Teaching application-orientated mathematics and developing didactic from the bottom up

Regina Puscher and Rüdiger Vernay

Contents

The MUED and its view to teaching

By the example of the MUED - a private association of mathematic teachers in Germany - we want to show the possibilities of didactic development from the bottom, from the work of practising teachers, and illustrate this with examples from the classroom.
The publication of the results of the international TIMS study in 1997/98 (which compared the mathematical skills of pupils of 41 countries and which were not very flattery for Germany) provoked a public debate about the need of change in regard of both - the methods and contents of the teaching of science and mathematics.
The demands resulting from that discussion have partly been met with the ideas developed and favoured by MUED (Mathematik- Unterrichts-Einheiten-Datei which means a sort of data bank for teaching-units) for some time. MUED is a private association of more than 700 teachers of mathematics. It is financed by the members. The starting point was dissatisfaction with the existing forms of structure-related teaching mathematics. Since its foundation about 25 years ago the MUED has been a platform where teachers can exchange and discuss their experiences and the teaching materials they have designed themselves. There are two MUED-meetings a year, sometimes special workshops and a newsletter 6 times a year.
Now the MUED offers for its members about 1000 different teaching units in a wide range of forms: from just a first draft of ideas up to published brochures. The published brochures have been taught several times by different persons in different schools and discussed more than once in working groups. So one can say that they are highly evaluated in a comulative process.
Due to the special working style of the MUED the latest developements in math-didactics can be discussed from the view of schoolteachers and put to use in the members own lessons. The important principle is to gain useful ideas from the experience of practising teachers in their own classrooms. MUED ideas have received more attention in the world of German math-didactics for about ten years. For some time now the competence of MUED - members is regularly in demand for in-service training. Some are members in curriculum commissions and co-authors of school-books.

Which are the ideas of the MUED?

Since the MUED is organized basic-democraticly, there is no set doctrine, but a range of different concerns brought forward by members. There are fundamental ideas shared by all menbers:
The pupils are the starting point of any course planning.Therefore the following questions must therefore be answered when selecting topics:
What do pupils need mathematics for?
Which themes are relevant for them?
Which kind of learning is the best fitted to their needs?
The orientation for teaching developing from the answers to these questions is first quite general: Teaching should assist and inspire pupils to develop abilities and skills that enable them to act reasonably, independently and as concerned and reflective citizens. When selecting topics it is important to point out how mathematics can be of help to understand society and enviroment of today. The pupils must realize that certain mathematical skills are useful when it comes to making decicions in a competent way. Last but not least they should learn to use mathematical terms (according to the situation) as a regulation aid that will enable them to find structures for the given situation.
Everyday problems are being used as the starting point. These problems should inspire pupils to carry out their own activities, to think independently and use their own methods. Pupils should meet mathematics

as an activity and not as a prefabricated product. These activities should be connected with meaningful situations and pictures in the minds of the pupils. Not all situations or applications are equally suitable, sometimes a fictious situation or story is better suited to learn a mathematical subject than a realistic one. The selection of topics ranges from application-orientated topics to pure mathematical ones.
If possible lesson plans should always be orientated towards activities carried out independently, with the emphasis on the process. (Unfortunately whole class teaching is very common in Germany.) As frequently as possible, a product should be the result of a learning activity (This may be a model of something, an exhibition, a newspaper article, ...). Errors should be regarded as most interesting and in the end useful parts of the learning process. Teachers are therefore more and more required to depart from the role of a guide that determes each step and move toward the role of an adviser.
We want to illustrate these theoretical ideas by five examples from the classroom. We can´t be too detailled in this lecture but please feel free to ask if you have any questions. And please ask at once, if you don´t understand the meaning of a sentence or a passage - that might happen because of our somewhat limited knowledge of the English language.

