WORLDVIEW THEORY AND THE CONCEPTUALISATION OF SPACE

Marc Schäfer, Rhodes University, Grahamstown, South Africa, M.Schafer@ru.ac.za

Introduction

The aim of this paper and presentation is to share with you a research project that I am currently involved in which looks at how young South African learners conceive of space. This study was inspired by the work of Kearney (1984) and Cobern (1991, 1993, 1994, 1997) who assert that an individual's world view, the epistemological macro-structure, is fundamental to the construction of meaning. The study incorporates world view theory, which has its origin in and is central to cultural anthropology (Kearney, 1984), within a Mathematics Education context in general and spatial conceptualization in particular. It is hoped that this research will contribute towards an alternative and fresh framework within which to position spatial learning and the construction of meaning about space and shape.

The incorporation of learners' epistemological macrostructures into the classroom practices will have implications for curriculum development in terms of recognizing prior learning and experiences.

Background

It is a common perception that spatial understanding is fundamental to the understanding and "appreciation of our inherently geometric world" (NCTM, 1989) in general and to mathematical cognition in particular. Battista *et al.* (1982) recognize this by suggesting that spatial thinking is an important aspect in mathematical performance. Although much research has focused on the relationship between spatial ability and mathematics achievement (Bishop, 1980), there is little understanding as to the role of spatial ability in the cognition process. Educators and policy makers the world over have intuitively recognized the role of spatial conceptualization and most mathematics curricula will insist on incorporating aspects of spatial development, usually in the form of formal Geometry.

There is little consensus about the concept of space, yet most western mathematics curricula are firmly rooted in a Euclidean paradigm which sees space as a measurable and rational system. Euclidean space is seen as ordered, where shapes are measurable and positioned in a definite manner (Heath, 1956). There is a sense of geometry – "the branch of Mathematics concerned with the properties and relations of points, lines, surfaces and solids; the relative arrangement of objects or parts" (Oxford Dictionary, 1995). Newtonian space is consistent with Euclidean space in terms of its tangibility. It affirms the "reality of space" (Gardner, 1999) and maintains that space exists independently of the subject's awareness. Kantian space on the other hand suggests that "space (and time) are not features of absolute reality but only forms of sensibility, elements of our subjective cognitive constitution, and that everything that has spatial properties – all the objects of our experience – are mere appearances as opposed to things in themselves" (Gardner, 1999). This implies that space should not be conceptualized in terms of objective features only. In Kant's concept of space, the space is independent from its content. This means that one's subjective understandings and experiences form an integral part of one's overall perception of space. Kant suggests that the order we find in nature is the order that exists in our minds, an order which is embedded or reflects the own structure of mind (Stumpf, 1994; Want and Klimowski, 1996).

There is a tendency in current thinking to embrace a broader view of geometry. The post-1994 curriculum for Mathematics in South Africa for example, sees space and shape within a context of social experiences. One of the specific outcomes of the mathematics curriculum suggests that learners need to be able to "describe and represent experiences with shape, space, time and motion, using all available senses" (Republic of South Africa, 1997). This apparent shift is consistent with a global epistemological paradigm move towards recognizing that cognition is an active and complex process of social interaction. The core epistemological principles that underpin this shift can be summarized as follows:

- Knowledge is actively constructed by the learner, not passively received from the environment (Matthews, 1992);
- The function of cognition is adaptive (Von Glaserfeld, 1990, cited by Taylor *et al*., 1993). This process organizes one's experimental world; it does not discover an independent, pre-existing world outside the mind of the learner (Matthews, 1992).
- The cognition process is not personal and insular, but one that relies on social interaction in general and, according to Vygotskian thought, on language in particular.

The notion that cognition is an active process and not a passive intake of information expresses a clear contrast with traditional didactic and expository teaching based on the metaphor of "teaching as transmission of knowledge" (Taylor *et al.*, 1993:3). In his earlier writing, Von Glaserfeld (1988), often labeled as a proponent of 'radical constructivism', makes the observation that in traditional theories of knowledge the activity of 'knowing' is taken as a matter of course, "an activity that requires no justification and functions as an initial constituent" (Von Glaserfeld, 1988:208). The learner is seen as a passive recipient of knowledge with very little say in his/her cognitive destiny. The "knowing subject" is conceived of as 'pure' entity "unimpeded by biological and psychological conditions" (Von Glaserfeld, 1988:208).

