Changes of teaching methods and mathematical content caused by new technologies. Changes of methods, objectives and content. Traps and threats.

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1. More and more teachers reach in their work for new technologies such as ordinary and graphic calculators and computers. Probably, they are not as popular as ball point pen, but certainly they became widespread. Classes where students are supposed to use e.g. graphic calculators are not something extraordinary. Change of the students' work style as well as the change of the class atmosphere is observed. Students may use real data instead of data carefully chosen to facilitate the calculations and in consequence they work more in groups and they are more confident. The scope of problems that can be presented to them to be solved develop in a natural way cooperation, increase the interest in the lesson. In other words, new tools change the style of the school work of student, as well as teacher.

In general, the changes caused by the introduction of new technologies in education can be observed in the social plane, with respect to the teaching style as well as the objectives and content of the education.

- Change of social and personal relations.

Use of technologies in the school teaching changes enormously the teacher-student relations. The teacher, introducing computer or calculator, accepts at the same time discussions among students and among teacher and students. It is a break of the traditional in Polish schools "silence in the classroom" and answering only to the questions asked by the teacher.

Students working with technological devices work individually and they experience the lesson personally and experience personally all experiments performed. It results in the identification with the work done and, in consequence, causes the feeling of personal responsibility for the work.

- Change of the teaching style.

Work with computer or calculator consist mainly of performing experiments, collecting and processing information. The teacher very often can only show the directions in which the students should go. Computers and calculators are individual working tools, thus it is not possible to close the students way to individual experiments and individual conclusions. In such a situation, the teacher becomes a real animator and partner in the discussion. It results in important increase of the individual learning and, in consequence, to better appreciation of every student. Therefore, one of the consequences of this approach is the method of students evaluation.

- Change of objectives and content.

When teaching mathematics by means of computers or graphic calculators, objectives of classes and content must be carefully analyzed. We do not realize very often that the main objective stayed aside, that the heart of the lesson for student was different from the one we assumed. This is strictly connected with the content. Further in the paper, this type of changes will be dealt with on examples of several lessons.

2. Let us look at one of the most popular lesson with the use of graphic calculator (or computer with the appropriate software).

Lesson 1. ***Graphs of a linear function y = ax + b.***

During such a lesson, the students can investigate themselves the role of coefficients a and b, create many examples of graphs of functions, learn about monotonous functions and many other function properties in a permanent way and in shorter time. Nevertheless, teachers very often do not realize that the skill that will not be learn by students during such lessons is how to draw graphs of a function. This is a skill considered by us as a basic skill and it is introduced in the traditional style of teaching from the very beginning of teaching students about linear functions. This skill is not trained during the lessons with calculators. Let us not, therefore, be astonished that our students will be in trouble while solving such problem as: *draw a graph of the function y = 3x – 4.*

Let us consider the next example (a basic issue concerning the square function in secondary school).

Lesson 2 ***Factorization of the trinomial of the form: $ax^2 + bx + c$***

All students have graphic calculators.

Let $f(x) = x^2 - 7x + 10$.

We start from a short discussion on the factorization . In common discussion we arrive on the statement that $f(x) = (x – x1) (x – x2)$.

Thus, what does it mean for a particular value of function, e.g. for f(10)?

The students write: $f(x) = x^2 - 7x + 10$ and calculate the value f(10). This value, 40 is presented in the factor form. For the number 40 the students can do it easily, without calculator, but for greater numbers it is convenient to use the calculator function that factor numbers in prime factors. It is not easy to conclude from one example, so we investigate the factors of f(11), f(12), f(13), f(14).

The following results are obtained on the calculator:

$f(10) = 5 \times 2^3$

$f(11) = 5 \times 3^3$

$f(12) = 2 \times 7 \times 5$

$f(13) = 11 \times 2^3$

$f(14) = 3^2 \times 2^2$.

Then, the students try to present such obtained factorizations in the form of factors of two numbers to obtain certain relations. After some tries, the students notice that:

$f(10) = 5 \times 8$

$f(11) = 6 \times 9$

$f(12) = 7 \times 10$

$f(13) = 8 \times 11$

$f(14) = 9 \times 12$.

The elements of factors differ by 3. Hence, $|x1 – x2| = 3$.

Thus, it is easy to guess that $f(x) = (x – 5) (x – 2)$. We check, by multiplying, that the obtained factorization is correct.

Then, students are asked to find factorizations of other square functions.

This lesson is usually of great interest to students. They eagerly play in guessing and looking for relations. They do not have to calculate anything, their work consists of the correct use of calculator commands and on proper reasoning. This reasoning is the most important part of the lesson. The factorization itself is the effect, the final result of the reasoning. The most important thing is to observe the proper regularities and, in consequence, to obtain the factorization. Thus, the students train here the skill of factorization of a number in prime factors, to notice the consistency, to generalize and to draw conclusions. The leading subject, which is the factorization of the binomial in linear factors and finding the roots is realized only by means of writing down the result, the final effect.

Such a lesson is very fruitful with respect to the notion of the factorization of a function in factors of the lower order. Attempts may be made to carry in a similar way the analogous reasoning for the polynomials of order 3, that is an additional advantage of this method, as examples of such generalization on the school level are not numerous.

We must, however realize that the demonstrated method will not be the one that will serve students to investigate the square function. The standard algorithm of calculating delta was not shown here, this skill was not trained by students. Moreover, the whole reasoning concerned the factorization of natural numbers in the prime factors, rather than the factorization of the square functions.

Lesson 3 ***Solution of square equations with a parameter***

Students are supposed to solve the following equation:

For which values of the parameter m, the equation $x^3 + (3 - m^2)\,x + m = 0$ *has two roots, one of which is greater than – 1 and the other is smaller than – 1?*

The students use a graphic calculator. The teacher suggest them to look at some graphs of square functions that satisfy the given conditions, and then poses a question. Do the graphs have a common feature? If the students observe a common feature, then the next step is to draw a graph of functions that do not satisfy the condition given in the problem. Thus, by observation, the students arrive at the solution.

Such a work style is very absorbing and brings a better understanding of the notion of the square function but it is not an analysis of the algebraic formula. The students did not make use of the Vieta formula, but rather concentrated on the use of algorithms and on algebraic transformations.

3. Similar situations can be used when using graphic calculators teaching mathematics The usage of calculators changes not only the style of work but effects also strongly the content. Such a situation occurs very often when new technologies are used together with the existing traditional structures and text book solutions. In general, being fascinated by novelties and tremendous possibilities of the hardware, we do not notice what our students really learn during such lessons.

The very important thing is that, beginning lessons with calculators, our students fully realize what kind of content we introduce and train – what we gain and what we loose. If we know all this, then we introduce the lacking new skills on the nearest occasion, or we sometimes give them up. Nevertheless, our decision should be fully conscious.

Summary. New technologies, such as graphic calculators or computers are very attractive challenges for many mathematics teachers. New tools cause various changes in mathematics teaching. Are these changes always positive? Can we, teachers, make good use of new tools, to prevent undesired changes? To answer these questions, a very good recognition of the changes is needed when new technologies are used in mathematics teaching. Is it only a change of methods and style of working? And what about the change of content, skills and in consequence the objectives of teaching mathematics?
In this paper, the problem of changes in teaching and learning mathematics with the use of new technologies, based on an analysis of some examples of lessons, will be discussed.