

## Using Web Sources to Promote Critical Thinking in High School Mathematics

Evan Glazer

University of Georgia, United States of America eglazer@coe.uga.edu

A pdf version of this paper can be found at: <http://www.arches.uga.edu/~eglazer/nime2001b.pdf>

**Abstract:** The United States educational system has been criticized for having a curriculum that limits critical thinking opportunities. One way to overcome this barrier and stimulate critical thinking is to create activities that use resources with dynamic capabilities and complex arrays of information. The World Wide Web incorporates numerous tools and information of this nature that can promote challenging and meaningful activities for high school mathematics classrooms (prealgebra through calculus). Web-based sources in the form of data sets and databases, predictions, simulations, and dynamic environments are discussed in this paper in order to present attributes that stimulate mathematical inquiry and exploration. Problem posing strategies are discussed to better enable teachers to create questions for student activities geared towards their own classrooms.

### A Need for Critical Thinking

If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war. We have, in effect, been committing an act of unthinking, unilateral educational disarmament. *A Nation at Risk, 1983*

The educational system in the United States has been criticized as an institution with an underachieving curriculum that leads students to memorize and regurgitate facts that do not apply to their lives (NAEP, 1983; Paul, 1993). Consequently, students have difficulty solving problems that require higher thinking skills (NCEE, 1983; Norris, 1985). Many claim that the traditional classroom environment, with its orderly conduct, didactic teaching methods, and teachers who view their jobs as dispensers of information, has greatly inhibited students' opportunities to think critically (Dossey et al, 1988; Goodlad, 1984).

International and national studies have concluded that learning in our schools is insubstantial. In 1982, the Second International Mathematics Study reported that K-12 students in the United States were outperformed in mathematics reasoning and problem solving by a vast majority of students in first world countries (McKnight et al., 1987). In 1983, the National Commission on Excellence in Education (NCEE) reported in *A Nation at Risk* that the United States educational system needs to reform its classroom environment so that all students acquire high-level skills in order to become critical consumers and participants in society. The development of critical thinking skills produces intellectual and socially competent citizens who effectively cooperate with other people and challenge real world problems (CTGV, 1993; Glaser, 1985).

In 1989, the National Council of Teachers of Mathematics (NCTM) released the Curriculum and Evaluation Standards for School Mathematics in an effort to empower students to become critical consumers of information. NCTM emphasized that our society has been transformed into an Information Age, where knowing how to access and use information is more important than memorizing information. The organization declared that "a climate should be established in the classroom that places critical thinking in the heart of instruction" (1989, p. 29). The proposed changes were an effort to promote mathematical literacy, including "the ability to apply mathematical ideas to problem situations and work with others to set up and solve problems" (Confrey et al., 1990, p. 1).

Since that time, many mathematics educators have been transforming their instruction and curricula to meet the societal need of establishing critical thinkers as suggested in the NCTM *Standards*. In order for this transformation to occur, teachers have been changing their classroom environments and searching for tools that promote problem solving, reasoning, communication, connections, and representation. Emerging technologies, such as computer software, graphing calculators, and the Internet, are some of the tools that have created stimuli for these cognitively enriched environments. In NCTM's Principles and Standards for School Mathematics (2000), the World Wide Web is noted as a source that can potentially enhance students' learning opportunities by utilizing data and resources in ways that promote graphing, visualizing, and computing. Web resources such as data sets and databases, predictions, simulations, and dynamic environments utilize these capabilities to provide unique ways of exploring mathematics. These tools and information are catalysts for designing inquiry-based

approaches to learning mathematics in an effort to stimulate critical thinking.

### **Web Sources that Stimulate Critical Thinking**

Critical thinking in mathematics can be interpreted in a variety of ways. Some consider it in an evaluative sense that is used to determine the quality of a decision or an argument. Others use the term in a generative sense that places emphasis on the creativity and ingenuity in designing a product or creating a solution to a problem. A working definition of critical thinking in mathematics that I use as a basis for designing Web-based and technology-enhanced activities is presented as follows (see Glazer, 2001, for a more detailed analysis and description of the term):

Critical thinking in mathematics is the ability and disposition to incorporate prior knowledge, mathematical reasoning, and cognitive strategies to generalize, prove, or evaluate unfamiliar mathematical situations in a reflective manner.