More recent researchers (Kuiper, 1991; Ernest, 1994) widely accept that learners develop understanding, ideas and beliefs about the natural world outside the 'formal' learning environment, and as Driver and Oldham (1986:105) suggest, "long before they are formally taught". Further, it is asserted that learners develop a sense about the world, albeit a 'wrong' sense sometimes, and move towards a cognitive perspective based on their past experiences. Driver and Oldham (1986) therefore maintain that an individual's knowledge is not considered a set of discrete 'bits' but a series of structures, and learning involves the development and change of such structures. In my view, learning occurs over a continuum and develops on the basis of continued reflection and evaluation within one's own experiential context - it is a cyclical process of continuous modification and adaptation, and I therefore concur with Taylor *et al's.* (1993) deduction that the learner's new understandings can be formed on the basis of his/her own prior knowledge and experience. The process of making sense of one's experience, the genesis of the individual's knowledge, can be a result of the individual's "purposeful and subjective interpretation of his/her experience of the physical and social world" (Taylor *et al.,* 1993:4).

Ernest (1992) emphasizes the 'social' dimension in the cognition process and suggests that 'social constructivism' regards the individual subjects and the realm of the social as "indissolubly interconnected". It is the social reality that creates or constrains the shared experience underlying physical or social understanding. This development in constructivist thought is strengthened by the work of Vygotsky who sees language as a key component in the cognition process. When considering the formation of concepts, Vygotsky (1986:107) suggests that "real concepts are impossible without words, and thinking in concepts does not exist beyond verbal thinking. That is why the central moment in concept formation, and its generative cause, is a specific use of words as functional tools." The above assertion appears dogmatic in its implication that the cognition process cannot be a personal and insular one. This in my view needs to be questioned, as it assumes that **all** cognition is based upon social interaction.

The implications that constructivism has for spatial conceptualisation are important, but Lerman (1992) warns against merely replacing one rhetoric for another. Although the debate around constructivism has succeeded in questioning and replacing the transmission metaphor with one that sees students constructing their own knowledge, the practical implications for the teacher need careful consideration. Lerman (1992) laments the emergence of terminology such as 'the constructivist teacher', the 'constructivist classroom' or of in-service courses that convince teachers to become 'constructivist' when the implications of 'constructivism' are not clearly understood. Although a supporter of constructivist theory, Solomon (1994) identifies with the dilemma that understanding the nature of 'constructivist teaching' is still difficult.

I identify with Lerman's (1992) reservations and the dangers of replacing one dogma with another. As I see it, constructivism has much to offer education in general, and the understanding of cognitive processes in particular. It provides a useful framework for understanding spatial conceptualization within a social context. Social constructivism recognises that past experiences, presuppositions and perceptions are central to the cognition process. It also reinforces the notion that untutored beliefs (within a socio-cultural context) influence the construction of meaning. Cobern (1991:7) refers to these presuppositions about the world as epistemological macrostructures and he asserts that "each person can be seen as having a fundamental, epistemological macrostructure which forms the basis for his or her view of reality. The common term is world view". I therefore argue that an individual's world view is fundamental to his or her conceptualization of space and shape.

World view theory has its roots in anthropology and Kearney (1984:10) suggests that there are three basic problems in the study of world view:

- What are the necessary and therefore universal types of images and assumptions which are part of any world view, and what are the specific contents of these universals?
- What relationship do these images and assumptions have with the world they represent?
- What influence does this world view have on behaviour?

