Construction of knowledge about measurement of area within Integrated Learning System (ILS) environments

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In this paper, visual representations from *RM Maths* and *SuccessMaker* instructional activities for measurement of area with non-standard units were analysed for levels of compliance with two principles for evaluating computer-based visual representations. The analysis proceeded in two stages: an epistemological analysis followed by a case study analysis. The visual representations in *RM Maths* were found to be superior to those in *SuccessMaker*.

Introduction

Visual representations (graphics plus accompanying text) are the principal tools utilised by two computer-based integrated learning systems (ILSs) (*SuccessMaker* (Computer Curriculum Corporation, 1996) and *RM Maths* (Research Machines plc, 1998-1999)) to facilitate the construction of mathematical knowledge. We have conducted a close analysis of the visual representations utilised by the two ILSs in their instructional sequences for a number of mathematics concepts. The analysis was informed by a set of seven principles for analysing visual representations within education software that were generated from a review of the research literature from the fields of mathematics education, cognitive science, computer-aided learning, computer graphic design and semiotics (see Kidman & Nason, 2000).

In this paper, we report on the component of our analysis that focused on each ILS's levels of compliance with the first two principles (see Table 1) during instructional sequences that concentrated on the measurement of area with non-standard units.

Table 1. Principles for evaluating visual representations (from Kidman & Nason, 2000)

1. Visual representations should be clearly displayed and explicitly understood by the student. This facilitates the process of stimulating relationships among the problem data and may also help students to recall knowledge and skills by making connections between prior internal representations and new situations.
2. Visual representations should enable the student to focus on the deep structural rather than surface structural aspects of the problems being investigated.

The measurement of area with non-standard units

Many students and adults appear have a limited understanding of the concept of area and to only comprehend area as formulae (Baturo & Nason, 1996) and not as a measure of the spread of surface (Foxman, Ruddock, Badger, & Martini, 1982). Many of the difficulties students have with measurement of area that have emerged from testing programs have appeared to reflect a lack of understanding of key ideas subsumed within the early stages of the teaching of area sequence (Baturo & Nason, 1996). This lack of basic understanding prevents them from applying their limited, often disjointed knowledge (Carpenter, Corbitt, Kepner, Lindquist & Reys, 1981). In particular, performance on items which involve rectangular regions has shown that problems in understanding the area concept and in calculating area and perimeter is more conceptual than arithmetical. It is due to formal knowledge not being built on existing knowledge, resulting in knowledge of set principles, but not the ability to use this in new situations (Hirstein, 1981; Foxman et al., 1982).

Another aspect that causes difficulty is lack of understanding of the multiplicative nature of the concept area (Kidman & Cooper, 1996). Many students have an additive rather than a multiplicative view of the relationship between length and area and thus tend to confuse perimeter with area. In order to alleviate this confusion, it has been suggested students' learning activities with non-standard units should involve the students in the process of producing: (1) shapes with the same area but different perimeters (see Figure 1a), (2) shapes which have the same perimeter but different areas (see Figure 1b), and (3) compound shapes by adding two shapes together to form a larger shape (see Figure 2a) *and* subtracting part of a shape from a whole shape (see Figure 2b)(Kidman, 1999; Baturo, Cooper & Kidman, submitted).

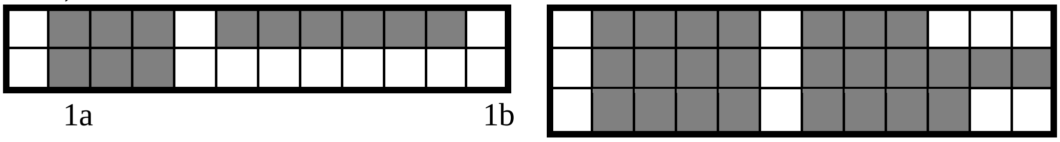

Figure 1. Shapes with same area/different perimeters (a) and shapes with same perimeters but different areas (b)

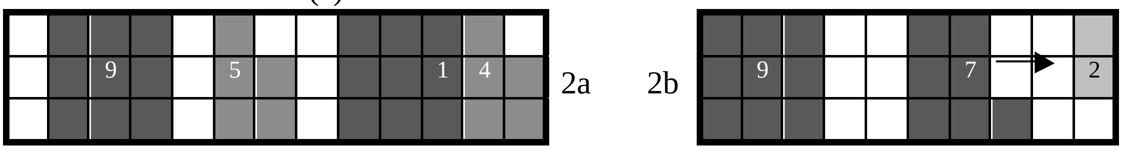

Figure 2. Compound shape created by adding two shapes together (a) and compound shape created by subtracting part of shape from whole of shape (b)

Analysis of the visual representations

The two stage analysis of the visual representations of each of the ILSs was informed by the two principles listed in Table 1. In Stage 1, an epistemological analysis was conducted. The visual representations were evaluated in terms of whether they would involve students in activities which would help them to learn how to measure the areas of shapes with non-standard units and also to gain deep structural knowledge about the concept of area (e.g., the multiplicative nature of the concept of area). This stage of the analysis was informed by the findings from Kidman (1999) and Baturo et al. (submitted) presented in the previous section.

