VIRTUAL MANIPULATIVES IN THE K-12 CLASSROOM

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Abstract: Innovations in technology, along with the growing prevalence of the Internet and its increasing availability in classrooms and homes throughout the world, have created a new class of manipulatives, *virtual manipulatives*. These "virtual manipulatives" offer a new, enhanced approach for teaching and learning mathematics using manipulatives and computers, one that combines the useful properties of existing computer manipulative programs while overcoming many of their disadvantages. Yet very little is known or written about them. The purpose of this paper is to establish a working definition of virtual manipulatives and discuss unique properties that make them particularly useful for the K-12 classroom.

What are Virtual Manipulatives?

No accepted, standard meaning or definition for the phrase *virtual manipulative* currently exists. There is, however, some circumscription in the use of the phrase *computer manipulative*. According to Clements and McMillan (1996) "computer manipulatives" are computer programs that allow the user to manipulate representations of concrete objects, such as base-ten blocks or geoboards, on a computer screen. In addition to computerized versions of concrete manipulatives, their examples of these "computer manipulatives" also include spreadsheets, databases, and Logo—items which have never functioned either traditionally or physically as "manipulatives." Unified by the computer screen interface, both static and dynamic representations, traditional and untraditional, qualify as "manipulatives." If one were to use this criteria, the only factor that distinguishes "virtual manipulatives" from "computer manipulatives' is their availability: "computer manipulatives" are primarily computer programs that must be purchased; virtual manipulatives are available to all Internet users.

On the World Wide Web there are essentially two types of representations that are being called "virtual manipulatives"—those that are *static* visual representations of concrete manipulatives, graphs, or worksheets, and those that are *dynamic* visual representations of concrete manipulatives. Since one of these is far more powerful and has much greater utility and potential for teaching, it is important to distinguish between them. And it is equally important to establish a name and definition that uniquely describes these dynamic images so as to avoid confusion.

Static, visual representations are essentially "pictures." As such, they provide the sorts of visual images ordinarily associated with pictures in books or drawings on an overhead. Though some of these representations "look" like concrete manipulatives, they *cannot* be manipulated in the ways a concrete manipulative can be manipulated. They may be virtual in the sense that they are available on the Internet, yet they are not manipulable (able to be moved, rotated, etc.). In addition, some are not objects at all, and more closely resemble a math worksheet or activity posted on the Web. These static, visual representations are *not* true virtual manipulatives.

On the other hand, dynamic, visual representations of concrete manipulatives are essentially "objects." They also present a computer screen version of the sorts of visual images associated with pictures in books or drawings on an overhead. But, unlike their static counterparts, these dynamic visual representations *can* be manipulated, just as a concrete manipulative can. That is, just as a concrete manipulative may be manually slid, flipped, turned, and rotated, the dynamic visual representation may be manually slid, flipped, turned, and rotated (using a computer mouse), as though it were a three-dimensional object. *This* kind of visual representation is truly a "virtual manipulative."

The real teaching and learning power of virtual manipulatives lies in their ability to be manipulated. True virtual manipulatives allow more than simply a viewing of objects on the computer screen. They allow increased engagement, forcing the user to interact with dynamic objects. Through this interaction students have opportunities to make meaning and see relationships as a result of their *own* actions. And it is this interactive engagement with a dynamic representation that is the key to the knowledge construction process. We believe the term "virtual manipulative" should be restricted to describing only those sites that offer this interactive capability and should distinguish such sites from virtual math sites where pointing and clicking results in the computer providing an answer in visual or symbolic form.