There has been growing interest in world view theory in science education research and Cobern (1993a, 1993b, 1994, 1997) in particular has embraced the notion of world view in arguing the importance of fundamental beliefs with respect to learning science. This research will draw from the experiences of Cobern in the context of learning mathematics with special reference to spatial conceptualization. The enabling framework which will facilitate the construction of world view profiles in this research, referred to as concept maps by Cobern (1993) and Slay (2000), is based on what Kearney (1984) terms logico-structuralism. According to logico-structuralism, world views are constructed on the basis of the following universal categories: Self, The Other (NonSelf), Classification, Relationship, Causality, Space, and Time (Cobern, 1993; Kearney 1984). This study will focus on Space and in order to establish individual (and corporate) profiles of the notion and conceptualisations of space, I will use Cobern's framework in which he made use of "vector pairs" such as: naturalism/religion, chaos/order, mystery/knowledge, function/purpose, mundane/special (Cobern, 1993) to establish concept maps of what individuals thought of nature. I will include additional vector pairs such as: mathematical/non-mathematical, measurable/random, plane/multi-dimensional in order to set up profiles for individual's world view on space. It is envisaged that more vector pairs will emerge as the research unfolds.

This study involves adolescent learners (Grades 11) from different cultural settings. By virtue of its subjective, implicit and personal processes, world view is culturally dependent (Cobern, 1991 and Kearney, 1984) and this study will make use of the rich cultural diversity in the South African educational landscape in its attempt to set up world view profiles about the nature of space.

Methodology Aligned with the objectives of the study, this research is grounded in an interpretivist-naturalistic paradigm (Schwandt, 1994; Lincoln and Guba, 1985; Denzin and Lincoln, 1994; Cohen and Manion, 1994) and makes use of multiple sources and types of data.

Sample selection

A sample of 30 Grade 11 learners have been selected from the following type of schools: Farm schools, Township schools, Ex-model C schools (co-ed), Ex-model C (girls), Ex-model C (boys)

(For clarity: Farm schools are situated in the rural areas of South Africa, often in very remote areas with only rudimentary facilities at their disposal. Township schools, the historically black schools, are located in the 'townships' of South Africa, usually on the outskirts of an urban area characterized by shanties and barely functional infrastructure. Ex-model C schools are the historically white state schools equipped with good and functional resources. They are situated in the historically white suburbs that are usually characterised by an adequate infrastrucure. Since the 1994 elections all state schools have become multiracial in theory. In practice mostly the ex-model C schools have transformed their racial profile whereas the township schools and the farm schools have remained essentially black. This research is situated in and around Grahamstown, a small University town in the Eastern Cape).

The research proceeds in five stages:

Stage 1: Spatial perceptual and spatial conceptual skills activity

Initially, the sample of learners were split into pairs and each pair undertook a series of different tasks. The objective of this was to identify trends and patterns in firstly, **what** spatial skills and combinations of skills were used to complete the tasks and secondly, to establish **how** these skills were utilized. The series of tasks consisted of a shape sorter activity, a puzzle building task, working with a soma cube, drawing observable objects and drawing a route map from memory. The choice of the apparatus was based on graphicacy research conducted by Wilmot (1998) and Boltt & van Harmelen (1995).

An initial questionnaire to establish a sense of the participants home background was administered. The general aim of the questionnaire was to construct individual profiles in terms of the participants home background, to establish a broad overview and obtain initial data of the participants 'understanding' and 'preconceptions' of space and shape, and to offer participants and initial opportunity for expressing themselves and reflecting upon their experiences and perceptions. The results of the questionnaire provided valuable guidelines for further in-depth exploration as articulated in the following stages. Face validity for this instrument was achieved by passing it before experts before revision and use.

Stage 2: World view profiles The second stage of the research involves the construction of world view profiles for each member in the sample. This is achieved by adapting Cobern's framework of conceptual mapping using vector pairs (Cobern, 1993a) as articulated above. It is based on a logico-structural approach which suggests that internal presuppositions and universal beliefs are structurally integrated (Cobern, 1991).