In Stage 2 of the analysis, involved the detailed the observations of two students' interactions with the ILS's teaching sequence for measurement of area with non-standard units. One of the children (a Year 4 female student) was selected because she had very limited previous classwork on the topic of measurement of area with non-standard units. The other child selected (a Year 5 female student) had just begun the study of measurement of area with non-standard units in her classroom. In addition to observing the two children's interactions with the ILS's learning activities, we also asked the children: (1) what they thought were the main idea(s) that they were supposed to be learning from the activities, (2) how they worked out their responses to the questions, and (3) what difficulties they had encountered during the activities. These questions were asked during and immediately after each activity.

RM Maths(RM) RM has two sequences of activities for measurement of area with non-standard units within Learning Progression O: Adding and subtracting areas. In Activities O1 and O2, students investigate as two shapes are joined to form a larger shape. In Activities O3 and O4, students investigate "finding the difference between two areas by cutting away". The three tasks in Activity O1 present students with shapes composed from unit squares (see Figures 3a and b) whilst in the three tasks presented in Activity O2, the shapes are composed from unit triangles. In Activities O1 and O2, the students are initially presented with two shapes and the number of units in each shape. The accompanying text and voice-over tells the students that the number on the shapes tells you their areas (see Figure 3a). *RM* then slides the two shapes together, then slides them apart back to their original positions. The student then is asked, "What will be the area of the big shape?" After the student has recorded their answer, *RM* slides the shapes back together again. Unit squares are then iteratively placed on top of the left hand shape without any accompanying voice-over counting. When the final unit has been placed on the left hand shape, the voice-over states the total number of squares (or area of the shape). Immediately following this, the unit squares continue to be iteratively placed, this time on the right hand shape accompanied by voiced-over iterative counting. For example, voice-over accompaniment for Figure 3b as the unit shapes are placed on the right hand shape would be, "8 and 9, 10, 11, 12, 13, 14."

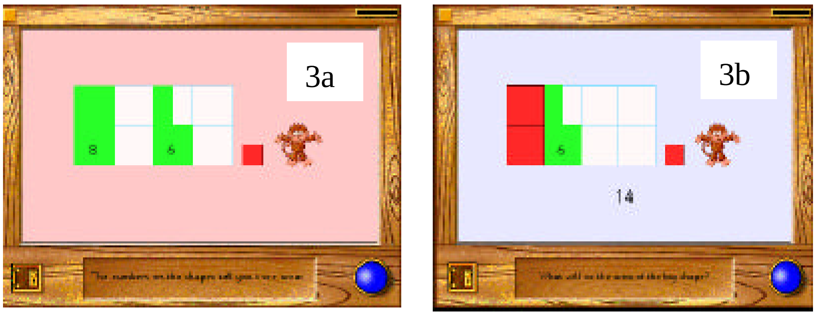

Figure 3. RM Maths O1 task

The three tasks in Activity O3 present students with shapes composed from unit squares (see Figures 4a and b) whilst in the three tasks presented in Activity O4, the shapes are composed from unit triangles. In Activities O3 and O4, the students are initially presented with two shapes and the number of units in each shape. The accompanying text and voice-over tells the students that the number on the shapes tells you their areas (see Figure 4a). *RM* then slides the smaller of the two shapes so that it becomes superimposed on the larger shape. Then it slides the smaller shape back to its original position. The student then is asked, "What area of [larger] paper will be left?" After the student has recorded their answer, *RM* again superimposes the smaller shape on the larger shape. Unit squares are then iteratively placed on the overlapping region of the larger shape (see Figure 4b). This is accompanied by a voice-over of each iteration. For example, voice-over accompaniment for Figure 4b would be, "12 take away 5 is 7, so the area left will match 7 squares. 1, 2, 3, 4, 5, 6, 7."