Kozma (1994) claimed that certain media contain unique attributes that stimulate particular instructional methods to influence learning. Thus, a variety of delivery mechanisms on the Web, such as information in a database or objects in a simulation, create opportunities for distinct learning experiences. Examples from four different Web-delivery methods follow, along with activity ideas that can stimulate curiosity and critical inquiry. A hyperlinked listing of several hundred Web sites of this nature can be found at <http://www.greenwood.com/glazer.htm>.

#### *Data Sets and Databases*

Data sets are groups of organized information presented on a page, such as census charts, temperature distributions, travel schedules, salaries, historical stock quotes, and agricultural prices. Values are often in a tabular format that can be imported to a spreadsheet for further analysis. Databases are groups of data sets that can be searched, sorted, and customized to the user's needs. For example, the US Naval Observatory displays astronomical data about the sun and moon's presence in the sky over a given time period ([http://aa.usno.navy.mil/data/docs/RS\\_OneYear.html](http://aa.usno.navy.mil/data/docs/RS_OneYear.html)). The user can adjust the variables of year, type of data (sunrise/sunset, moonrise/moonset, etc.), and location on the globe to produce a table of information. For example, a student can search for a yearly listing of the times when the sun will rise and set in order to predict a pattern in the amount of sunlight a city receives over the course of a year. A mathematical analysis of the data in this investigation gives meaning to the astronomical terms *equinox* and *solstice* because they are critical points on the sinusoidal model of the data. An extension to this investigation might be to form comparisons between the equinox and/or solstice and the vertical and horizontal position at different locations across the globe. In other words, does every city experience the equinox and solstice at the same time? If so, why is this true? If not, which variables influence the times at which these events occur?

#### *Predictions*

Predictions are groups of forecasted data extrapolated from trends in existing data sets. Examples include predicting someone's life span based on a series of lifestyle characteristics, the performance of a baseball player in a different era, or an updated world population every five seconds. Prediction sites should raise questions of wonderment since the data is represented by events that have not occurred, will never occur, or are unrealistic to measure. For example, the Top 100 Ever Adjusted Web Site (<http://www.the-movie-times.com/thrsdir/Top10everad.html>) presents a listing of predicted gross movie profits that are adjusted for inflation. The highest grossing film of all-time, *Titanic*, is listed fifth because the site predicts that four older movies earned a larger gross at the box office after adjusting for inflation. Data about the title of the movie, the release year, the total gross, and the adjusted gross, are provided for the US's top 100 films of all-time. Students can tackle a variety of questions with the inflation-adjusted data, such as: What is the inflation rate? Is the inflation rate constant? How was the inflation rate determined? How was the inflation rate used to determine the adjusted gross amounts? Students can extend this activity by creating a new table that will predict the films' adjusted gross amounts at a future date, maybe when the students' favorite film exceeds an inflation-adjusted gross of one-billion dollars.

#### *Simulations*

Simulations enable the user to modify variables or repeat trials in order to formulate patterns and generate hypotheses about a phenomenon or concept. Simulations can display mathematical shapes, such

as dropping random balls in an array to illustrate the normal distribution, or they can recreate real events that are not easily observable, such as air traffic control and train transportation systems. Simulations can also represent environments that are either too dangerous or costly, such as testing the relationship between amounts of voltage, current, and resistance in a circuit and examining when a light bulb will light up or blow up. Another example of a simulation of this nature is taking a simulated shopping trip to acquire a series of animal cards placed in boxes of cereal (<http://www.mste.uiuc.edu/reese/cereal/default.html>). In this simulation, the student is asked to determine the number of boxes of cereal needed to acquire all of the different animal cards. The user can vary the number of different cards and open random boxes until all of the cards have been acquired. After multiple trials, the student can formulate conjectures regarding the relationship between the number of different animal cards available and the number of boxes of cereal one would need to purchase in order to collect all of the cards. Students can extend this activity to discuss variations of the situation, such as allowing students to trade duplicate cards or collecting multiple sets of cards.