Stage 3: Investigation of mathematical spatial capacity and performance

This stage investigates the performance of the individual members of the sample in an adapted version of a combination of the Monash Space Test (Wattanawaha, 1977) and the Mathematical Processing Test (Suwarsono, 1982). The tests will be adapted to suit the conditions for this research in terms of language, content, context, grade and general accessibity. These tests will initially be piloted. In constructing this spatial capacity and performance test I will further draw on the research work done by Pallascio *et al. (*1993), Leeson *et al*. (1997), van Niekerk (1998), van Niekerk (1995), Clements and Battista (1992), Shar and Geeslin (1980), Tartre, (1990), Fennema and Tartre, (1985), Michaelmore, (1983), Parzysz (1988), Battista (1994), Senk (1989). I will make use of Surwarsono's (1982) strategy in including a mathematical processing questionnaire which involves the participators having to articulate how they solved a particular problem and what strategies they adopted.

Stage 4: Exploring theories of consistency The fourth stage of the research involves searching for consistency and relationships between the data collected in stages 1, 2 and 3. This will involve analyzing the qualitative data obtained in stages 1 and 2 and matching that up with the data obtained in stage 3. Additionally, the data will be examined to determine consistencies between the results identified by the educational research literature and the findings of this study.

Stage 5: The South African context This stage will contextualise this study within the South African experience in terms of curriculum and classroom practice implications. This will involve an analysis of the objectives and outcomes of the new South African mathematics curriculum in terms of its spatial understanding and commitment. The analysis will be related to the findings of the research and, with the involvement of school teachers, the implications for classroom practice will be discussed.

Data collection For Stages 1 and 2 data was collected by means of "systematic observation" (Merriam, 1998: 95), the "qualitative interview" (Weiss, 1994) and a questionnaire. An integral part of the spatial capacity and performance test referred to above is the mathematical processing questionnaire. This questionnaire will be used to ask participators to articulate the manner in which they solved the problems in the test with particular reference to spatial competencies. Teachers opinions and experiences are crucial to Stage 5 of the research and a semi-structured interviewing process will be used.

Overall paradigm

The overarching paradigm within which this research is grounded is an interpretivist-naturalistic one (Schwandt, 1994; Lincoln and Guba, 1985; Denzin and Lincoln, 1994; Cohen and Manion, 1994). Key features within this paradigm that characterize this study include: Interpretation is key to understanding any situation; understanding a situation is based on the perceptions and points of view of those who live in it; this research thus starts with individuals and seeks to understand their interpretation of their world; both the researcher and the participants are involved with and participate in the research; the research is seen as a process that emerges and grows, and is reflexive in nature; the methodology of the research is multi-dimensional.

The use of multiple sources and types of data (Le Compte *et al*.1993), as suggested by the above framework, will ensure credibility (Lincoln and Guba, 1985). This is often referred to as triangulation and is an essential validation technique for conclusions and recommendations (Le Compte *et al.* 1993).

At the time of writing this paper, the study had just completed Stage 2 of the data collecting. Below some examples of the data. I am not in a position yet to draw any conclusions or make any inferences about how the various data relate to each other. I am merely highlighting some examples to stimulate debate and arouse interest.

Some initial data and findings

The tasks in Stage 1 which, at this stage, revealed some interesting issues, were those which involved drawing. One of the tasks for example asked of the participants to draw a map that indicated the route that would take them from their school premises to the local Standard Bank building. The variety of responses was quite overwhelming. Many of the participants were meticulous in their construction of the map and very successfully oriented and mapped out their route, whereas others struggled immensely to symbolically articulate their route and spatially express their orientation.

Another task asked them to firstly draw the front view of two objects placed on a table in front of them. The objects consisted of a small rectangular box and a fruit tin placed at the front left corner of the box. After completing this task the participants then had to draw the same two objects as if they were sitting on the opposite side of the table. A good number of the participants were not able to imagine and hence draw the correct orientation of the fruit tin relative to the box for the second part of the task.

A similar task asked of the participants to draw a bird's eye view (an aerial view) of an arrangement of objects. This arrangement of objects was presented to them in the form of a side view photograph. Problems encountered with this activity were two fold: some were not able to visualize the aerial view of the objects and others were not able to correctly position the objects relative to each other. Apart from establishing a sense of the participants home background, the aims of the initial questionnaire was to offer participants an initial opportunity for expressing themselves and to establish a b road overview and obtain initial data of the participants 'understanding' and 'preconceptions' of space and shape. One of the items for example, asked that the following statement be completed: *When I think of space I …* The responses were fascinating:

… have an imagination of a big hole which is round

…space is everywhere because like now I'm in a classroom in this class there is a space. And outside it also is a big space.