Unusual houses

In the center of the following example you´ll find the idea of project and the idea of reality orientation. To solve the placed realistic problems the pupils need mathematical tools and mathematical skills which they have to acquire in the course of the unit. They don´t learn mathematics as a supply for future problems but situation-related. Thus they won´t ask the question "Why are we supposed to learn that? " - they´ll know why. Unfortunately pupils in conventional teaching too often get answers like "You`ll need it for later topics" - which is completely unsatisfactory for them and at the same time it is evidence of a let´s say improvable teaching competency.
I would like to present to you parts of the project " Unusual Houses ". Joining in I´ll show to you some photos of the objects which have been manufactured by pupils during this unit.
A newspaper article about " flying houses " was the starting point of this project: Houses with hexagonal base and nearly triangular rooms, which you can transport with a heavyload helicopter from one place to another. This idea can be used as a class room unit, I thought, pupils can plan such houses by themselves and manufacture models. Of course you need mathematics for doing that: To calculate the room area, you need the formula for the area of the triangle. But who says that only triangles are possible shapes for the base of the rooms? Parallelograms and trapezoids are possible bases as well. For the calculation of their areas you have to learn some more geometry. The hexagon area is necessary for calculating the volume; you deal with prisms.
What do we have now? A daily life problem, which can only be tackled properly/adequately by using or inventing some parts of mathematics - in my opinion very well suited for a project for my 14 -year-old pupils. On the one hand in parts well-known (however forgotten) contents and skills have to be activated and applied in a new context. On the other hand new contents and skills are asked for and can be studied in a meaningful context. It was quite evident for me that a practical task should be included in this unit: The planning, drawing and constructing of a model of the building.
To prevent working above pupils capacity they would need a warming-up-phase to lead them carefully to the center of the topic. During this warming-up-phase former contents should be activated and trained and perhaps one or two new contents should be introduced.
At this point the so-called only-roof-houses were a good choice, houses with roofs down to the soil. Sometimes you find them among holiday houses. To deal with them you have to calculate triangle areas and prism volumes and at the same time you have to look at former contents and get some training in them.
We started by manufacturing simple models of only-roof-houses, a folded sheet of paper served as a roof and half a sheet as base. At first we estimated the height of the house in reality, and determined the scale our model had. The concept of scale should be known but of course quite a lot of the pupils will have no clear understanding of the true meaning. A good chance to re-activate and train the concept of scale. Such houses use to have two floors. In which height should the ceiling be? In which parts of the house should you be able to stand upright? How can you make intelligent use of the space, in which you can´t stand upright? The fantasy of the pupils was asked for. They were very eager to tackle these questions. A lot of ideas have been developed, a lot of calculations were done. A controversial discussion in a small group occured: Is it enough, if the lower floor is in a height of 2,30 m? Which furniture could be placed

in the rooms and where? Quite a lot of arguments had to be studied and supported with calculations. That means a lot of practising the use of scale.
After this creative phase I put a focus on triangle areas and triangle prisms. Window area in a gable or the area of wood panels in gable-walls were the starting point. Calculating the volumes of roofs with different pitches meant to calculate the volume of triangle prisms. After learning the formulas a training sequence followed, at first without applications, later in realistic contexts. There were several problems in dealing with only-roof-houses or roofs, which requested more activities than just filling in numbers into one of the new formulas: Making sketches and drawings, picking out the necessary informations from worksheets with a lot of not necessarily needed informations, calculations of rectangle areas and right parallelepiped volumes.
After approximately 7 lessons the second part began, the " flying houses ". In the meantime I had managed to get more detailed informations from the technical designers of these buildings, which I presented to the class, connected with the announcement that their job would be designing such a house and manufacturing a model from it. All necessary calculations had to be documented. Documentation and model should be presented 3 weeks later as result of a long-term homework, assessed as a classroom test. The pupils were inspired by this job. First own designs were quickly presented. The shapes of their rooms were quite different from the shapes I had expected. The necessity to be able to calculate areas of parallelograms and trapezoids was soon realized, some pupil managed to solve the problem satisfyingly by cutting the shapes into triangles and rectangles.
For some room shapes the pupils needed to do calculations that were more complex than they were able to. Therefore in a lot of houses, there predominated shapes which could be calculated easily.
At first attempt some pupils had difficulties in chosing realistic sizes of the rooms. For example they designed a bathroom of 30 m2 whereas the living room had only 15 m2. The tip to measure the sizes of rooms in their own flats or houses to reach suitable sizes and proportions was helpful.
This project "flying houses" has been accepted to be printed in an innovative school book, titled “math live”, first of its type in Germany. It was hard work to transfer a quite open project-like situation into a chapter of a school book, a lot of problems had to be solved for that, not everything could be arranged in the same way as it was done in the original situation. Nevertheless I am quite content with the result. We produced an English version of this chapter, which is available for all who are interested . Additionally we´ll discuss this project in more detail in our workshop.
After all this you may already have noticed: In our view teaching mathematics means more than just enabling pupils to solve situation-referred problems. They have to be able to establish connections between the actual problem and former knowledge. For several times a year we have established in our school phases for collection and consolidation: all the facts pupils have learned in dealing with different projects and problems are collected, analyzed and systematized. Thus in these phases revision and consolidation take place at the same time.

