MODELLING WITH ECONOMICS

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Abstract The paper concerns the methodology of modelling. Focusing on qualitative, visual and analytic modelling and their symbiosis, microeconomic models are presented to describe realistic situations taken from "everyday" reasoning. Typical models of economic behaviour are used to point out important aspects of practical didactic of mathematical modelling. Relevant mathematical problems to solve are mentioned.

Introduction

In the sequel, a *model* is to be understood as a simplified representation of a real situation (analogous to [1], [2] among others]. A *mathematical model* (briefly a *model*) is a model created using mathematical tools. Accordingly, by *mathematical modelling* (briefly *modelling*) we understand a procedure of creating models. From the viewpoint of practical didactic a certain classification of modelling types has been suggested, namely *quantitative, qualitative, visual, analytic and abstract* (for the details see [10]). All the mentioned types are not sharply separated, they exist in a fruitful symbiosis. Abstract and quantitative modelling are rather out of scope of our further considerations. Traditional quantitative modelling works with concrete data (mostly numbers) and the resulting quantitative model is a certain mathematical structure (equation, function, etc.). Unlike this, qualitative modelling works with variables (or quantities), which are not represented by their number characteristics, but by their sign characteristics. Consequently, employing qualitative modelling, we are able to grasp trends of behaviour. In *qualitative modelling* , in order to describe a time trajectory of some variable , four basic "qualitative" values are considered - positive(+), negative(-), zero(0) and unknown(*). Each variable X given by a function x (t) of time t is represented by a (n+1)-tuple [X, DX, D^2X,..., D^nX] , where X is the qualitative value of x (t), DX is the qualitative value of derivative of x (t),..., D^nX is the qualitative value of n- th derivative of x (t) (usually n is at most 2 and it is naturally supposed that all mentioned functions do not change their sign over given interval); then (n+1)-tuple [X, DX,..., D^nX] is said to be the *qualitative model* (*of X*) . Quantitative models may be transformed to qualitative ones by means of so called *degradation* (see [3]). *Visual modelling* is based on shape characteristics. The result of visual modelling, *a visual model* , is a geometric structure (mostly a curve) of given properties ; required properties are usually formulated by a mathematical (or at least by a semantically correct) statement (or declaration). The resulting structure is depicted graphically (its analytic form is not essential at the moment) . Apparently, visual modelling grasps the shape ; it is more of qualitative than quantitative nature. The use of visual tools in mathematics instructing is constantly spreading and is connected with the problems of *visualization* in mathematics (see [5]). *Analytic modelling* is based on analytic characteristics. The result of analytic modelling, an *analytic model*, is a mathematical structure (mostly an elementary function).

To Methodology of Modelling, Microeconomic Models

The role of modelling in mathematics instructing is very diverse (see e.g. [1], [2], [5], [6], [7], [11]). Several research studies have been carried out dealing with the impact of different types of modelling on the mechanism of understanding or the ability of creating models, respectively. As usual, the "flow diagram" of modelling starts with some verbal declaration describing the situation and ends with the "solution" as a mathematical model. The result is mostly expected in an analytic form, i.e., it is an analytic model. This final point claims some preceding stages, particularly those based on visual modelling, resulting in a visual model. It is apparent, that the order of employing various types of modelling plays an important role and their influence on the final result is different. As to visual modelling, our conclusions (in accordance with our experience and initial hypothesis) indicate, that the process of finding the model and accordingly the process of understanding is significantly sensitive to visual modelling. In more general form, the understanding is elastic to visualization. Hence, emphasis is layed on tools of visual modelling in didactics. In the sequel we will present concrete cases showing practical realization of the mentioned principles. As a side effect, some mathematical problems , which could immediately flourish didactics, are formulated.

