

# **Effectiveness of Problem Posing Strategies on Prospective Mathematics Teachers' Problem Solving Performance**

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## **Introduction:**

Problem solving has long been viewed as an important topic in mathematics education. It is a focus issue concerning students' learning of mathematics (NCTM, 1989;1991). Contemporary reform efforts not only place a heavy emphasis on problem solving but also on problem posing. In the Curriculum and Evaluation Standards, problem solving is included as the first in the list of standards across all grade levels. In particular, it suggested the "investigating and formulating questions from problem situations" by students themselves. (NCTM, 1989, 70). In the Professional Teaching standards, it proposed that "students should be given opportunity to formulate problems from given situations and create new problems by modifying the conditions of a given problem" (NCTM, 1991, 95). The suggestions in both standards imply that problem posing is an integral part of problem solving and should not be emphasized separately from problem solving.

One of our major goals in mathematics teaching is to encourage our students to be a good problem solvers. To achieve that goal we teach mathematical problem solving strategies with more practices on it. We as a mathematics educators tend to neglect the other side of the coin in mathematical problem solving in our mathematics teaching program, that is problem posing (Gonzales, 1994, 78), in spite of its importance in developing our students' mathematical thinking. New trends in mathematics education (principles, 2000) recommend a changing from asking students to solve problems, to developing problems through changing their questions, adding new data, eliminating some data, changing variables or constructing a new problem based on the original idea.

In my discussions with teachers I observed that their abilities in solving non-routine problems were very weak. But they had a good attitude to pose questions from a given problem. I did tried to give more attention in mathematical problem solving and posing as a topic in "Methods of Teaching Mathematics" for prospective teachers in College of Education.

## **Objectives of the Study:**

- (1) To identify the effectiveness of using problems posing strategies on performance of mathematics prospective teachers for problem solving.
- (2) To identify problem posing skills needed to be included with polya's four steps to improve mathematics prospective teachers in problem solving performance.
- (3) To develop educational activities for mathematical problem solving and posing as a part of mathematics education program for prospective teachers.

## **Background:**

The first recommendation in "An Agenda for Action" produced by NCTM in the US. It recommended: problem solving be the focus of school mathematics in the 1980s. School Mathematics should Contains problem solving as the main activities in all mathematics aspects, also teachers should offer their students rich problems, often based in the real world, which would challenge and excite them, that is because problem solving is an effective way to introduce and explore new mathematics. Through problem solving, the students could develop much of the mathematics for themselves. Student-teachers are prepared to teach mathematics with a problem solving approach, to help their students in solving mathematical problems. Their educational program to do that doesn't reflect their abilities to solve problem. Abilities to use different problem posing strategies, may effect their problem solving performance. Relation between problem solving performance and problem posing still need to explore it as (Silver, 1993) mentioned "there is a need for further research that examines the complex relationship between problem posing and problem solving, there is also interest in exploring the relation of posing to other aspects of mathematical knowing and mathematical performance. In Silver's researches, he found different results of that relation: Silver and Cai (1993) found a strong positive relationship between posing and solving performance. While Silver and Mamona (1989) found no overt link between the problem posing of middle school mathematics teachers and their problem solving there is no clear, simple link has been established between competence in posing and solving (Silver, 1993). It is possible to improve student, teachers' performance in problem solving, by using problem posing strategies?, Kilpalrick discussed that and suggest that by drawing students attention to the reformulating process and given them practice in it,

we can improve solving performance (p.130). Given a mathematical problem to student, means we put him in a new thinking situation; thinking of the given information in the problem statement, thinking of best strategy to solve it using his own questions that lead him to solution and thinking of more information related to the given information.

The given information given explicitly in a problem statement is almost never adequate for solving the problem. The problem solver has to supply additional information consisting of premises about the problem context. (Kilpatrick 1987, 125). For example, to solve a word problem about the distance between two cities, student need to understand that distances can not be negative numbers. The idea of improving mathematical problem solving performance has been discussed in the light of Polya's four steps for problem solving. Through problem posing in Polya's steps, problems can themselves be the source of new problems, the solver can intentionally change some or all of the problem conditions to see what new problem result, and after a problem has been solved the solver can Look Back to see how the solution might be affected by various modifications in the problem. In Making a Plane to solve a problem, Kilpatrick showed that student may take Polya's heuristic advice to see whether, by modifying the conditions in the problem, a new, more accessible problem might result that could be used as a stepping stone to solve the original one.