…think of outer space somewhere out there or right next to you. Sometimes one say I need some space. It's being alone and having no one to confine you. It's being free in outer space. There are millions and millions of light years free space.

…think of loneliness and nothing around me. I like things around me.

…think of aliens and stars and everything else up there. All the mystery.

…think of endless green grass and then my small room at the hostel. I think colour makes you aware of how much space there is. I think of irregular shapes trying to fit into small openings

…about the sky and the place around me. At home I have my own space where I do my homework and hang out. When I think about space I think about the place beyond the limits of the earth.

The responses to the questionnaire then formed the basis for the interviewing process. In order to assist in setting up worlview profiles, the following 'vector pairs' were used as a framework for discussion: Space in terms of –

- Naturalism - Religion
- Chaos - Order
- Mystery - Knowledge
- Function- Purpose
- Mundane- Special
- Cartesian- Non-cartesian
- Finite - Infinite
- Tangible - Intangible
- Internal - External

Generally the participants found it difficult to articulate their understandings and perceptions of space. Many found it difficult to discuss beyond the common notion of space being the universe with its stars and planets. Others on the other hand were very eloquent in their explorations and spoke very freely about their personal space for example. At the end of each interview, the participants were asked to choose three pictures from a portfolio of eleven. The pictures consisted mainly of Escher drawings depicting aspects of space and spatial orientation. The criterion of selection was simply that the pictures had to either be consistent with or contradict their view/perception of space. This facilitated good discussion and enabled them to illustrate their ideas and views. Figure 1 shows three examples of the portfolio.

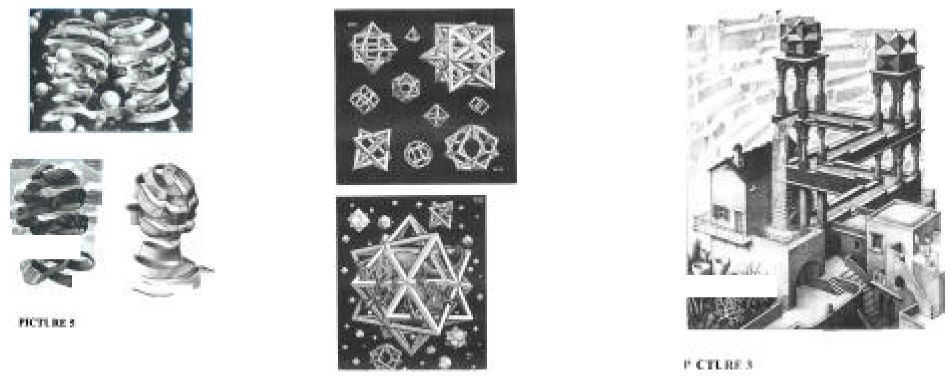

Fig 1. Three examples of the picture portfolio.

On the basis of the interviews and discussions with them, I was then able to set up tentative space frameworks for each participant. It is envisaged that these frameworks will ultimately assist in establishing individual space profiles of the participants. Table 1 is an example of such a framework:

	TOTALLY CONVINCED	STRONGLY CONVINCED	SLIGHTLY CONVINCED	NO STRONG FEELINGS	SLIGHTLY CONVINCED	STRONGLY CONVINCED	TOTALLY CONVINCED	
Naturalistic					X			Religious
Chaos				X				Order

Mystery		X						Knowledge
Function			X					Purpose
Mundane						X		Special
Mathematical					X			Non-mathema
Finite							X	Infinite
Tangible						X		Non-tangible
Internal			X					External

Table 1. An initial framework for one of the participants that will facilitate the establishment of world view (space) profiles.

