InterMath[1]--Professional and Cognitive Development through Problem Solving with Technology

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Abstract *InterMath is an Internet-based (http://www.intermath-uga.gatech.edu/) project with the goal of designing and implementing a series of workshops and ongoing support programs that feature contemporary applications of* ***technology*** *and* ***mathematics pedagogy*** *in the* ***middle-grades****. Technology is used to deliver the curriculum through web-based materials and to explore the mathematics using cognitive tools such as dynamic geometry software, spreadsheets, and graphing calculators. Objectives of InterMath include*

- *strengthening the middle school teacher's knowledge and understanding of mathematics,*
- *providing a support structure (on-line & in-school) to aid teachers in implementing and integrating technology tools for doing mathematics, and*
- *providing a structured inservice curriculum that follows Georgia's Quality Core Curriculum objectives as well as reform efforts expressed in publications by the National Council of Teachers of Mathematics.*

InterMath is a collaborative effort among the University of Georgia, Georgia Institute of Technology, and nine regional technology centers in the state of Georgia. InterMath, a five-year effort to design and implement a series of field-based workshops and ongoing support programs to assist both teachers and administrators in effecting mathematics reform, is funded through the National Science Foundation.

A Vision for School Mathematics

The pedagogical shifts embodied in a series of documents published by the National Council of Teachers of Mathematics (NCTM) emphasize vastly different approaches to mathematics teaching and learning than are typical in today's classrooms (NCTM, 1989, 1991, 1995, 2000). Rather than static knowledge and skills detached from both other domains and everyday events, mathematics is viewed as problem solving, reasoning, and communicating so that students are empowered to confidently "explore, conjecture, and reason logically [about the world around them]" (NCTM, 1989, p.5). This change in learning philosophy reflects a need for mathematics that is based in an information-rich and technology-based society. Learning goals should incorporate values that reflect mathematics for life, mathematics as a part of cultural heritage, mathematics for the workplace, and mathematics for the scientific and technical community (NCTM, 2000).

One way to support these goals is to provide teachers extended opportunities to experience and do mathematics in an environment supported by diverse technologies (Dreyfus & Eisenberg, 1996). The development of mathematical understanding occurs when technology is used as a cognitive tool that supports thinking, reasoning, and problem solving (Jonassen & Reeves, 1996). The use of cognitive tools such as dynamic geometry, graphing calculators, spreadsheets, and symbolic processors, can provide opportunities and experiences for exploration, developing understanding, interpreting and communicating about mathematics (see Bransford, et al, 1996; Schoenfeld, 1982, 1989, 1992; Silver, 1987). Our approach in the InterMath project focuses on using these tools to develop mathematical power--understanding, using, and appreciating mathematics.

Project Overview

Description and Goals

InterMath (http://www.intermath-uga.gatech.edu/) is an Internet-based project with the goal of designing and implementing a series of workshops and ongoing support programs that feature contemporary applications of technology and mathematics pedagogy in the middle-grades.

InterMath has two primary teacher components:

- workshops comprised of in-class portions and a "follow-along" component in which participants create curriculum for use in their own classrooms.
- an ongoing system to support teachers beyond the initial laboratory/workshop.

Intensive support is provided throughout the workshops under the close tutelage of InterMath facilitators distributed throughout the state. The site-based component focuses heavily on scaffolding in-school reform efforts. As participants near completion of the laboratory portion, they transition to the ongoing support system--a peer community to ensure continuity beyond the

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laboratory.

Project goals and objectives reflect multiple targets aimed at involving teachers and administrators in technology-enhanced mathematics reform. They link the epistemological, pedagogical, and logistical activities designed to support state and national standards. The primary goals of InterMath include:

- Promote innovative practices among workshop participants in the tool uses of technology in middle-grades mathematics teaching and learning *(teacher as learner)*.
- Support teachers in their development of instructional activities and strategies to facilitate standards-based reform and to promote meaningful learning in middle-grades mathematics classrooms *(teacher as facilitator)*.
- Establish the human and technological infrastructure needed to sustain meaningful reform of middle grade mathematics instruction (*teacher as community member*).