Testing to recognize a sickness/disease(?)

In many cases in which you want to know something about health, you can use special tests which show if you have the sickness, the test is designed to indicate or not. You use a test for sicknesses like AIDS or Hepatitis C or cancer or you use one to diagnose malformations of the foetus. Tests are also used in agriculture for animals for example to verify or to exclude BSE .
These examples demonstrate: There are themes discussed in public and in this discussion the question of the validity of the tests is quite important . So testing is a relevant topic in daily life, but it may also become personally relevant if a person has to reach a certain decision in this domain.
So mathematical comprehension is crucial in helping to understand some parts of the public discussion, in broadening ones knowledge and in reaching an appropriate decision in personal matters.
Example Hepatitis C
Information: In Germany the Hepatitis C - Virus causes more deaths than AIDS.
Main way of infection: contaminated blood - infusions or blood- products.
(In our school we had a person who became infected with Hepatitis C. This stirred up the discussion about the ways to become infected.)
With the antibodies-test ELISA you can find out if you are infected with Hepatitis C or not.
We wanted to obtain clarity on the question: What exactly does a positive or negative test-result mean?
For this you need 3 informations:
• In Germany in 1998 the estimated infection-rate was 0,00235.

(and two informations about the test:)
• test-sensitivity: 99% - that means: 99% of the truly infected persons get a positive result if tested.
• test-specificity (?): 98% - that means: 98% of the truly not infected persons get a negative result if tested.
What does this mean for yourself in case of you got yourself tested and got a positive result?
 (Baumdiagramm)

$P(HCV^+/T^+) = (0.00235 \times 0.99) : (0.00235 \times 0.99 + 0.99765 \times 0.02) = 0.1044$
Because you have a very small probability to be infected you have an astonishing result: If the test shows a positive result, you are infected with the probability of only 10.4%.
You can ask other questions in this situation. For example a second test will increase the validity:

$P(HCV^+/T^+/T^+) = 0.8523$

You have the same mathematical situation (with other values for the probabilities) in the case of brest-screening and brest-cancer - an example which is very interesting and important especially for girls. Same situation is if you are testing cattle with a quick-test for BSE. BSE has been a big theme of public discussion not only in Germany but in many European countries in the last year. We tried to get data about the test, but we only got this information from the ministry: "The data about the validity of the BSE- Test are very sensitive. They are not released to be published at the moment."
This was an experience, too - and a new quality of test-sensitivity..
I also planned a project about prenatal diagnosis of malformations within the subjects biology, ethics and mathematics.
(transparency) - it shows the validity of test for spina bifida and Down Syndrom - from the mathematical side the same theme as above - Information: In green: These are the women for whom the test shows a higher risk to get a child with down syndrom, but only two of them are concerned. In Blue: .... (spina bifida) For the red one the test has shown no risk, but the child has one of the mentioned malformations)
This theme too is of great importance especially for young women. I taught a group of young adults, and a short time after we started to discuss the unit we got to know that one young women just was involved with these problems and was not able to face them. So we had to drop the theme.
That can happen if you touch the things of life.