Polya was looking towards problem solving as a major theme of doing mathematics, and "teachign students to think" was of primary importance. The other aspect of problem solving that is seldom included in our textbooks is problem posing. Polya didn't talk specifically about problem posing, but much of the spirit and format of problem posing is included in his illustrations of Looking Back (Wilson; 1993, 61). Looking Back may be the most important part of problem solving. It is the set of activities that provides the primary opportunity for students to learn from the problem. Polya identified this phase with admonitions to examine the solution by such activities as checking the result, checking the argument, deriving the result differently, using the method for some other problem, reinterpreting the problem or stating a new problem to solve. Teacher's skills during the uses of Polya's four steps in problem solving should go consistency with their abilities to use suitable problem posing strategies to generate more questions and problems for students.

#### **Mathematical Problem Posing Strategies:**

Mathematics teachers might use one or more strategies to formulate new problems or encourage their students in mathematics classes to be a good problem posers as they are a good problem solvers. Strategies could be used depending on the most suitable conditions (mathematics content, students levels, learning outcomes and mathematical thinking types). Problem posing situations classified as free, semi-structured or structured situations.

[1] Free problem posing situations: Situation from daily life (in or outside school). Student Can use them to generate some questions leading him to Construct a problem, students are simply asked to pose a problems encouraging them to 'make up a simple or difficult problem" or "Construct a problem suitable for mathematics Competitions (or a test)" or "make up a problem you like". It is more useful if the teacher tries to relate the real life situations to the mathematics content being taught and ask students to pose new problems from it. This will be more effective in developing students mathematical thinking. Problem posing situations might take these types: Every day life situations, free problem posing, problem I like, problems for a mathematics competition, problems written for a friend and problems generated for fun.

[2] Semi-Structured problem posing situations: Students are given an open situation and are invited to explore it using knowledge, skills, concepts and relationships from their previous mathematical experiences and it takes the following forms: Open-ended problems (i.e. mathematical investigation), Problem similar to given problems. Problems with similar situations, Problems related to specific theorems, Problems derived from given pictures, Word problems

This strategy was developed with student-teachers as the following (Abu-Elwan, 1999):

A semi-structured situations from daily life was presented to all students. Students were asked to complete the situations using their perspective to be able to pose problems from that formed situation. Students can generate problems by omitting the questions from given situations.

[3] Structured problem posing situations: Any mathematical problem consists of known data (Given) and unknown (required). The teacher can simply change the known and pose a new problem, or keep the data and change the required. Brown and Walter (1990, 1993) designed an instructional problem formulating approach based on the posing of new problems from already solved problems,

but they have also recommended varying the conditions or goals of given problems. This reformulation approach appears to be the most effective method for introducing structured problem posing activities in mathematics classrooms.

In order to create teaching / learning situations that provide a good problem posing situations, (Lowrie, 1999) recommend the mathematics teacher to:

Encourage students to pose problems for friends whom are at or near their own standard until they become more competent in generating problems.

Ensure that students work cooperatively in solving the problems so that the problem generator gains feedback on the appropriateness of the problems they have designed.

Ask individuals to indicate the type of understanding and strategies the problem solver will need to use in order to solve the problem successfully before a friend generates a solution.

Encourage problem – solving teams to discuss, with one another, the extent to which they found problems to be difficult, confusing, motivating or challenging.

Provide opportunities for less able students to work cooperatively with a peer who challenged the individual to engage in mathematics at a higher level than they were usually accustomed.

Challenge students to move beyond traditional word problems by designing problems that are open ended and associated with real life experiences.

Encourage students to use technology (Calculators, CD, ...) in developing their mathematical thinking skills, so they can use this technology to generate a new mathematical situations.

**Research Questions:** This study attempt to answer the following questions: How effective is the teaching of problem posing strategies to enhance student-teachers performance in problem solving? Is there any differences in mathematical problem solving skills between student-teachers whom study problem posing strategies and those whom just study problem solving strategies?

**Hypotheses:** The study included three hypotheses which are listed below:

There would be a statistically significant differences ( $p < 0.01$ ) between student-teachers' mean scores of experimental group and control group in mathematical problem solving part of the achievement text in favor of the experimental group.

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**Method, Subjects:** 50 student-teachers participated in the study, All of them were in grade three in College of Education, (Sultan Qaboos University) and their specialist were mathematics / computer, They were enrolled in "Methods of Teaching Mathematics 2" Course. They were divided in two groups; group E as an experimental group and group C as a control group, each group consists of 25 student teachers. The study was conducted over the period extending from November 12, 2000 to December 17, 2000.

**Instruments:** An achievement test on "*Mathematical Problem Posing-Solving*" has been developed to determine the effectiveness of using problem posing strategies to enhance student-teachers' performance in mathematical problem solving. The main topics and ideas of the achievement test are relevance in everyday life and are reinforced through common activities like shopping and vacationing problems were developed based on the three problem solving strategies:

Look for pattern, Make a list, Work Backward.