Conclusion

This research is still in its early phases and there are many issues still to be thought through. I am currently adapting and modifying the appropriate 'tests' for Stage 3 of the research. The challenge will then be to tease out links and relationships between the results of the tests and the world view profiles. It is hoped that this study will shed light on the cognition process of spatial thinking and the development of spatial competencies. If we are to take seriously the notion of prior learning and experience then the recognition of individual world view profiles is central to cognition.

References

Battista, M.T., Grayson, H.W. and Gary, T. (1982). The importance of spatial visualisation and cognitive development for geometry learning in preservice elementary teachers. *Journal for Research in Mathematics Education*, 13(5), 332 –340.

Battista, M.T. (1994). On Greeno's Environmental/Model view of conceptual domains: a spatial/geometric perspective. *Journal for Research in Mathematics Education*. 25(1), 86 – 93.

Bishop, A.J. (1980). Spatial abilities and mathematics education– a review. . *Journal for Research in Mathematics Education*, 11(2), 257 – 269.

Boltt, G., and van Harmelen, U. (1995). Text illustrations: and aid or an obstacle to learning? Experiences with South African teachers. Unpublished paper. Education Department, Rhodes University, Grahamstown.

Clements, D.H. and Battista, M.T. (1992). Geometry and spatial reasoning. In Grouws, D.A. (Ed.)(1992), *Handbook of research on Mathematics Teaching and Learning* (pp. 420 – 465). New York: Macmillan Publishing Company.

Cobern, W.W.(1991). *World view theory and science education research*. NARST Monograph No.3. Manhattan, KS: National Association for Research in Science Teaching.

___________ (1993a). College students' conceptualisation of nature: an interpretive world view analysis. *Journal of Research in Science Teaching*. 30(8), 935 – 951

____________ (1993b). World View, Metaphysics, and Epistomo logy. Paper presented at the 1993 annual meeting of the National Association for Research in Science Teaching, Atlanta, G.A. Retrieved July 17, 2000, from the World Wide Web: http://wmich.edu/slcsp/106.1

___________ (1994). Word view, culture, and science education. *Science Education International*. 5(4), 5 – 8.

_____________ (1997). Distinguishing Science-Related Variations in the Causal Universal of College Students' Worldviews. *Electronic Journal of Science Education*. 1(3). Retrieved November 1, 2000, from the World Wide Web: http://unr.edu/homepage/jcannon/ejse/cobern.html

Cohen, L. and Manion, L. (1994). *Research Methods in Education* (4th ed.). London: Routledge.

Denzin, N.K. and Lincoln, Y.S. (1994). Entering the Field of Qualitative Research. In Denzin, N.K. and Lincoln, Y.S. (Eds.)(1994), *Handbook of Qualitative Research*. London: SAGE Publications.

Driver, R. and Oldham, V. (1986). A constructivist approach to curriculum development in science. *Studies in Science Education*. 13, 105 - 122

Ernest, P. (Ed.) (1994). *Constructing Mathematical Knowledge: Epistemology and Mathematical Education*. London: The Falmer Press.

Ernest, P. (1992). Putting the Social back into Constructivism. In *Proceedings of the PDME (Political Dimension in Mathematics Education) International Conference in South Africa*. Belville: University of Stellenbosch.

Fennema, E. and Tartre, L.A. (1985). The use of spatial visualization in mathematics by girls and boys. *Journal for Research in Mathematics Education*. 16(3), 184 - 206.

Gardner, S. (1999). *Kant and the Critique of Pure Reason*. London: Routledge.

Heath, T.L. (1956). *The thirteen books of Euclid's Elements. Volume I.* New York: Dover Publications.

Kearney, M. (1984). *World view*. California: Chandler & Sharp Publishers.

Kuiper, J. (1991). *Ideas of Force: A Study of the understanding of the concept of 'force'* *of Secondary School Students in Zimbabwe*. Unpublished Doctoral Thesis, Vrije University, Amsterdam.

Leeson, N., Stewart, R. and Wright, R.J. (1997). Young children's knowledge of three-dimensional shapes: four case studies. In *Proceedings of the MERGA 20 Conference, Australia, 1997,* 310-316.

