Communication in the Secondary Mathematics Classroom: Exploring New Ideas

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In recent years, communication has received considerable attention as an "essential part" (NCTM, 2000) of the secondary school mathematics experience. For example, since 1999 in the province of Ontario, Canada, mathematics teachers have been required to assess and report on student achievement regarding the communication of mathematical understanding. The provincial curriculum document (Ministry of Education and Training, 1999) states that

> [t]he importance of communication in mathematics is … a highlight in secondary school…. This curriculum assumes a classroom environment in which students are called upon to explain their reasoning in writing, or orally to the teacher, to the class, or to other students in a group. (p. 4)

In this paper we describe three recent experiences that the authors have had in exploring innovative approaches to communication in the secondary mathematics classroom. In the first, we discuss some significant outcomes of a three-day communication working group for mathematics educators, co-led by one of the authors, in which novel ideas were encouraged. In the second, we describe the activities of one of the authors and a small group of mathematics teachers who worked together for almost three years, exploring ideas for non-routine written communication in their classrooms. In the third, we describe how students communicated their mathematical understanding by oral, written, and visual means in a series of integrated mathematics and visual arts assignments given by one of the authors. The order of presentation takes the reader closer to the classroom with each experience, but all three highlight the complexity, the playfulness, and the practicality present in the process of communicating mathematics. From these experiences, we draw some general conclusions about the possibilities for new ideas in communicating mathematical understanding.

Experience One

In 1997 a group of mathematicians and mathematics educators from across Canada, as part of an annual experience, met to participate in working groups on various aspects of teaching and learning mathematics. One of these groups focused on the subject of "communicating mathematics" (Pirie & Franks, 1997). The meetings, nine hours in total, extended over three days and were structured to challenge participants' thinking about the "intersection of representation, communication, and mathematics" (p. 53). Where, for example, is the mathematics situated? In "communicating mathematics" are we inevitably communicating a representation of mathematics or is it the mathematics itself? For example, the notion of "triangle" may be communicated in a variety of forms: The word "triangle," a sketch on the blackboard, a well-drawn and labelled diagram in a text, a paper cut-out, a construction of straws, a compass and straightedge construction on paper, or a dynamic figure on the computer screen are just a few of the possibilities. The meaning(s) intended by the communicator--the teacher, the author, or the student, as the case may be--and attributed by those to whom each form is communicated, may very well not coincide, or even intersect. The mathematical knowledge, the sense of the context, and the assumptions that each participant in the communication experience brings to the moment are all critical factors.

In response to an initial request to brainstorm ways in which communication of mathematics could happen the full group of approximately 16 produced a list of fifty representational forms. A sample of these forms is listed in the following table.

Diagrams	Tables	Objects
Picture	Visualizations	Pile of buttons
Simulations	Video	Journal writing
Computer screen image	Physical models	Singing

Poetry	Algebraic expressions	Story
Talk (math language)	Facial expressions	Numbers
Talk (informal language)	Thoughts	Mental images
Silence	Curriculum guides	Proof
Symbols	Graphs	Text
Algorithmic procedures	Matrices	
Body language	Metaphors	

The appropriateness of these ideas as representational forms of mathematics was not evaluated; the list was accepted as given, and served as the basis for the activities that followed. The fifty items were assigned, in mixed clusters, to six smaller groups, who first reflected on and then reported what they thought each of their forms could and could not mathematically communicate. The groups found themselves confronted by such questions as what different meanings might be assigned to each form, whether mathematics content, affect, or both could be communicated, and when was it difficult to tell. Participants had to tackle taken-for-granted assumptions and hitherto little-considered qualities of communicating mathematics.

Each group's next task was to try to communicate meaningfully the mathematics of a predetermined topic using only the representational forms that had been assigned to it. Thus, for example, one group was asked to communicate the *cosine law* while having available only tables, action, journal writing, mental models, mental images, and algorithmic procedures, while a second group attempted to communicate the *calculus of differentiation* having as representational forms only facial expressions, video, numbers, test scores, algebraic expressions and analogies (that is, no 'voice'). We as leaders deliberately chose topics that we thought would represent a challenge. Groups reported feelings of both frustration and liberation; frustrated with the constraints of dealing with unusual or restrictive representational forms and liberated by being challenged to communicate the mathematics with those very same forms. The 'audience' also felt challenged by the experience of being "communicated to" in unusual ways. The experience left participants with a deeper sense of what was involved in communicating mathematics, and more questions upon which to reflect.