The test consists of eight open-ended problems, each one of it contains:

A statement of the problem. A question asking student to solve it. Another question asking students to pose an extension to the original problem, then to solve it. Four referees were asked to give their opinions regarding the validity of the test, Based on their suggestions, all modifications were developed. Reliability of the test was established using a group of 20 mathematics student-teachers. A reliability Coefficient mean value of 0.69 was secured. The achievement test in its final form was ready to use in the experimental design. (App. 1).

**The Experimental Design:**

First: Students-teachers in groups E and C were involved in a quasi-experimental design as the following: Students-teachers in Group C (Control) studied a problem solving strategies based on

Polya's four steps; several problems has been presented to students-teachers in this group, they were requested to solve it using suitable problem solving strategies, problems has been chosen from:

School mathematics textbooks (middle and secondary stages)

Problem solving experiences in Mathematics as a source book.

Internet web: [www3.actden.com/math-den](http://www3.actden.com/math-den)

Second: Student-teachers in group E (Experimental) studied problem solving strategies based on polya's four steps (polya, 1973). Problems has been chosen from the previous sources like group C. But students in group E, studied it under other treatment for polya's four-steps. Student-teachers explore the intent of each step in the process and attained a basic understanding of the process.(fig. 1).

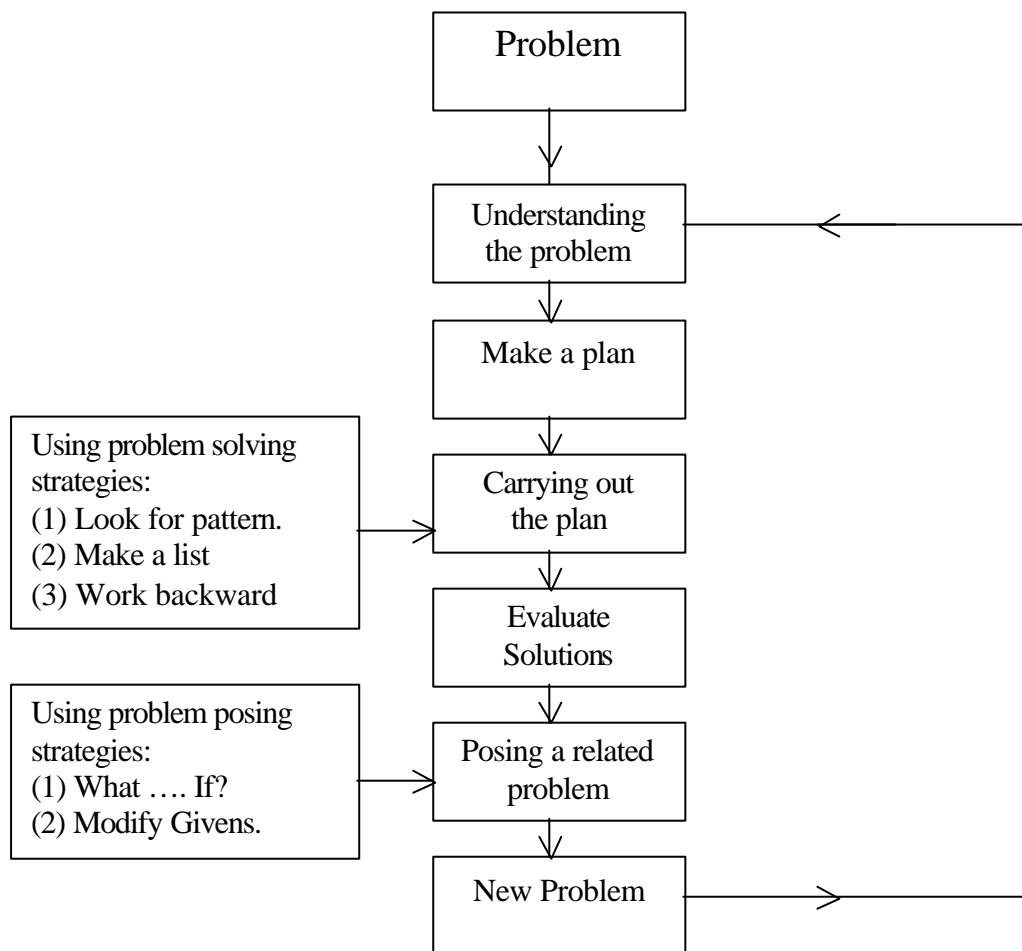


Fig (1): Framework for cyclic of activity Problem (solving – posing)

They are introduced to solve problems by traversing each of Polya's steps in the problem solving process.