In our view, a "virtual manipulative" is best defined as an interactive, Web-based, visual representation of a dynamic object that provides opportunities for constructing mathematical knowledge (Moyer, Bolyard, & Spikell, in press). Concrete manipulatives commonly used in schools (i.e., pattern blocks, tangrams, Cuisenaire rods, fraction bars, geoboards, and geometric solids) currently inform the design of many computer-based representations. While their availability through the Web may make them *virtual*, their interactivity—that is, the user's engagement and control of the movement of these objects—combined with opportunities to discover and construct mathematical principles and relationships makes them *virtual manipulatives*. Virtual manipulatives are created and placed on the Internet in the form of "applets" – that is, a smaller version of an application program. Presently a computer mouse is the most common way to manipulate the representations, but technological advancements (such as voice commands and infrared signals) should offer new ways of moving and interacting with virtual manipulatives.

Unique Properties of Virtual Manipulatives for School Mathematics

Virtual manipulatives and virtual manipulative Web sites promise to be great assets for classroom use. Already they can link iconic and symbolic notations, highlight important aspects or features of individual manipulatives, provide links to other resources on the Web, and record and store user's movements and other work tasks. They also provide as much engagement as physical manipulatives (Dorward and Heal, 1999).

One especially appealing advantage is their availability. Because virtual manipulatives are accessible over the Internet, they offer teachers and students inside the wired classroom free and unlimited access to interactive manipulative experiences (see examples at the National Library of Virtual Manipulatives, http://matti.usu.edu/nlvm/). Additionally, because they are Web-based, manipulative experiences are no longer confined to the classroom: parents and students can access them from their home computers. Teachers who may be reluctant to send concrete manipulatives home for student use may be more likely to give assignments to students who have access to the virtual manipulatives through their home computers.

Another unique feature of virtual manipulatives is their ability to be altered. For example, because some virtual manipulative sites allow users to color parts of the visual object, students could add color for one-to-one correspondence when counting objects. Other sites make it possible for students to add lines or points to figures, which would be useful for counting or marking the sides of a polygon to determine shape or perimeter. Students could also mark edges, vertices and faces in order to highlight these aspects on platonic solids (see, for example, http://www.illuminations.nctm.org/imath/3-5/GeometricSolids/GeoSolids1.html).

For students in the upper elementary and middle school grades, the use of concrete/physical manipulatives is sometimes viewed as "playing with blocks," something "little kids do." With virtual manipulatives this connotation is removed because students view the use of the computer as more sophisticated than the use of the manipulatives in their physical form.

When using concrete manipulatives, teachers sometimes express concerns that they do not have enough manipulatives for everyone in the class and that it takes time for distribution and clean-up of the materials. With virtual manipulatives, students control the number of blocks available with the click of a computer mouse (see for example the pattern blocks at www.arcytech.org/java/patterns/patterns_j.shtml). Putting away or cleaning up virtual manipulatives is as fast as the click of the mouse on a recycling bin or a broom icon (see, for example, the base-10 blocks at www.arcytech.org/java/b10blocks/b10blocks.html).

Virtual Manipulative Web Sites

Distinct differences in interactivity exist among sites on the Internet labeled as "virtual manipulatives." The best sites are those with a variety of dynamic features and applications to different content areas. Many of these sites allow users to extensively manipulate and even alter the representation in order to explore a concept. Yet some sites that claim to offer virtual manipulatives permit little to no user interaction. As with virtual manipulatives, teachers looking for interactive Web sites where their students can construct mathematical knowledge must learn to distinguish between which sites contain virtual manipulatives and which merely present math lessons and activities in a virtual format. The important factor to consider is whether or not the user can actually *move* the object, and whether, through that movement, the user has opportunities to explore relationships and construct mathematical meaning.

Utah State University's National Library of Virtual Manipulatives (http://matti.usu.edu/nlvm/index.html) contains one of the most extensive collections of virtual manipulatives currently available on the Web. Funded by the National Science Foundation, this site features a variety of virtual manipulatives and tutorials, mostly in the form of Java applets, intended primarily for mathematics instruction at the elementary and middle grade levels (http://matti.usu.edu/nlvm/nav-asp/projectinfo.html).