Pippi Longstocking

I want to clarify another aspect of topic-oriented learning by the following example for 12-year-old children. It also deals with learning in a daily life situation. Different from the examples described before the central topic is a fictitious story, within that story the concept of angle is introduced and practiced.
In many countries Pippi Longstocking - youth-novel figure of the Swedish authoress Astrid Lindgren - is wellknown. In Germany almost each/every(?) child knows Pippi Longstocking from books or from television. She lives as a 12-year-old girl without her parents in a house together with her horse and an ape in a small Swedish village . She is unusually strong, which again and again produces strange situations .
The story for class room instruction refers to situations from the novels. For the selection of the figure it was crucial that the leading character is an active girl - normally in such stories adventurous persons are rather boys -, and that for the purpose of teaching new situations - which remain in familiar contexts - could be created easily. In the conceived story Pippis father, a captain, comes home from a long sea voyage and wants to celebrate a party with his daughter and her friends. He plays a mysterious game not telling the children where the party will take place. The children must work out that for themselves. The only help are several hidden messages.
The decoding of the notes requires knowledge of turns. Carefully an early stage of the concept of angle is introduced and the handling of it is practiced in some different ways.
Two examples should clarify that:
(1) In the dark cave a further message is hidden which the children have to find. They only have a drawn description of the way but no means to light the cave. Pippi leads Thomas, one of her friends, through the cave by shouting to him directions and number of paces from outside the cave. The situation is played on the schoolyard. The child, who plays the role of Thomas, gets blindfolded. The person, who represents Pippi, receives the description of the way, but has no visual contact with Thomas. Only by shouting orders it leads him through the cave. The other pupils watch the situation. Afterwards you can

clarify quite easily that it is necessary to determine certain fundamental ideas. What is a half turn, what a quarter turn?

(2) In the meantime the children have got a simplified geo triangle as an aid for measuring and drawing turns. Now the handling should be practiced.

The children have found the next message in the cave, the description of the way to the fairground. They have to reconstruct this way on a map and thus will find the fairground.

You see: all parts of this unit operate in the same context. Even the exercises are context-referred. It is obvious, that the children have the possibility to form a clear notion of the new concept, which is connected with adventures and pictures in a positive way and so can be re-activated later easily.

Remaining in one context and forming an idea of angles, in this example go together very well.

In the further process of the unit it will become necessary to measure turns or draw more exactly. At that point the graduation on the geo-triangle is useful.

As an exercise, they have to navigate between South Seas islands. The idea for that was also taken from the original story. Pippi Longstocking´s father is not only a captain, but at the same time king of a South Sea island.

Newspaper- Math

These are original newspaper-reports from German newspapers - we translated them. When we read the first, we thought about coming to Australia at once - it seems that it could be really a cheap holiday...

Olympics

Sidneys hoteliers have been sadly mistaken. In the Olympic town you find an over-supply of hotel-beds. The result: The prices slump partly for more than 100%.

But okay , you should not believe all you read ...

A second example:

Fast-drivers

Some years ago every tenth driver ran too fast, today it is only every fifth. But even 5% are too much and so speed checks will go on.

We do not know, how much Australian journalists know about mathematics, but in Germany sometimes their mathematic comprehension seems to be very small.. So you don´t have difficulties to start a unit about newspaper- math with incorrect reports. And the pupils like very much to find these mistakes.

This is not a teaching unit which is finished by doing it one time. You can take it up again to develop the skills and you can use newspaper-reports of topical interest in other math-units. The aim of this unit is:

• to critically analyze information with number specification or mathematical arguments or at least to realize what they mean

• to use mathematical information as starting point for further questions and to give one´s own position on the basis of the mathematical knowledges

-> in this way to train applying mathematical knowledge in everyday life

In one part of the newspaper-math-lessons students can learn to analyze a given newspaper-report or graphic or advertisement in a mathematical way:

•Which techniques can I use to analyze the article? Which is the important information? (Folie)

Doctors train more helpers .... This article deals with jobs for apprentices. It is showed to underline the important information, there is an example for an accompanying question. And it is shown which analysing steps you can take:

a) to notice the important information

b) to make the calculation

c) to evaluate (judge?) the solution

•Which questions are possible? (transparency)

Here is an advertisement poster. In this example we don´t look at the subject matter but to the shape. One possible question is here: How large is the poster? This should increase students awareness that you can use mathematics mathematics not only when numbers occur.

• What do the numbers / graphics / pictures really say? (transparency) This graphic illustrates a newspaper-report mentioning that the costs for pensions for civil servants are rising enormously. But this impression is strengthend by compressing time-axis-scale in the last section? / segment?

