics) may learn more effectively than those in typically sedentary classrooms, regardless of the activity being developed. In addition to movement, manipulations and experimentation, they should be able to talk about what they are doing and learning, write about it, relate new learning to previous experiences, and apply it on a daily basis. What they learn must become intrinsic (Chickering & Gamson, 1987). To accomplish these ideas we propose the gallery walk. It is a classroom strategy that allows students, individually or in groups, to have the opportunity to present through a poster, for instance, the resolution of a problem, situated around the classroom, in a perspective very similar to that of the artists when exposing their work in a gallery (Fosnot & Dolk, 2002). In general, in a gallery walk dynamic, a task is proposed for the students to solve, collaboratively, in small groups. Each group has the responsibility to organize their work in a poster that will be later displayed in the classroom. With all the posters distributed around the classroom walls or in another space outside the classroom (e.g. halls, library), they are observed by all the students, who must carefully analyze the content, formulating comments and/or questions that are placed on each poster or in a notebook. Later, the posters are collected and displayed to generate a collective discussion, taking into account the comments and questions formulated by the teacher and the students.

Considering all the phases involved in a gallery walk, we consider that this strategy favors discussion, critical thinking, communication, collaborative learning and teamwork, fundamental skills that students must develop, and may be especially attractive for kinesthetic students. It is also a way for students to get feedback about their work in a "non-threatening" way.

Methodology

As previously mentioned this paper reports an experience that is part of a larger study about the potential of visual strategies in solving problems with preservice teachers. The main purposes were to: identify and understand the strategies used when solving problems with multiple resolutions, in particular to identify the use of visual strategies; analyze/discuss peer strategies; and characterize the reaction to the gallery walk as a learning and teaching strategy. In order to do it, we adopted a qualitative methodology, of exploratory nature.

In the teaching experience here described the participants were 14 students, future teachers of elementary education (3-12 years old) that attended a unit course of Didactics of Mathematics, where a gallery walk was implemented as a teaching strategy for solving problems approaching different contents (e.g. geometry, fractions). Data was collected in a holistic, descriptive and interpretative way and included classroom observations, written productions to the proposed tasks, and a written report where these future teachers commented the gallery walk experience on the following questions: How did you feel about solving the tasks proposed in a gallery walk environment? What kind of advantages do you identify in this activity in relation to the acquired learning and attitudes? Is it possible to use this teaching strategy at any level/theme of teaching?

The gallery walk used in this study consisted of the following steps: 1) *Problem solving* – students, in pairs/trios, solved the proposed problems; 2) *Construction of posters* – next they discussed among themselves their resolution proposals and the way to present comprehensively in the poster; 3) *Presentation and Observation of the posters* – posters were displayed in the walls of the classroom. Each student went through the different posters to analyze the presented resolutions.; 4) *Elaboration of comments* – walking through the gallery, each student wrote their personal comments, doubts or questions, in post-its, placed in the several posters. It should be noted that in steps 3 and 4, while students discussed their colleagues' solutions, the teacher circulated around the classroom, evaluating students' observations and discussions, and also clarifying issues raised by the students; 5) *Group discussion* – After this round, the students take their own poster and analyze the contents of the post-its, making a small report; 6) *Collective discussion* – with all posters displayed once again in the classroom wall, the groups orally present their resolutions and respond to the previously made comments. This moment allows the teacher to highlight some of the ideas to include in the discussion and clarify doubts and errors, facilitating also the elaboration of a final synthesis of the fundamental knowledge emerging from the whole experience.

The gallery walk and the tasks

According to Presmeg (2014), tasks can be classified as visual problems, due to their context, the way they are presented, or because the visual solution is more powerful. In this paper, we chose two examples of geometric nature, that the students involved solved through a gallery walk (Figure 1).

Task 1
The area of the square is 4 cm^2. The points P and Q are midpoints of two opposite sides. Given the conditions of the figure, what is the area of the shaded part of the figure? Present more than one resolution.

Task 2
The figure shows three adjacent squares. The dimensions of the sides of each square are respectively 5, 4 and 3 cm, as indicated in the figure. Discover the area of the region in black. Present more than one resolution.

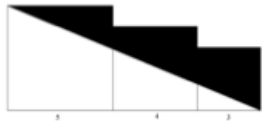

Figure 1. Tasks proposed.