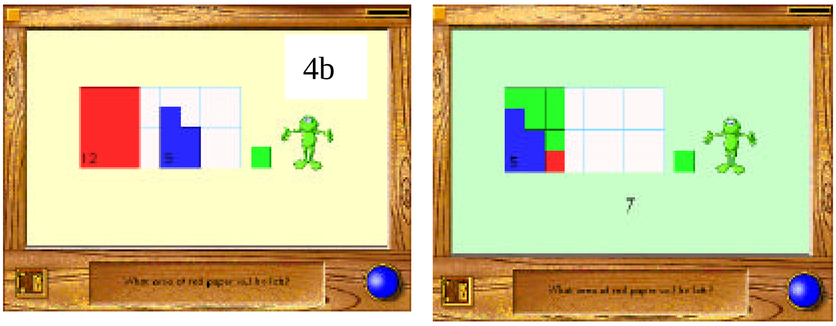

Figure 4. RM Math O3 task

Analyses of the diagrams in these tasks, indicated that it was highly likely that the visual representations in *RM* would facilitate the process of learning how to measure the areas of shapes with non-standard units and the gaining of deep structural knowledge about the concept of area. The structure and dynamic nature of the animated graphics contained within the visual representations should facilitate the establishment of cognitive connections between the visual representation and the concept of area. For example, the iterative covering of the shapes with the animations should enable students to see how the measurement of area can be ascertained by counting the number of units needed to cover the area. This is reinforced by the close synergy between graphics, accompanying text and voice-overs. The *RM* visual representations also enable students to engage in activities where "real" compound shapes are composed by either joining two shapes together to form a larger shape or subtracting part of a shape from a whole shape. Research conducted by Kidman (1999) has found that activities such as these are crucial in ensuring that students construct multiplicative rather than additive viewpoints of the relationship between length and area.

It was found that some of the animations and their accompanying voice-overs caused some problems for the younger child who had limited experiences with the measurement of area with non-standard units. For example, with the task illustrated in Figure 3a, the initial animation of joining the two shapes and then separating them together with the question, "What will be the area of the big shape?" led the child to generate an answer of 22. This was obtained by adding the 8 and the 6 from the two shapes plus the size of the region between the two shapes (8). When she got feedback that her answer was incorrect, she focused on the 8-unit region between the shapes and got an answer of 8. On the third try, she selected the area of the bigger shape and generated an answer of 8 again. When the system indicated that she was again wrong, she became most puzzled by the task. However, this puzzlement quickly dissipated when the system utilised the iterative counting on of units to explain why the answer was 14. When subsequent O1 and O2 tasks were administered to her, she confidently and correctly answered all the questions. During the post-activity interviews, it was quite evident that she had clearly understood the main idea of these tasks. The older child immediately got the idea behind these activities and experienced no problems with any of the adding of areas tasks.

The younger child also experienced some difficulties with the O3 and O4 activities. In both activities, the major purpose was to have students investigate "finding the difference between two shapes by cutting away". According to the authors of *RM*, the major purpose of this activity to help students construct the notion that the areas of compound shapes can be found by subtraction. However, the operation represented by the visual animations was not one of "cutting away". Instead, it was one of iteratively "counting on" from the area of the smaller shape until the part of the larger shape not covered by the smaller shape had been covered by the unit squares. This visually represented a "counting on" model of subtraction. After completing all six tasks in the two activities, the younger child had not constructed the notion that the area of a compound shape can be found by subtracting a part of a shape from the whole shape. The older child experienced none of these problems. She was able to utilise her previous knowledge about visual representations of area and the measurement of area with non-standard units to make sense of the activities and successfully complete all of the tasks in both activities. She also had multiple models of subtraction - take away, missing addend and comparison. This particularly helped her with comprehending the major purpose of the Activities O3 and O4. During the post-activity interviews, it became quite evident that the younger child's knowledge about subtraction was limited to "take away" and she did not relate the process of counting on modelled in these activities with the operation of subtraction. Her limited knowledge about subtraction together with the ways in which the visual representations scaffolded her solution process for each of the tasks therefore seemed to prevent her from abstracting from the main idea of the O3 and O4 activities, namely that the area of a compound shape can be found by subtracting a part of a shape from the whole shape.

SuccessMaker (SM) The *SM* activity for measurement of area with non-standard units (Activity ME2.58) consisted of a one screen presentation with a compound shape task where students are asked to join a rectangle consisting of one column of two unit squares to a rectangle consisting of three columns of two unit squares (see Figure 5). The students are asked the question, "How big together?" If they correctly answer 8, they get immediate verbal feedback that they are correct. If they answer incorrectly, they are given another go. Then if they again fail, *SM* verbally tells them the correct answer before proceeding randomly onto a new, possibly unrelated topic. The epistemological analysis of the visual representation in this learning activity indicated that it was highly unlikely that the visual representation would facilitate the process of learning how to measure the areas of shapes with non-standard units, and the gaining of deep structural knowledge about the concept of area. The structure and static nature of the visual graphics and the ambiguous nature of the accompanying language contained within the question, "How big together?" makes it problematic whether students would be able to make a cognitive connection between the visual representation and the concept of area (Principle 1). It seemed more probable that many students would perceive the activity as another addition task. The static nature of the visual representation also would prevent students from engaging in activities which research conducted by Kidman (1999) and Baturo et al. (submitted) has found are crucial in ensuring that students construct multiplicative rather than additive viewpoints of the relationship between length and area, namely activities where students produce with unit squares: (1) shapes with same areas but different perimeters, (2) shapes with same perimeters but different areas, and (3) "real" compound shapes by joining two shapes together to form a larger shape *and* subtracting part of a shape from a whole shape.