They are required to write a description of each step as follow:

(1) *Understanding the problem:*

Ask yourself questions such as: “What is the problem all about?”, “What am I given and non-givens?”, “What do I need to find?”

(2) *Make a plan:*

What strategy of which you know (look for pattern, making table or work backward) you will use.

(3) *Carrying out the plan:*

Perform the necessary computations and describe the steps that you take.

(4) *Evaluate Solutions:*

Check if there might be other solutions or other strategies which will yield the same solution.

Student-teachers must indicate all questions, attempts, frustrations or any restrictions they may have placed on a problem. Within the context of solving a given set of problems, probing questions

are posed such as: Is all the given data relevant to the solution? Do any assumptions have to be made? Are there different ways of interpreting the given information or conditions? As the questions are posed, students reach a good understanding of each problems. The most important step is to encourage students to “generate an extension of the given problem” or “posing a related problem” as it is suggested by Gonzales (Gonzales, 1994, 81). She suggested a fifth step which is:

(5) *Posing a related problem:* Use the given problem and modify it to obtain a variation of the given problem. Student is poses a related problem by changing the values of the given data and by changing the context of the original problem, that doesn't mean he has to modify or change the solving strategy used in the original problem.

Student-teachers may used any of the following techniques in writing a new related problems:

Change the values of the given data,

Change the context and

Change the number of conditions.

Third: Group C has studied “problem solving activities” during the time of Sept. and Oct. 2000 (5-weeks) while Group E has studied “problem solving-posing activities during the time of Sept. and Oct. 2000 (7-weeks), the extra two weeks because of the techniques used in problem posing during and after solving of the problem.

Fourth: Achievement test on “Mathematical problem solving-posing test” has been presented to student-teachers in group C and E in same time on October 29, 2000 as a post test.

\* **Results:** To determine the effectiveness of problem posing strategies on prospective Mathematics teachers’ problem solving performance, t-test was used as a measure of comparison between the mean scores on: mathematical problem (solving-posing) test for both experimental group (E) and control group (C), SPSS V10.0 has been used and it is shown in table (1).

Table (1)

Means, Standard deviations and the value of T of scores in Mathematical problem (solving-posing) test

Variable	Experimental (n=25)		Control (n=25)		Mean Difference	t-value	P<
	M	SD	M	SD			
Problem – solving	4.16	.75	3.64	1	.52	2.09	0.01
Problem – posing	2.32	.69	1.76	.66	.56	2.93	0.01
Problem (Solving-Posing)	8.16	1.43	6.72	1.62	1.44	3.33	0.01

\* Maximum score in “problem-solving part” of the test = 5 marks

\* Maximum score in “Problem-posing part” of the test = 3 marks

\* Maximum score in “Problem (solving-posing) test” = 10 marks

It is evident from table (1) that the level of problem solving performance has significantly improved for student-teachers of the experimental group comparing with performance of other student-teachers of Control group in problem solving, with means of 4.16 and 3.64 respectively. The resulting t value of 2.09 is significant at  $p < 0.01$ . This improvement in the level of problem solving performance for experimental group is consistent with the significant increased in problem posing performance for student-teachers in Control group, with means of 2.32 and 1.76 respectively. The resulting t value of 2.93 is significant at  $p < 0.01$ . Student-teachers of experimental group were able to exhibit a significantly higher level of problem-solving performance as compared to that exhibited with student-teachers of Control group. In consistent with previous result, student-teachers of experimental group were able to exhibit a significantly higher level of problem-posing performance as compared to that exhibited with student-teachers of Control group. Overall, the level of problem (solving-posing) performance has significantly improved for student-teachers of the experimental group comparing with performance of student-teachers of control group in the test of problem (solving-posing) with means of 8.16 and 6.72 respectively. The resulting t value of 3.33 is significant at  $p < 0.01$ .

**Discussion:** This study aimed at examining the effectiveness of problem posing strategies on prospective mathematics teachers’ problem solving performance. The study sought to develop polya’s four-steps to include a generated more questions in “make a plan” and “Carrying out the plan” or make an extension to the original problem to enhance student-teachers performance in problem solving. Participants of experimental group has studied problem solving using polya’s four-steps as it

is developed by problem posing strategies. Results from this study supported the hypothesis and a significant improvement in student-teachers problem solving performance was observed, as well as there is a significant improvement in student-teachers problem posing performance in the achievement test. An alternative explanation for the present findings is that student-teachers has opportunities to discuss each step in their solving a presented problem with more emphasize on the uses of problem posing strategies used to develop new problem, that support the same findings of (Gonzales, 1994) and (Leung, 1993). More researches needs to be performed on the relationship between problem solving performance and problem posing abilities for students in all stages.

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