Another important item of the unit is this:

The students have the task to find a newspaper-report (or an advertisement or something else ...) to analyze it on their own using their current mathematical knowledges. The side effect of this job: The students read newspapers!

The pupils have to do a written elaboration or some and give a lecture to the class. Here are 3 examples of their topics to show the wide range of the themes:

• One pupil compared mobil-phone tariffs (rates?). When you teach mathematics with applications, comparing tariffs is a theme which suggests itself. But here the pupil found by himself the article and the task and prepared it for the others. (transparency)

• Another pupil dealt with the increase of the petrol - prize caused by ecological taxes. There has been a great political discussion in Germany about taxes raised for example on petrol to prompt the people to save energy and to behave ecofriendly. In the article chosen by this pupil was noticed that you have to compare the rising of the petrol price with other price-risings to have a factual discussion. So she compared the increase of the petrol price with the increase of wages and prices for bread since 1950 and wrote an essay about the "petrol-lie" with mathematical arguments.

• In the year 2000 the EXPO took place in the German city Hannover. This international exposition was very nice, but at the end there existed mountain of debt of 2.4 billon German marks (about 2 billion Australian dollars). The children tried to illustrate, what this mountain of debt means... (transparency)

Stopping distance

In Germany an important point in the discussion about traffic politics was and is the question whether the number of accident victims can be reduced by speed limiting. One aspect of this discussion referred to a general speed limit in residential areas. Instead of the previous regulation speed 50 (30 mph) speed 30 (20 mph) now is prescribed in many residential areas.

To be able to judge the arguments, it is necessary to know facts about the stopping process of a car. The stopping distance on a dry asphalt road amounts to about 31 m at a speed of 30 mph, at 20 mph however it´s only approximately 15 m. The pupils should learn that the stopping distance can be determined experimentally, as well as by calculation. Beyond that they should know that the computational method permits e.g. conclusions between the braking distance and the speed before braking. This method is used among other things by the police for clarifying causes of accidents.

A short theoretical outline:

The stopping distance consists of the reaction distance and the braking distance. The time of detecting the danger up to responding the brakes is called reaction time, the covered distance the reaction distance. This can be discribed by a linear function. The braking process, the way up to the complete stopping of the vehicle, can be described by a square function. Methodically regarded, you can revise linear functions and introduce square functions in this unit. The connection to application is clear, the reference to the experience of the students is obvious. They are at an age, in which the longing for the driving licence is not too far away (in Germany one may acquire the driving licence at the age of 18). As well as alcohol is a theme for them, which is going to gain importance for them in near future.

All this causes questions. Answering them it shows that mathematics is helpful or even necessary for the solution. They learn to vary function equations by parameters. That is necessary, if the external influences on the brake process are changed, maybe by different weather conditions or by different road surfaces. At the same time students should detect that the selection of the parameters is also influenced subjectively.

I´ll explain that with an example: The reaction distance is decribed by the equation $s = t \cdot v$, in which t is the reaction time of the concerned person. It varies from 0.8 seconds by a professional driver up to approximately 2.5 seconds by a drunken driver. This factor varies even with so-called normal drivers between 1 and 1.5 seconds. In court, too, they argue and jugde with different factors.

This is a good example to show how decisions can be supported by mathematics, but it also shows the limits.

Finally I would like to present some context-referred exercises to this topic. The first ones are simple training exercises, the others more complex.

(transparency)

Mr. D.Runk drives home carefully after a bar hop (speed 20 mph). A bike- rider crashs to the ground 30 m away. What will happen?

An accident in Bremen, time 19:45 crossing Celler street – Osterdeich

Mercedes out of the Celler street, measured braking distance 16 m

Mitsubishi on the Osterdeich with right of way,

measured braking distance 30.5 m

Who is guilty, the Mercedes driver, the Mitsubishi driver or both?

Riding bicycle you have a feeling for the stopping distance. Nevertheless calculate stopping distances for reasonable velocities. Do the results fit together with the behaviour you watch on the streets?

Manager Mrs. S.Tress gets into a fog-area with her car. The visibility is down to 40 m. How fast should she drive?

This was an example out of the curriculum for year 9. This topic too is a chapter from the above-mentioned school book " maths live ". Unfortunately we cannot offer a translation of it, because this volume will not be published until February next year.