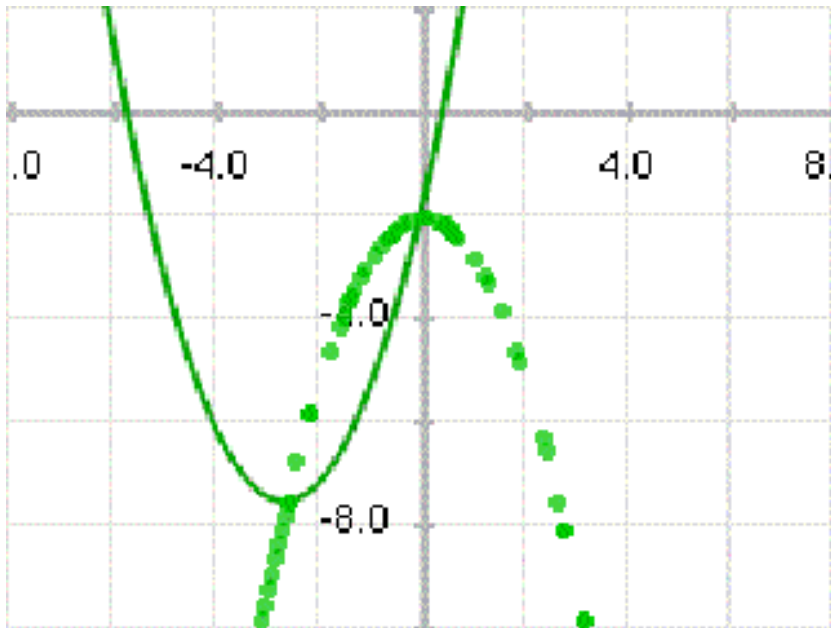
#### *Dynamic Environments*

Dynamic environments are interactive systems that enable users to visualize consistencies and variations in a system by clicking or dragging objects on the screen. A mathematical game is an example of a dynamic environment because the student makes decisions about how she will interact with the environment, consequently changing that environment on-screen. Geometry objects created in Java also represent dynamic environments because students can drag points on the screen to change the size and shape of objects while detecting changes in various calculations. Explore Math has an interactive quadratic function grapher ([http://www.exploremath.com/activities/Activity\\_page.cfm?ActivityID=13](http://www.exploremath.com/activities/Activity_page.cfm?ActivityID=13)) that illustrates changes in the shape and position of a parabola as adjustments are made in its corresponding equation. Vertex and intercept data are available and dynamically change with adjustments in the system, allowing students to see connections between characteristics of the equation and positions on the graph. In addition, the user can trace the vertex of the parabola as coefficients of the equation are modified. For example, when the second coefficient of the quadratic function varies ( $b$  in  $y = ax^2 + bx + c$ ), then the locus of the vertex will trace a quadratic function (see Figure 1). Students might try to justify why the locus is quadratic and if there is a relationship between the coefficients of the equation of the locus and the coefficients of the equation of the function. An extension to this activity could be to ask students if similar locus patterns appear as different coefficients vary or to examine locus patterns with different functions (see <http://jwilson.coe.uga.edu/EMT668/EMAT6680.F99/Glazer/essays/locus/criticalpoints.html> for a more dynamic environments and detailed analysis related to this topic).

#### **Designing Questions to Promote Critical Thinking**

By themselves, Web resources do not foster critical thinking. Asking a student to visit the sites mentioned in this paper will likely not place them in a situation to think critically. Instead, these sites are tools that can serve as catalysts for opportunities to create questions for exploration or extensions based on students' curiosity. Hence, the Web is a source for creating new learning opportunities, but it does not necessarily scaffold learning. The teacher's instructional strategies are the crucial link between the medium and the content. They ensure that learning activities address goals and objectives that are challenging and feasible for students to explore given available human and physical resources. I use the following strategies to design questions that use Web-based sources and promote critical thinking:

- The problem is loosely structured and does not guide students through a predetermined series of steps.
- The situations are meaningful to the students and relevant to the curriculum.
- The learning environment promotes experimentation and making conjectures.
- Student analysis leads to generalization, proof, or evaluation.