The National Council of Teachers of Mathematics (NCTM) recently launched an electronic version of the Principles and Standards for School Mathematics (NCTM, 2000). The new Standards include Electronic Examples (under the Standards) and I-Math Investigations (in the Illuminations section of Teacher Resources) with interactive activities organized by grade bands (K-2, 3-5, 6-8, 9-12) and keyed to specific standards (http://standards.nctm.org). Several of these examples include virtual manipulative experiences for users.

Virtual Manipulatives for K-12 Classroom Use

The many virtual manipulatives currently available for classroom use are too numerous to mention. Therefore, we will focus on one particular virtual manipulative in this section, the virtual pattern blocks, and describe a variety of ways classroom teachers might use this manipulative for mathematics instruction at various levels.

The virtual pattern blocks consist of the following shapes: regular hexagon, parallelogram, rhombus, square, equilateral triangle, and trapezoid. Two Web sites that feature the pattern blocks are found at the National Library of Virtual Manipulatives site (http://matti.usu.edu/nlvm/nav-asp/navigation.asp?g=&t=2) and at Jacobo Bulaevsky's site (http://www.arcytech.org/java/). A variety of mathematics concepts can be explored using the virtual pattern blocks.

In the early grades children explore the concepts of repeating and growing patterns. At the virtual pattern blocks site, children can create various patterns by clicking on the blocks, sliding them with the mouse, and rotating the shapes to a selected position. Students can arrange the blocks to create tessellating patterns that have no spaces or gaps between the blocks.

In the upper elementary grades students can use the blocks to create a variety of regular and irregular polygons. Polygons constructed from the square block pieces can be explored to find area and perimeter by changing the color of the blocks and highlighting the units on the edges of the figures to count the number of units in the perimeter. Questions can be posed such as: "Find a figure that has an area of 5 units and a perimeter of 12 units" or "How many different perimeters can you create with an area of 5 units?" Students will discover many relationships between the perimeter and area of figures through these types of explorations.

The pattern blocks can also be used to explore fraction relationships. For example, if the hexagon is equal to 1, then 2 trapezoids placed together will also equal 1, and therefore the relationship of the trapezoid to the hexagon is that the trapezoid is "half" of the hexagon. Using these same relationships students will find that the rhombus is equal to 1/3 of the hexagon, and the triangle is equal to 1/6 of the hexagon. Different blocks or combinations of blocks can be identified as the *whole* and other fraction relationships can be explored.

In the middle grades, students might explore the properties of the four quadrilaterals in the virtual pattern blocks (square, parallelogram, trapezoid, and rhombus). To identify attributes of these quadrilaterals, students might be asked to complete a task in which they construct one of the quadrilaterals – such as the parallelogram – using more than one block or using two or more blocks of different shapes. These investigations can be extended to examining the properties of similar figures.

Virtual Manipulatives as Representations

In the National Council of Teachers of Mathematics Principles and Standards for School Mathematics (2000), representation is identified as one of the important processes in the teaching and learning of mathematics. Virtual manipulatives can be thought of as one of the many representations teachers have at their disposal for teaching mathematics. Being able to teach mathematics effectively requires educators to choose the kinds of representations that will support meaningful mathematics learning in classrooms.

The kinds of external representations (Pape, 2001) the teacher uses during instruction for demonstration or for student exploration have a direct impact on students' learning of mathematics. If teachers choose a dynamic representation such as a virtual manipulative, rather than one that is static, the kinds of cognitive abstractions students develop may be markedly different. Because it is advantageous for students to internalize their own representations of mathematics concepts, interacting with a dynamic tool during mathematics experiences may be much more powerful for internalizing those abstractions.

Conclusion

As the number and sophistication of virtual manipulatives increases, their potential use for school mathematics is limited only to the creative ideas of the teachers and students who use them. By using these sites during classroom instruction and sharing their insights and experiences with others, teachers can contribute significantly to our growing understanding of virtual manipulatives and how best to use them with today's technologically savvy students. The wireless classroom where students all have laptops and access to their own collection of virtual manipulatives is just around the corner.

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