Professional Development

Workshop Procedures

The workshops are intended to immerse teachers in active problem solving with technology. Participants explore different concepts each class meeting by working through various InterMath investigations and writing about one in-depth. Each participant builds a personal web page using artifacts and productions from the workshops to compile an electronic portfolio. Write-ups and projects, reflecting participants' synthesis and reflection about their explorations, will be submitted electronically for workshop credit. The purpose and focus of a write-up is to communicate and synthesize investigations involving exploration, solving a problem, or working with an application. The key elements of a write-up consist of the learner's synthesis, communication, mathematical ideas, interpretation, and utility of an investigation. Final projects, focusing on a technology-enhanced mathematics investigation of the individual participant's determination, are submitted and discussed at the end of the workshop. Participant productions are placed on the InterMath website for public sharing.

The laboratory leader presents demonstrations and explanations, clarifies problems, and demonstrates alternative solutions using a projected image from the leader's workstation. In a typical session, a leader might allocate one-third of the time in whole-group mode, and during the balance of the meeting provide direct support for participants working on their projects or units, either individually or in groups. In addition to the 45-hour workshop, the 55-hour "follow-along" course will promote the use of technology to enhance mathematics teaching in their home school and to extend each participant's expertise. This additional component to the workshop promotes reflective practice among the participants, emphasizing realistic applications of technology in middle school teaching. Each participant's web page contributions includes conceptual work, projects, activities for their classroom, and links to related teaching-learning resources in order to establish a highly connected framework of resources.

Our Use of Technology

The InterMath workshop illustrates *how* and *when* technology can be used appropriately in the mathematics classroom. The literature describes two distinctly different approaches in the use of technology in classrooms: using the computer as a tool for exploration or problem solving and using the computer as a tutor that delivers instruction and provides feedback. Research on the use of computers in mathematics as a tutor and a tutee are usually not situated in problem solving environments. Most tutor-based technologies are in the form of drill and practice software, which tend to rely on lower ordered skills, and are often negatively related with student achievement (Jonassen & Reeves, 1996; Wenglinsky, 1998). Jonassen and Reeves (1996) argued that higher-order thinking occurs in environments where the student is learning with, and not from, the computer. It is this approach that InterMath promotes and intends to develop among its participants.

Many studies investigating technology-enhanced environments include an emphasis on conceptual development situations. For example, when calculators and computer software perform calculations and simplifications, teachers have more time to emphasize why something is happening, instead of focusing on algorithms (Grassl & Mingus, 1997; Heid, 1988; Maury, 1987; Palmiter, 1991). Moreover, the imperfections in calculator graphs and computations also provide opportunities for conceptual development. For example, Dion (1990) found cases where the graphing calculators' resolution caused certain functions to appear differently than they are supposed to. In addition, Goldenberg (1998) found that the graphing calculator window can provoke critical inquiry because different functions can appear to look the same if they are on different domain and range windows. Finally, Burrill (1992) noticed that the calculator has difficulty simplifying computations with extremely large and small numbers, consequently producing an incorrect answer. Used appropriately, these situations expose misconceptions and help students develop a richer understanding of the mathematics being studied.

Cognitive Development

Rationale of Workshop Activities

In designing the workshops, we have kept in mind the work of Malone and Lepper (1987) concerning the design of instructional environments that are intrinsically motivating. They have identified four sources of intrinsic motivation in learning activities: (1) gives an appropriate level of challenge, (2) appeals to the sense of curiosity, (3) provides the learner with a sense of control, and (4) encourages the learner to be involved in a world of fantasy in which learners can experience vicariously rewards and satisfactions that might not be available to them otherwise. While a workshop leader may not be able to incorporate all of these sources of intrinsic motivation into every learning activity, incorporating at least one appears to increase the likelihood that the activity will be intrinsically motivating.