We will concentrate to modelling of dependencies formulated verbally by means of declarations (statements). Verbal declarations are of particular didactic value. We will focus on higher educational levels (engineering mathematics courses), although in a simplified fashion it may be of use also on lower educational levels. With a view to teaching aims we will restrict to the situations, which can be expressed as functional correspondence between two sets of numbers (usually reals). So, we will silently assume, that the relationship between variables under consideration is functional.The resulting models will be functions of one real variable. In particular, visual models will be sets of points in the plane forming continuous curves, analytic models will be elementary functions. So, continuity will be typical property of such models. It tackles one of crucial points of modelling, namely the problem of discreteness and continuity. More precisely, continuous models of real situations with discrete character. Handling (mainly calculating) nearly exclusively elementary functions at secondary schools, students brought in mind the perception of the function concept as continuous. On the other hand, they hold continuity for ideal, i.e. practically non existing. In engineering courses, they encounter real actions that are evidently more continuous rather than discrete (e.g. electricity), but also those that are very far from being continuous (e.g. economics). To our experience, it claims no small effort of educators for the students to cope with. The problems may effectively be overcome by doing modelling. When employing modelling, mathematics instructors should also realize that (see also [1]) students must face a new phenomenon - *heuristics*. Up to now, students were led to find a unique solution (or for example in geometry small finite number of solutions) of a problem using step by step defined process (algorithm) of deterministic nature. Therefore it is desirable to lay emphasis on different models of a given problem (real situation). Moreover, we respect important principle, that the given problem is theoretically simple and currently comprehensible. From this viewpoint the problems taken from everyday "economics" seem to be useful.

Case 1. The general standard version of the law of decreasing demand in economic theory is usually given in the form of statement S : ”*as the price of a good rises, demand falls*”. The validity of this law (under ”normal” conditions) is quite convincing. The task is to find : first visual, then qualitative and in the end analytic model of statement S. The initial point in the reasoning of a student is to realize, that statement S describes the relationship between price P and demand Q. Which of the variables P or Q is independent or dependent, respectively, and why ? It is important to refer to the fact, that the change of demand Q (consumer´s behaviour) is the cause of the change of price P, so P is independent and Q dependent variable. Now, the student should be able to express it formally as a function of one real variable Q = f(P), where f is called a *demand function*. The next step is to invent further properties of demand function f, given by statement S to approach visual model. What is the domain of f ? Apparently P and Q attain nonnegative (mostly positive) values. Further, from statement S it follows (why?), that f must be decreasing . At this stage, there is enough information to attempt to draw a graph of f – any decreasing function in the first quadrant, which is the visual model of S. Students usually draw a decreasing convex curve with asymptotic behaviour to both axes (like in economic literature) (Fig.1). In case that a student draws a decreasing concave curve, he must immediately solve the question – what happens with values of Q for very small and very large values of P ? It evokes very interesting and beneficial eco-mathematics analysis. If concavity of f is to be preserved in the first quadrant, then there must exist P_0 , Q_0 with $Q_0 = f(0)$ and $0 = f(P_0)$ (Fig.2).

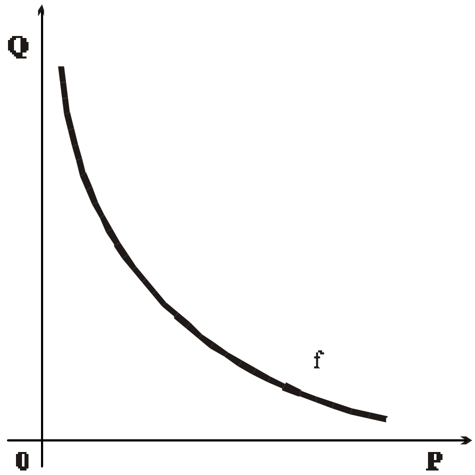

Fig.1

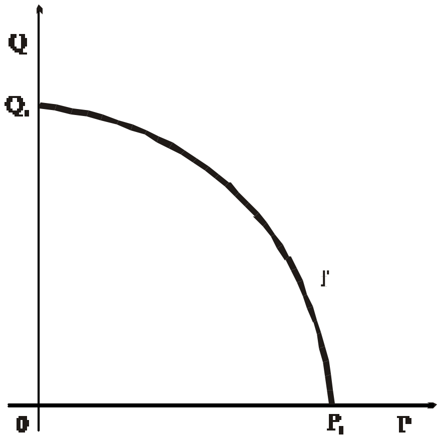

Fig.2

If there are no Q_0 or P_0 of the mentioned properties, then f must change concavity to convexity at some points. Then there are several possibilities of behaviour of f, two of them are depicted in Figures 3 and 4. For all existing alternatives economic analysis may be performed concerning real possibility of corresponding behaviour of consumers. For example, what is the economic explanation for inflexe points P_1, P_2 like?

