New ideas in teaching the Multiplication Table in Primary Mathematics Education

Mr Yeung Kim Wai Thomas & Mr Leung Hing Keung

Hong Kong Institute of Education, Maths Dept.

10, Lo Ping Rd., D3-2F-16, Tai Po, N.T. Hong Kong, tkwyeung@ied.edu.hk

Abstract This paper presents a new teaching technique which aims to lessen children's traditional pain and frustration in learning the multiplication table successfully. Most Mathematics textbooks & teachers uses the traditional ways of memorizing the multiplication table column by column. By the time children get to the last two columns, their memory of the first few columns diminishes and all kinds of errors and mix-ups will surface. This often leads to loss of interest, self-confidence and self-esteem. This article will show Mathematics teachers an effective and interesting way of teaching the multiplication table by understanding and using the communicative property of multiplication. It also makes use of number theories, 'personification' of numbers, story telling techniques and songs. The writer has confidence that Mathematics teacher will like it. The world of Primary Mathematics will be a better place if all Primary Mathematics teachers use this new teaching technique. More children in the world will then enjoy learning mathematics, feel less pain and frustration, gain more self-confidence and appreciate the wonders of Mathematics.

1. Introduction

"Teaching does not occur until learning does."(A Patterson, 1999). How can we say that we have taught children the multiplication table until we are sure that they have really learnt it? In Hong Kong, school children were taught the multiplication table in Primary two. However, the majority of students in Primary three, four or even five would still make silly mistakes in simple multiplication or division as they have trouble with the multiplication table. This often leads to frustration, low self-esteem, lack of confidence and loss of interest in Mathematics. Something must be wrong and something must be done. Why do children find it so difficult to learn the multiplication table well while they can learn computer games so fast? Were they fearful of the multiplication table? Was it because of the boring traditional method of memorizing the table column by column? Can teachers make the learning of the multiplication table more interesting and easier?

2. Elimination of almost half of the multiplication table

If we look at the whole multiplication table from 2 to 9, there are a total of 64 cells students have to memorize (Table I). However, because multiplication is commutative, the cells are symmetric along the diagonal. For example, 3x6 = 6x3, 4x7 = 7x4, 5x8 = 8x5, etc... Thus, by using the commutative property of multiplication, the actual number of cells really needed to be memorized can be reduced to 36 as shown in the diagram below (Table II). This is a major breakthrough psychologically for many students. Their eyes would glow when you tell them how much memory hard work can be reduced by knowing and understanding the commutative property!

3. The Palm-method for the elimination of column nine

Multiplication of nines was one of the most difficult for most students. By use of number theory, we can make it simple with our palms. Let students have their palms facing up and fingers stretched. From left to right, imagine the fingers being labeled from one to ten. Bending finger one (the left thumb), we have 1x9, the results is shown by the nine unbent fingers. Bending finger 2 means 2x9, the result is shown by the unbent left thumb(1) and the unbent eight fingers on the right side of the bent finger(8) as the result should be 18. 3x9 can be shown by bending finger 3. Two fingers unbent on the left side of the bent finger and seven fingers unbent on the right side of the bent finger. The result is 27. 4x9 can be shown by bending finger 4. Three fingers unbent on the left side and six finger unbent on the right-side showing the result as 36. By bending finger 5 for 5x9, 45 is shown by the four fingers unbent on the left and five fingers unbent on the right. 6x9 is done by bending finger 6. The result 54 is shown by the five unbent fingers on the left and the four unbent fingers on the right. Similarly, the multiplication for nines can be done up to 10x9 and can be remembered easily for all students. The whole column nine in the multiplication table can now be eliminated.

The above method can be explained by number theory. For numbers which are divisible by nine, the sum of their digits must be divisible by nine. For multiples of nine below 90(9x10), the sum of digits equals 9

which must be the number of unbent fingers since we all have ten fingers and that we bent only one finger each time(10 - 1 = 9).

4. The Fist-method for multiplication of 8, 7 & 6.

Have the students hold their fists. Releasing 3 fingers (the thumb, forefinger and the middle finger) while holding two other fingers bent would represent the number 8 as shown in the figure below.

(For representation of eight)

Thus, using both fists, 8x8 would show a total of 6 unbent fingers (counting both hands) and 4 bent fingers. The 6 unbent fingers multiplied by ten would represent 60. Two bent fingers multiply by two bent fingers on each fist represent 4 (2x2=4). The sum: 60 + 4 = 64 shows the result of 8 x 8. Releasing 2 fingers from the fist (thumb and the forefinger) would represent 7 as shown in the figure below.

(For representation of seven)

Thus, 7x8 would show 5 unbent fingers, three(3) bent fingers on one fist and two(2) bent fingers on the other. The 5 unbent fingers(5x10=50) represent 50 and the product of the bent fingers(3x2) is 6. The result of 7x8 is: 50 + 6 = 56. Releasing just the thumb from the fist would represent the number 6 as shown in the figure below.

(For representation of six)

For 6x8, we have 4 unbent fingers and two bent fingers times four bent fingers(2x4=8). Thus the result is 48. 6x7 is the most difficult because there are only three(3) unbent fingers showing thirty(30). Three(3) bent fingers times four(4) bent fingers gives twelve(12). The result of 42 is done by adding 30 to 12. (30+12=42)

5. Eliminating the 2 column by counting in two's. Multiplication by two's have always been the easiest for most students. Teachers can let students do counting by two's like: two, four, six, eight, ten, twelve, fourteen, sixteen and eighteen. Thus, the two-column can be eliminated.