Following the steps previously described in this paper, students started by solving the tasks in small groups of 2/3 elements. They began the approach to each problem by discussing possible strategies to reach the solution (Figure 2). The collaborative work facilitated the exchange of ideas and decision-making, in particular in finding the most effective strategy. After realizing what would be the best path to reach the solution, students planned the structure of the poster that they would be displaying in the classroom walls. In group, they decided which strategy to present, which were the most appropriate representations, how to write the text, among other aspects. There was some concern with the content of the poster and if it would be clear to those who observed it, showing the relevance of written communication in mathematics, including the representations used (Figure 2).

Figure 2. Students solving the tasks and constructing the posters

After the posters were finished, they were displayed around the classroom walls. Each student went through the space freely, carefully observing all the posters. Individually they made comments that they considered relevant and also raised questions when doubts were raised. This feedback was written in post-its that were associated to each of the presented posters (Figure 3).

Figure 3. Presentation/Observation of the posters and Elaboration of comments.

This step was followed by the analysis of the comments and questions associated to each poster by the respective groups. The students collected their poster, with the post-its, and read the feedback given by their colleagues. In group they discussed the pertinence of the comments, what they could do to improve their work, how to clarify their ideas, and, in some cases, they detected errors that had not been noticed (Figure 4). The feedback given by the peers was a positive contribution to promote significant reflection.

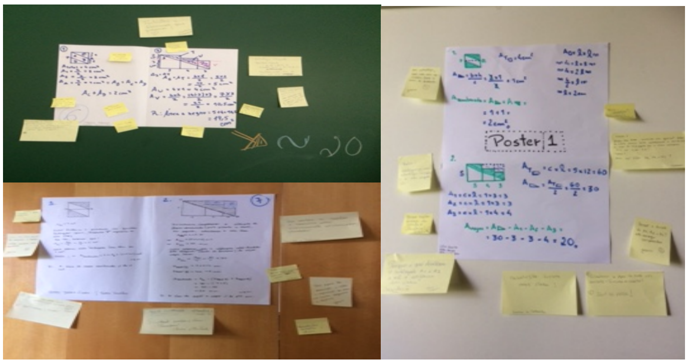

Figure 4. Posters comments/Group discussion.

Finally, after each group analyzed the comments and questions posed and decided if they wanted to clarify some aspect of the poster or even rectify some step, the collective discussion began. The posters were displayed in a central part of the classroom, so that they would all be visible in the same space. All the groups had the opportunity to synthesize their work, explaining the impact of peer feedback, clarifying some aspects that had become less clear and correcting some errors. This discussion was mediated by the teacher responsible for the curricular unit who, in addition to what was already mentioned, was also careful to focus on the diversity of strategies used by the different groups and highlight the contents emerging from the tasks. Although at this stage the role of the teacher was more interventional, in the previous phases all the work of the students were supervised and, whenever requested, the teacher intervened to support the students.

Analyzing, the resolution of the proposed tasks, it can be said that there were no significant difficulties showed by these students. They used different approaches: only visual or only analytical; visual complemented with analytical. In the phase of the collective discussion, associated to the gallery walk dynamic, the opportunity arose to analyze in more detail each of the resolutions presented by the different groups as well as some aspects that were not so clear. In particular, there was a resolution of the second task that raised some doubts. Although the numeric and algebraic manipulations were correct, the result presented did not correspond to the expected solution. The students knew that this could not happen, but they did not understand why, they weren't able to identify the error. This was an interesting situation because it allowed to discuss the dangers of reasoning based only in the appearance of an image, motivating the analysis of the mathematical justification that proved the impossibility of the presented solution.

Some conclusions

In general, this experience allowed, on one hand, to identify strategies used, but also to realize the potential of the gallery walk for a more effective teaching of mathematics. We concluded that the resolution of these tasks allowed to identify the strategies used by the students and to verify that, although they continued to use routinized formulas and procedures, visual resolutions also appeared. The gallery walk strategy engaged students in their peers' resolutions, in discussions with each other ("without fears and reprisals") and in classroom discussions, allowing them to increase their repertoire of resolution strategies more effectively than only in traditional discussions. This experience had a positive impact in the participants that showed enjoyment and will to implement it with their future students. These evidences confirm the potential of the gallery walk as an active learning strategy (e.g. Edwards et al., 2014; Fosnot & Dolk, 2002; Nesin, 2012). We believe that these ideas can be transposed all contents in mathematics but, more than that, also to other content areas.

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