LeCompte, M.D. and Preissle, **J.** (1993). *Ethnography and Qualitative Design in Educational Research* (2nd ed.). London: Academic Press.

Lerman, S. (1992). The position of the individual in radical constructivism: In search of the subject. In *Proceedings of the 7th International Congress on Mathematics Education, Quebec, Canada, 1992.* Quebec: International Congress on Mathematics Education.

Lincoln, Y.S. and Guba, E.G. (1985). *Naturalistic Inquiry*. London: SAGE.

Marriam, S.B. (1998). *Qualitative Research and Case Study applications in Education.* San Fransisco: Jossey-Bass Publishers.

Matthews, M.R. (1992). Constructivism and empricism: An incomplete divorce. *Research in Science Education,* 1992, 22, 299 - 307.

Michaelmore, M.C. (1983). Geometry and spatial learning: some lessons from a Jamaican experience. *For the Learning of Mathematics.* 3(3), 2 - 7.

NCTM, 1989, *Curriculum and Evaluation Standards for School Mathematics* Reston, Va.: The Council.

Pallascio, R. , Allaire, R. and Mongeau, P. (1993). The development of spatial competencies through alternating analytic and synthetic activities. *For the Learning of Mathematics.* 13(3), 8 - 15.

Parzysz, B. (1988). Knowing vs Seeing. Problems of the plane representation of space geometry figures. *Educational Studies in Mathematics.* 19, 79 - 92.

Republic of South Africa, (1997).*Senior Phase (Grades 7 to 9. Policy Document.* Pretoria: Government Publication.

Senk, S.L. (1989). Van Hiele Levels and achievement in writing geometry proofs. *Journal for Research in Mathematics Education.* 20(3) 309 - 321.

Schwandt, T.A. (1994). Constructivist, interpretivist approaches to human enquiry. In Denzin, N.K. and Lincoln, Y.S. (Eds.)(1994), *Handbook of Qualitative Research*. London: SAGE Publications.

Shar, A.O. and Geeslin, W.E. (1980). Children's spatial-perceptual preferences: a cross-cultural comparison. *Journal for Research in Mathematics Education.* 11(2), 156 - 160.

Slay, J. (2000). *Culture and conceptualizations of Nature: An interpretive analysis of Australian and Chinese perspectives.* Unpublished Doctoral Thesis, Curtin University of Technology, Perth.

Solomon, J. (1994). The rise and fall of constructivism. *Studies in Science Education,* 23, 1 - 19.

Stumpf, S.E. (1994). *Philosophy, History and Problems.* (5th ed.). New York: McGraw-Hill.

Suwarsono, S. (1982). Visual Imagery in the mathematical thinking of seventh grade students. Unpublished Doctoral Thesis, Monash University, Melbourne.

Tartre, L.A. (1990). Spatial orientation skill and mathematical problem solving. *Journal for Research in Mathematics Education.* 21(3), 216 - 229.

Taylor, P., Fraser, B., and Fisher, D. (1993). *Monitoring the Development of Constructivist Learning Environments.* Paper presented at the Annual Convention of the National Science Teachers Association (NSTA), Kansas City, MO, 1993.

van Niekerk, R. (1995). From spatial orientation to spatial insight: a geometry curriculum for the primary school. *Pythagoras.* 36, 7 – 12.

van Niekerk, R. (1998). What is happening to primary school geometry in South Africa? *Pythagoras*. 46/47, 63 – 70.

Von Glasersfeld, E. (1988). *The Construction of Knowledge: A Contribution to Conceptual Semantics*. New York: Intersystems Publication.

Vygotsky, L. (1986). *Thought and Language,* (Revised Edition, Kozulin, A. (Ed.). Cambridge: MIT Press.

Want, C. and Klimowski, A. (1996). *Introducing Kant*. Cambridge: Icon Books.

Wattanawaha, N. (1977). *Spatial ability, and sex differences in performance on spatial tasks*. Unpublished Master's Thesis, Monash University, Melbourne.

Wilmot, P.D. (1998).*Graphicacy as a form of communication in the primary school.* Unpublished Masters Thesis, Rhodes University, Grahamstown.