Most of the limitations of the visual representations in the *SM* activity identified during the epistemological analysis were confirmed during the observations of the two children's interactions with the activity. This was especially so in the case of the younger child. In her first attempt, she gave seven as her answer to the question, "How big together?" She also gave an incorrect answer the second time and only seemed to understand the nature of the problem after *SM* gave her the correct answer. When asked what she thought the purpose of the activity was after she had completed it, she said, "How big area of small blocks gunna be." It was only when she looked more closely at the two rectangles and identified the number of unit squares in each column, she was able to "see" that there were 3x2 + 1x2 altogether.

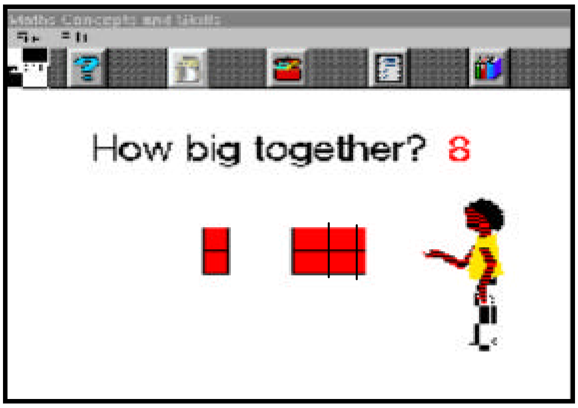

Figure 5. SM Activity ME2.58

The older child who had had experience with measurement of area with non-standard units immediately recognised that the visual representation was presenting an area problem. She indicated that the *SM* visual representations were similar to those she had seen before on measurement of area textbook and worksheet activities presented to her in school. During the interview, this child pointed out that the visual representation had an ambiguity that "could be confusing to younger kids." If putting the two rectangles "together" meant side-by-side, then the correct answer was eight. However, if "together" meant superimposing one rectangle on another, then what you would see would be six unit squares. She pointed out that many mathematics software packages with animation moved shapes together by superimposing rather than placing them side-by-side; she therefore reasoned that many young students would incorrectly give an answer of 6.

Discussion and Conclusions

The results from the analysis of the visual representations utilised by *RM* and *SM* in their measurement of area with non-standard units activities are summarised in Table 2.

Our epistemological analysis of the *RM* visual representations indicated that they highly complied with Principle 1. They displayed the topic being investigated and presented clear and explicit instructions that were both well structured and sequenced. Furthermore, the animations linked the measurement area activities to the operations of addition and subtraction. However, data from the case study analysis indicates that the levels of compliance of the visual representations with Principle 1 were heavily predicated by a child's prior knowledge not only about the topic being taught but also by other mathematical concepts and processes embedded in a task. The younger child's lack of prior knowledge interfered with her understanding of the task. Because of this, we modified our initial evaluation from high compliance to moderate compliance.

Our epistemological analysis of the *RM* tasks indicated that because they would enable students to engage in the types of activities (producing compound shapes with unit squares by joining shapes or by "cutting away" from a whole shape) enabling them to focus on the deep structural aspects of measurement of area with non-standard units, we initially evaluated them as having high compliance with Principle 2. The evaluation was altered to moderate compliance because the younger child lacked background knowledge about (1) measurement of area with non-standard units and (2) missing addend subtraction. Therefore, she was unable to focus on the deep structural aspects of measurement of area with non-standard units.

Table 2. Analysis of visual representations

Principle #	*RM Maths*	SuccessMaker
1	Moderate	Low
2	Moderate	Low

Our epistemological and case study analysis of the *SM* visual representations were consistent. The limitations identified in both types of analysis resulted in an evaluation of low compliance with Principles 1 and 2. The visual representations in *SM* Activity ME2.58 did not present clear and explicit instructions nor facilitate the construction of deep structural knowledge about the topic.

We have conducted a close analysis of the visual representations utilised by the two ILSs in their instructional sequences for a number of common topics other than measurement of area with non-standard units. Our findings in these other studies have produced findings identical to those reported in Table 2. Based on our cumulative findings from all of these studies, it would seem that *RM* would have greater success, than *SM*, at facilitating the construction of mathematical knowledge, and this success can, in part, be attributed to the higher quality of its visual representations. Another consistent finding that has emerged from this and related studies is that when one is evaluating the levels of compliance of an ILS's visual representations with Principles 1 and 2, one needs to carefully consider the prior knowledge students bring to ILS activity. This finding is consistent with Mayer and Gallini (1990) who found the cognitive conditions for effective illustrations includes the students' prior knowledge.

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