Experience Two

In early 1994, Franks began a "writing in the secondary mathematics classroom" project with a group of five high school teachers from two local school boards. At the time, although the use of non-routine writing in classrooms was certainly not new, its widespread encouragement in mathematics was a relatively recent phenomenon. In its 1989 *Curriculum and Evaluation Standards* document, for example, NCTM endorsed the continued development of language and symbolism to communicate mathematical ideas so that all secondary students can express such ideas orally and in writing, and be able to reflect upon and clarify their thinking about mathematical ideas and relationships (p. 140). The published literature describing various forms of non-routine mathematics classroom writing, and generally extolling the benefits of such activities, was increasing (e.g., Artz, 1994; Gordon & MacInnis, 1993; Nahragang & Petersen, 1986; Sawada, 1992). Nevertheless, teachers of mathematics, particularly at the secondary level, generally appeared reluctant to incorporate such ideas into their classes.

The local project was intended to explore mathematics classroom writing possibilities with teachers, to provide support as they adapted writing strategies to suit their individual situations. The goal–certainly the hope–for this experience was that over time, with common interests to help build cohesiveness, the group would achieve a sense of community that would in turn sustain and even increase the desire for, and the practice of, innovative approaches to secondary mathematics teaching (Franks, 1995). There was no agenda of strategies, instead, a variety of writing approaches were discussed at group meetings. The teachers then made choices from these ideas, tried them out in their classrooms, and at subsequent meetings described what they were doing, how the students were reacting to it, and how well they were doing.

In order for a sense of community to take hold, the meetings needed to be held frequently. Ideally this should have meant getting together perhaps every two weeks, but realistically, this was not possible to maintain. The project began midway through the school year, and for the rest of the year and well into the second, the group met every four to six weeks.

During the first half year, and into the second full year, the teachers attempted a wide variety of writing (and reading) strategies. Some had been exploring writing activities before the project began, but for others it was quite new. As a result, the choices they made tended to be highly personal and contextual: comfort level, grade level, and course type were major factors. The choices ranged from somewhat more cautious and structured ideas such as creating a "dictionary" and completing a crossword to the more substantial and creative, such as personal journal writing, short and long writing assignments or projects, mathematical autobiographies, and the use of literature, for example, the books by Anno.

As the months went by the group began to make a number of important observations. The first was an affirmation that, indeed, oral and written communication between the teacher and students in the secondary mathematics class was important. In the classroom, teachers found themselves giving greater emphasis to the notion of mathematics as a language.

Students' level of language comprehension was clearly a factor in how well each did, with some, particularly those who liked to write, doing well, while others showed more limited ability to develop a thoughtful response to the assigned work. They sometimes had difficulty understanding what was required of them, and would miss the point of the assignment. The teachers realized that they needed to be clearer in their expectations and more aware of assumptions in the choice and level of language they used in describing their assignments. They also needed to insure greater student relevance and interest when making assignment choices. This was generally new territory for many of the participants, and students and teachers alike were learning how to deal with this added level of complexity in the mathematics class.

These were lessons that members took into the second and third years of the project, as individual teachers refined their classroom activities. The group also began to explore other means of communication not directly related to the classroom, an action that could be taken as a sign of growing community. For example, group members created a reporting form for describing books and other print items that would be of interest to teachers of mathematics at all levels. These forms were to be circulated to all schools in the area, with completed responses kept in a central database accessible to teachers. The group also helped to establish a small periodical, in affiliation with the local mathematics education association, for which area teachers were encouraged to write.

This growing cohesiveness and expanding sense of what it meant to "communicate" also fostered a sense within the group that in terms of student writing activities, members needed to move beyond the stage of using and reporting on individual student activities in their own classrooms. By the end of the second year, plans had been made to develop an extended project on linear equations, incorporating significant writing opportunities, which could be used by anyone teaching that topic. Throughout the fall and winter of the third year the group worked on the unit's development and in early spring some of the teachers implemented the unit or portions of it, with some success, in their classrooms.

That effectively marked the end of the group. While most members were able to take part in a well-received presentation at a provincial mathematics education conference that spring of 1997, the overall project did not have a sufficiently strong central core to hold the group together after that, as other priorities began to take hold in group members' lives. Although it was not originally planned that way, the mathematics unit became a culminating activity. Nevertheless, remaining together for over two and one-half years was considered quite a feat in itself.

Experience Three

In September 2000, Jarvis, a mathematics and visual arts teacher, began a graduate research study involving two Grade 9 Academic mathematics classes. This Action Research study (Jarvis, 2001) examined students' perceptions of integrated learning and documented their performance on three such math/art assignments. Utilizing both a qualitative and quantitative research design, information was gathered during a secondary school semester through a variety of research instruments.