6. Eliminating the 5 column. Most students find multiplication by five easy as the results have either 5 or 0 as the last digit. Some can even count by five's like: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, etc... Another

method is to picture in our imagination that we have many palms. One palm shows 5 (fingers). Two palms show 10 (fingers). Three palms show 15. Four palms show 20 etc... We can imagine that we get help from other people's palms.

7. The Song method for the nine remaining cells.

After the above eliminations, there are actually only 9 cells left for memorization: 3x3, 3x4, 3x6, 3x7, 3x8, 4x4, 4x6, 4x7 and 4x8 as shown in Table III. Teachers can use any singing tunes familiar to the students and put in the numbers at will.

7.1 The writer has used the famous X'mas carol: Jingle Bells for the multiplication of three's:

Familiar Tunes	Number Song
Jingle bell,	3 3 nine,
jingle bell,	3 3 nine;
Jingle all the way;	3 8 twenty four;
Oh what fun,	3 4 twelve;
It's to ride,	3 6 eighteen;
On a one-horse open-sleigh.	3 7 twenty one.

7.2 'Dashing through the snow' can be used in the multiplication of four's:

Dashing through the snow: "4 8 thirty two";
On a one horse open sleigh: "4 7 twenty eight";
4 6 twenty four;
4 4 sixteen.

7.3 Since the local mother tongue for Hong Kong is Cantonese which has nine tones in the language, the writer has composed the following rhymes for learning the nine cells in Cantonese:

Tune of Rhyme	Number Rhyme
1.1 So so la me me	4 8 3 0 2
1.2 La so me me so	3 8 2 0 4
1.3 La me la me so	3 6 1 0 8
2.1 So so la me me	4 4 1 0 6
2.2 So la me me so	4 7 2 0 8
2.3 La la me me la	3 7 2 0 1
3.1 La so la m me	3 4 1 0 2
3.2 So me me me so	4 6 2 0 4
3.3 La la-----so la	3 3 -----9

8. Personification & Story telling method

The numbers 3 to 9 can be imagined to be people like:
3 = 3rd uncle, 4 = 4th aunt, 6 = 6th brother, 7 = 7th sister, 8 = 8th nephew, 9 = 9th niece.

Now, for 3x3=9, we can say that 3rd uncle bought himself a cat with 9 lives.

For 3x4=12, we can say that 3rd uncle went with 4th aunt to see the 12 knights of the round table.

For 3x6=18, we can imagine that 3rd uncle went golfing with 6th brother & finished all 18 holes.

For 3x8=24, we can say that 3rd uncle went with 8th nephew to 'Seven-Eleven' which opens 24 hours a day.

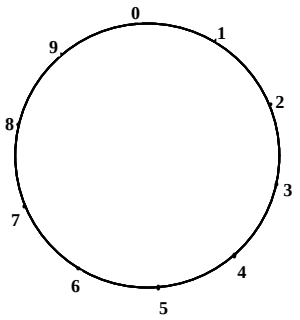
For 4x6=24, we can say that 4th aunt went with 6th brother to a gas-station which also opens 24 hours a day.

Similar incidences or short stories can be invented. Students can be asked to make up stories on their own to help their memorizing of the multiplication. Sometime, the crazier the stories, the easier they can

remember them. Students & teachers should have lots of fun doing this exercise. Cartoons and comics characters like Donald Duck, Mickey Mouse, Snoopy, Charlie Brown, Hello Kitty, Snow White, etc. are also excellent figures for representing the numbers 3 to 9.

9. A diagrammatic game for drilling what has been learned.

A circle is divided into 10 equal segments numbering from 0 to 9 as shown below:



A number from 1 to 9 is picked and its multiple is calculated. Starting from 0 in the diagram, straight lines are drawn connecting the last digit of each multiple in consecutive order until a completed figure is drawn.

10. Conclusion

High sounding researches and theories on education are mostly useless unless they can improve the effectiveness of the learning & teaching process in classroom interactions. This paper attempts to put theory into practice. Students and teachers can see for themselves how number theory can be put into action in the learning of the multiplication table. Learning can be made easier and less painful if educators can use their creative minds more often. Slight innovations can make a world of difference and learning can be even joyful. The teaching technique described in this paper is just one of the many ways in which the learning of Mathematics can be fun and that students and teachers can experience the sheer joy in the learning and teaching of Mathematics. Mathematics teachers are encouraged to innovate with their students and share their innovative ideas with their peers. They can put their quality products on the internet and spread around in the global village.

Multiplication table (Table I)

		2	3	4	5	6	7	8	9
2		4	6	8	10	12	14	16	18
3		6	9	12	15	18	21	24	27

4	**8**	**12**	**16**	**20**	**24**	**28**	**32**	**36**
5	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**
6	**12**	**18**	**24**	**30**	**36**	**42**	**48**	**54**
7	**14**	**21**	**28**	**35**	**42**	**49**	**56**	**63**
8	**16**	**24**	**32**	**40**	**48**	**56**	**64**	**72**
9	**18**	**27**	**36**	**45**	**54**	**63**	**72**	**81**

Multiplication table (Table II)

Multiplication table (Table III)

		2	3	4	5	6	7	8	9
2		4	6	8	10	12	14	16	18
3		6	9	12	15	18	21	24	27
4		8	12	16	20	24	28	32	36
5		10	15	20	25	30	35	40	45
6		12	18	24	30	36	42	48	54
7		14	21	28	35	42	49	56	63
8		16	24	32	40	48	56	64	72
9		18	27	36	45	54	63	72	81

		2	3	4	5	6	7	8	9

2									
3			**9**	**12**		**18**	**21**	**24**	
4				**16**		**24**	**28**	**32**	
5									
6									
7									
8									
9									

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