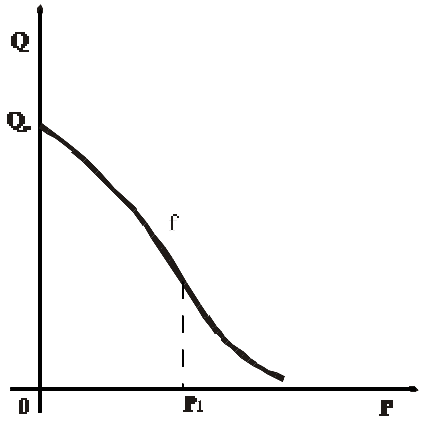

Fig.3

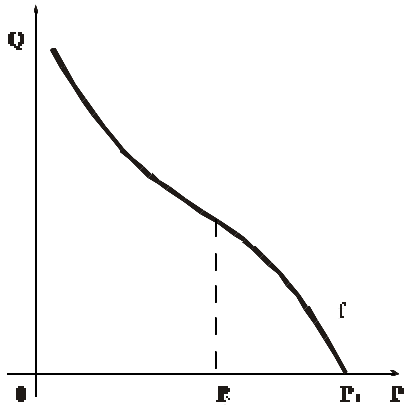

Fig.4

Now, with the help of visual model, it is simple to state qualitative model of S - the triple [+, - , +] for basic model (Fig. 1) , the triple [+, - , -] for the model depicted in Fig. 2 (with a slight modification of f from positivity to negativity) and the couple [+, -] for models depicted in Figures 3, 4 (also + meaning nonnegativity). As to analytic model of S, a student must be familiar with the shapes of curves - graphs of elementary functions. He may offer a hyperbol-type curve $Q = a/P$, $a > 0$ for the model in Figure 1. For the model in Figure 2 a parabola $Q = aP^2 + bP + c$, $a < 0$, or with "drastic" simplification a line $Q = aP + b$, $a < 0$. Such function may serve as analytic models of S. To find analytic models corresponding to visual models in Figures 3 and 4 slightly more mathematics is required.

***Problem 1** :* To characterize classes of analytic models of typical demand curves within the class of elementary functions.

Remark : There is a historical practice in economic literature to depict price P on vertical axis and the remaining variable on horizontal axis, which is often confusing for the students in case that price P has the role of independent variable. In Case 2 we keep this practice.

Case 2. Let us consider basic version of statement S (as given in Case 1) , firstly with the amendment (a) " a good under consideration is non-essential "- denoted by S´and secondly with the amendment " a good under consideration is essential-unavoidable"-denoted by S ´´. In both cases (due to economic theory) the structure of the consumers´ group changes at extreme high prices (similarly at extreme low prices - for the details see [4]). In (a), there is naturally such price P_0 at which there is no demand. Then visual model of S´ is of the shape as depicted in Figure 5.

Qualitative model of S´ is [+, - , +]. For analytic model of S´ we have at the disposal for instance function $P = P_0 e^{-aQ}$, $a > 0$. In (b), visual model may be of the shape as depicted in Figures 1 or 6. Qualitative model of S´´ is [+ , - , +] and for analytic model of S´´ we may choose for instance function $P = c(b + \log_a Q)$, $0 < a < 1$, $c > 0$.

Problem 2 : To characterize classes of analytic models of S´ , S ´´ within the class of elementary functions.

Both problems 1 and 2 may be concluded by constructing software-generators of associated analytic models.

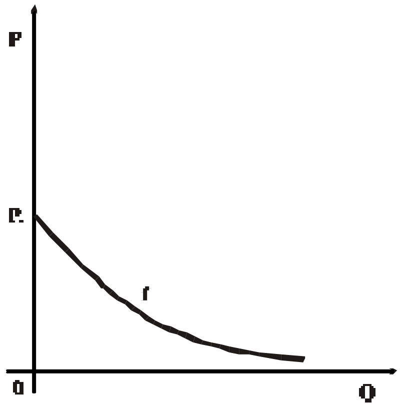

Fig.5

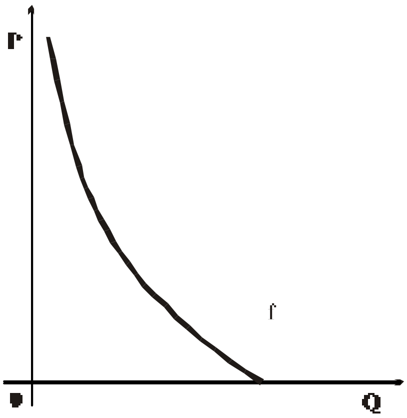

Fig.6

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