Each of the three integrated assignments was created by combining ideas from an historical time period, a character(s) from that period, a visual arts component, and a mathematical skill, or skill set, that was to be reinforced in the learning. Furthermore, each assignment required students to partake in math research, problem solving, practice worksheets, the creation of a final *SMARTWORK (Secondary Math & Art Work)*, and the written and oral communication of this final project.

The first assignment sought to reinforce expectations from the *Number Sense & Algebra* strand, particularly those dealing with ratio and proportion. This assignment was centered on the character of Pythagoras, the historical period of ancient Greece, and the math/art concept of the famous *Golden Section* or *Divine Proportion*. It required the students to use the *Golden Section* ratio accurately in a creative piece without boundaries.

The second assignment sought to reinforce expectations from the *Analytic Geometry* strand, particularly those dealing with slope, equations, and the Cartesian plane. It fancifully supposed that the Italian artist/polymath, Leonardo da Vinci, and the French mathematician/philosopher, René Descartes, could somehow have met each other (chronologically impossible), discussed their respective sciences of linear perspective drawing and analytic geometry, and collaborated on a project. Using large sheets of grid paper, students were required to create a Renaissance *SMARTWORK* that featured the Cartesian Plane, two prisms drawn in perspective and with labelled slopes and equations for selected construction lines, coffee stains, singed edges, and a reversed artist's signature (in the style of Leonardo's left-handed mirror writing).

The third and final assignment sought to reinforce expectations from the *Measurement & Geometry* strand, particularly those dealing with interior/exterior angles and transformational geometry. This assignment was centered on the character of M.C. Escher, 20th century Europe, and the math/art concept of tessellations or regular plane division. Students were given the opportunity to construct tessellations using a variety of methods including cardboard templates, paper grids, *Dynamic Geometry* software, and then their own choice of a final medium.

Notwithstanding a general sense of discomfort among students regarding the oral presentations, the majority indicated that the communication of math concepts (written and verbal) throughout the three integrated assignments helped to reinforce the new math learning. During the interviews one student stated, "You explain it and then others explain it – so you take yours and theirs and put it together and understand it a lot more." Another student reflected on the written communication aspects of the assignments, "I think the writing part helped reinforce the learning. I actually understood it better after I wrote it, because I had to put it into words." A third student made the following comment, "When you do something creative and visual, you remember it more easily."

Perhaps the most fascinating communication occurred during the Open House for parents and guardians. Over 60 people attended this special event that featured an exhibition of student *SMARTWORKS*, refreshments, music, contextual information, and sample resources. As families were led around the classroom, the researcher quietly watched and listened as students explained math concepts, pointed out favourite pieces, and simply enjoyed the unique experience.

Both the pilot study, involving Grade 11 Advanced students (May 2000), and the extension study, involving Senior Calculus students (June 2001), featured similar positive student perceptions to those of the Grade 9 study, yet with higher levels of proficiency and articulation regarding the creation of finished *SMARTWORKS* and the communication of these projects.

The historical, philosophical, and epistemological connections between Mathematics and the Visual Arts have been shown to be plentiful (Bickley-Green, 1995; Jarvis, 2001; Schlain, 1991). Newman and Boles stated that although these two disciplines are often viewed as polarities, they are in fact "the left and right hand of cultural advance: one is the realm of metaphor, the other, the realm of logic. . . . Their union allows the human spirit freedom" (1992, p. xiv). This study attempted to explore Einstein's claim that, "The supreme art of the teacher is to awaken joy in creative expression and knowledge."

Concluding Remarks

In this paper we have just begun to offer a sense of the communication experiences embedded in the three contexts. We hope, and believe, however, that the complexity, potential playfulness and practicality of communicating mathematics through non-routine means are evident. Communication is a complex act, yet much is taken for granted in the daily classroom discourse on mathematics. Whether an educator or a student, one is constrained by the nature of the representational form(s) with which one chooses to communicate. The potential for a multiplicity of meanings is high and, consequently, so is the potential for confusion and frustration. Nevertheless, choosing to push beyond what might be considered the typical forms of communication to a more inclusive approach can be both motivating and liberating. Teachers, and certainly students, often express a desire for a traditional structure, but we, as authors, have found that both groups, given sustained support, have had rewarding experiences when asked to become playful yet thoughtful risk-takers. We also claim that with the 21st Century has come the feasibility of communicating mathematics in an ever-expanding, diverse set of forms. If the classroom is to serve not only as the centre for comfortable and "tried and (possibly) true" practices, but also as the ground for exploring boundaries, then educators, including mathematics educators, have a responsibility to engage learners in non-routine, innovative communication experiences. To remain committed solely to the traditional is to miss a grand opportunity.

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