**Figure 1.** Trace of a vertex of a parabola when varying  $b$  in the function  $y = ax^2 + bx + c$ .

#### *A Web-Based Investigation that Promotes Critical Thinking*

Realtor.com (<http://www.realtor.com/>) is a database that enables users to search for available homes for sale in the United States. The user selects a state and geographic region, then sorts the available homes according to desired sale price, number of bedrooms and bathrooms, and available space. In the search return, a picture of each home is presented with a variety of information, including its sale price and area. Teachers who want their students to compute the area of the home based on its floor plan should look at the Log Home website at <http://www.homebuyerpubs.com/foorplans/floorplans.htm>. After viewing several listings, the student might explore whether there is a relationship between the price of a home compared to its area. The following scenario uses these features and concepts with the strategies presented earlier:

Your long lost uncle has willed your family an undisclosed amount of money to use as a down payment on a house. Your parents express two concerns in your search for this property:

1. They do not want to pay mortgage insurance; and
2. Ideally, they would like as much space as possible.

Based on the patterns in the neighborhood housing sale prices, recommend two homes that your family should consider purchasing.

**Figure 2.** Critical thinking investigation using a Web-based source.

The investigation is loosely structured so that students are encouraged to develop their own plan with strategies to find, organize, and analyze information and data related to the problem.

#### **Conclusions**

As educators, we need to better understand how the World Wide Web can provide opportunities for critical thinking in mathematics. This medium is providing our students with information and tools that would otherwise be difficult or impossible to access. A beautiful characteristic of the Web is the ability of Web sites to regularly change and update their content.

Consequently, countless learning opportunities become available to teachers and students because the number of pages on the Web that can be used for educational purposes grows each day.

As the number and quality of resources on the Web continue to grow, increased attention needs to be placed on instructional development so educators can optimize the potential of Web within their

classrooms. Therefore, we need to encourage teachers to use the Web sources and the strategies presented in this paper and elsewhere in order to design Web-based learning activities that align closely with their curricular and instructional goals. If more teachers become designers of their instruction, then increased opportunities for sharing creative and innovative ideas and resources across the Web will become inevitable.

#### References

- Cognition and Technology Group at Vanderbilt. (1993). Toward integrated curricula: Possibilities from anchored instruction. In M. Rabinowitz (Ed.), Cognitive science: Foundations of instruction (pp. 33-55). Hillsdale, NJ: Erlbaum.
- Confrey, J., Piliero, S. C., Rizzuti, J. M., & Smith, E. (1990). High school mathematics: Development of teacher knowledge and implementation of a problem-based mathematics curriculum using multiple presentational software. Apple Classrooms of Tomorrow (Report #11). Cupertino, CA: Apple Computer Inc.
- Dossey, J., Mullis, I., Lindquist, M., & Chambers, D. (1988). The mathematics report card: Are we measuring up? Trends and achievement based on the 1986 national assessment. Princeton, NJ: Educational Testing Service.
- Glaser, E. M. (1985). Critical thinking: Education for responsible citizenship in a democracy. National Forum: Phi Kappa Phi Journal, 65(1) 24-27.
- Glaser, E. M. (2001). Using Internet primary sources to teach critical thinking skills in mathematics. Westport, CT: Greenwood Press.
- Goodlad, J. L. (1984). A place called school. New York: McGraw Hill.
- Kozma, R. B. (1994). Will media influence learning? Reframing the debate. Educational Technology Research and Development 42(2), 7-19.
- McKnight, C. C., Crosswhite, F.J, Dossey, J. A., Kifer, E., Swafford, J. O., Travers, K. J., & Conney, T. J. (1987). The underachieving curriculum: Assessing U.S. school mathematics from an international perspective. Champaign, IL: Stipes Publishing Company.
- National Assessment of Educational Progress. (1983). The third national mathematics assessment: Results, trends, and issues (Report No. 13-MA-01). Denver, CO: Educational Commission of the States.
- National Commission on Excellence in Education. (1983). A Nation at Risk. Washington, D. C.: U.S. Department of Education.
- National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: Author.
- National Research Council (1989). Everybody counts: A report to the nation on the future of mathematics education. Washington, D. C.: National Academy Press.
- Norris, S. P. (1985). Synthesis of research on critical thinking. Educational Leadership, 42(8), 40-46.
- Paul, R. W. (1993). Critical thinking: How to prepare students for a rapidly changing world. Santa Rosa, CA: Foundation for Critical Thinking.