Pertaining to the first source of intrinsic motivation, we have included a variety of problems on a continuum of difficulty levels. By posing challenging problems within a familiar context, teachers will develop confidence in problem solving and thus will more likely engage in the activities. The context of the problems enables teachers to safely sample and reflect on their own approaches to problem solving. The second source of intrinsic motivation is appealing to the sense of curiosity.

Activities can stimulate curiosity by introducing ideas that are surprising or discrepant from the learner's existing beliefs and ideas. While the mathematical problems posed in the workshop center on middle-school curriculum, they are more open-ended and generative than is typically seen in a traditional middle-school curriculum. Problems can be used as a springboard for ideas and investigations that participants find personally intriguing. Furthermore, teachers are able to choose among several activities in which to actually engage. They can choose activities that are most applicable to their classroom needs and relevant to their mathematical understanding. Since participants can choose activities based on their preferences, the third source of intrinsic motivation (providing the learner with a sense of control) will be reflected throughout the laboratory.

The fourth source of intrinsic motivation is encouraging engagement through fantasy. As an example of a task using fantasy, consider the following problem requiring the use of the Pythagorean theorem:

> The learner needs to calculate the distance from point *a* to point *b* in order to inform Captain James T. Kirk about how to set the transformer beam on the Federation Starship *Enterprise* so they can pick up the necessary dilithium crystals directly below on the planet's surface. Kirk only knows the distances of the ship and the crystals from a third point where his scouting party has stopped (Lepper & Hodell, 1989).

Fantasies are more intrinsically motivating when they employ characters and situations with which the learner can identify. Faced with either this fantasy-like problem or a series of abstract problems in which learners are asked to find the length of one side of a triangle, one can imagine which type of problem learners would prefer.

The philosophy permeating InterMath is that teachers must relearn mathematics in a more open-ended, generative manner so they may come to understand what reform documents intend by "meaningful learning." Furthermore, by encouraging teachers to create and modify their own curriculum units, InterMath attempts to avoid what Howson, et al. (1981) warn may be a cause for failed reform -- teachers failing to assume ownership of reform.

Workshop Content

The mathematics content and concepts of InterMath reflect curriculum that would enhance a teacher's understanding of middle-grades mathematics. The laboratory centers on the middle-school mathematics curriculum per Georgia's Quality Core Curriculum (QCC) and the NCTM Standards (1989, 2000). The InterMath curriculum is meant to engage teachers and is intended to deepen teachers' understanding of mathematical concepts related to the middle school curriculum. Thus, the investigations would likely need to be modified for use with middle school students. There are 13 units that can be used for InterMath workshops. Thirteen units are called Fraction and Decimals, Integers; Ratios, Proportions & Percents; Quadrilaterals, Triangles, Polygons, Probability, Statistics, Solids, Circles, Graphs, Patterns, Functions & Equations. The following criteria have been used to highlight recommended investigations for teacher exploration:

- Multiple cases can be investigated using technology.
- Pre-Algebra students can rely on technology to investigate the situation.
- The investigation promotes generalizability or can be used as a springboard for further exploration.
- Multiple methods can be used to explore the situation.
- Multiple solutions are possible.
- The investigation, based on middle school mathematics, is easy to start exploring.
- The investigation can be modified for use in a middle school classroom.

Next Steps

InterMath is at the end of its second year of a five year project. Over the past year, the web-based InterMath materials have been developed and tested with various teachers in the state of Georgia through teacher workshops. A community of teachers has already technology-enhanced materials for their classrooms that can be accessed on the InterMath website (http://www.intermath-uga.gatech.edu). In addition, we are in the process of developing an ongoing support system that will encourage a sustained effort among teachers in the InterMath program. For example, an online discussion forum has been built from the InterMath website so that teachers can share ideas, collaborate on projects, ask questions, and obtain technical assistance. The goal at the end of the five year project is to have a self-sustaining system of resources, tools, and people with a common goal of enhancing mathematics education using technology as a catalyst for change.

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