Autoreport of the dissertation of Bruno D'Amore

Introduction

In this dissertation I will demonstrate a consequence at times manifest in the semiotic transformations involving the treatment and conversion of a semiotic representation whose sense derives from a shared practice. The shift from one representation of a mathematical object to another via transformations, on the one hand maintains the meaning of the object itself, but on the other can change its sense. This is demonstrated in detail through a specific example, while at the same time it is collocated within a broad theoretical framework that poses fundamental questions concerning mathematical objects, their meanings and their representations.

The current situation concerning the problem studied in the dissertation

The current situation concerning research in this field is complex and I will divide it into complementary areas.

Ontology and knowledge

In a number of studies in the late 1980s and 1990s I sustained the position that, while the mathematician can avoid debating the question of the *sense* of the *mathematical objects* he uses and of the sense of *mathematical knowledge*, this question is of vital importance for the researcher in Mathematics education (D'Amore 1999, pp 23-28, and elsewhere). Such a position is amply supported by Radford (2004): «One can very well survive doing mathematics without adopting an explicit ontology, that is, a theory dealing with the nature of mathematical objects. This is why it is almost impossible to infer from a technical paper in mathematics its author's ontological stand. The situation has become very different when we talk about *mathematical knowledge*. (…) Theoretical questions about the content of knowledge and the ways such a content is transmitted, acquired or constructed, has led us to a point in which we can no longer avoid taking ontology seriously».

This conviction has led me to dedicate much time to the study of conceptual knowledge, after having established an ontological belief on the basis of the way in which human beings *know* concepts (D'Amore, 2001a,b; 2003a,b). The debate is long-standing and can be traced back to Ancient Greece, but Radford makes every effort to pose the question in modern terms: «Men, he said, have a prior intellectual knowledge of conceptual things thanks to an autonomous activity of the mind, independently of the concrete world» (Radford, 2004) [the reference "he said" is to the mathematician Pietro Catena (1501-1576, Professor at the University of Padua and author of *Universa Loca*, in which he asserted that «mathematical objects are ideal and innate entities» (Catena, 1992)].

The debate becomes truly modern with the distinction between (human) "intellectual concepts" and "concepts of objects" proposed by Immanuel Kant (1724-1804) in his *Critique of Pure Reason*: «[These] concepts of the pure intellect are not concepts of objects; they are logical skeletons without content; their function is to make possible a regrouping or *synthesis* of intuitions. The synthesis is the responsibility of what Kant identified as the cognitive faculty of Understanding» (Radford, 2004).

Anthropological approach
For many researchers, an anthropological approach necessarily comes before a pragmatic choice (D'Amore, 2003b, and elsewhere). Once again, the position of Radford is clearly within this tradition: «In this line of thought, an anthropological approach cannot avoid taking into account (…) the fact that the manners in which we use the diverse kinds of signs and artefacts during our acts of knowing are subsumed in cultural prototypes of sign and artefact usage (…). What is relevant is that the use of signs and artefacts alter our modes of reception of the objects of the world, that is to say, signs and artefacts alter the way in which the objects are given to us through our senses (…). To summarize: From the viewpoint of an anthropological epistemology, the way in which I see that the riddle of mathematical objects can be solved is to consider mathematical objects as fixed patterns of activity embedded in the always changing realm of reflective and mediated social practice» (Radford, 2004).
There is general convergence of opinion concerning this position: «Mathematical objects must be considered symbols of cultural units which emerge through a system of uses connected to mathematical activities practiced by groups of people and thus evolve with the passage of time. What determines the progressive emergence of "mathematical objects" is the fact that certain types of practices are typical of specific institutions and that the "meaning" of these objects is intimately linked to the problems faced and the activities conducted by human beings, thereby rendering impossible the reduction of the meaning of a mathematical object merely to its mathematical definition» (D'Amore, Godino, 2006).

Systems of practice
This convergence can be further exemplified: «The notions of 'institutional (and) personal meaning' of mathematical objects have led to those of 'personal practice', 'systems of personal practices', 'personal (or mental) object', useful instruments for the study of 'individual mathematical cognition' (Godino, Batanero, 1994; 1998). Each of these notions has a precise institutional collocation. Clarifying these points is essential in order to define and render operative the notions of 'personal and institutional relationship to the object' introduced by Chevallard (1992)» (D'Amore, Godino, 2006).
Our idea of "system of personal practices" is consistent with Radford's anthropological semiotic approach (ASA): «In the anthropological semiotic approach (ASA) the ideality of the concept of the conceptual objects is directly connected to the historical and cultural context. The ideality of mathematical

objects – i.e. what makes them general – is entirely dependent on human activity» (Radford, 2005).
The sociological aspects of this dependence on human activity and social practice is thus expressed: «The mathematical learning of an object O by an individual I within the society S is nothing more than the agreement of I to the practices that other members of S develop with reference to the object O» (D'Amore, in D'Amore, Radford, Bagni, 2006) and: «classroom practices can be considered as systems of adaptation of students to society» (Radford, in D'Amore, Radford, Bagni, 2006).

Learning objects

During my attempts to define learning difficulties concerning the concepts and the knowledge of objects, I have often made use of *Duval's paradox*: «(…) on the one hand the learning of mathematical objects cannot but be a conceptual learning, while on the other activity involving mathematical objects is only possible via the use of semiotic representations. This paradox can constitute a definite vicious circle for the learning process. How can learners avoid confusing mathematical objects with their semiotic representations when the former cannot but be related to the latter? The impossibility of direct access to mathematical objects without semantic representations makes their confusion practically inevitable. Moreover, how can learners fully acquire mathematical practices, necessarily linked to semiotic representations, without a previous conceptual learning of the objects represented? The paradox becomes even greater if mathematical and conceptual activity are considered as one and the semiotic representations are then considered secondary and extrinsic» (Duval, 1993, p. 38).
These questions can be mainly referred to a certain way of construing the idea of semiotics.
Once again, I agree with Radford: «The epistemological problem can be summarised in the following question: how can we know these general objects when our only access to them is through the representations that we make of them?» (Radford, 2005).

The representation of objects

As regards the representation of objects, Radford makes reference to Kant: «In a famous letter to Herz, written in February 21, 1772, Kant questions the efficacy of our representations and asks: 'On what basis do we construct the relationship between what we call representation and its corresponding object?' (…) In this letter Kant questions the legitimacy of our representations in presenting and representing objects. In semiotic terms, Kant reflects on the adequacy of the sign. (…) Kant's doubt is of an epistemological order» (Radford, 2005).
The question posed particularly concerns the idea of the sign, since for Mathematics this form of representation is specific. The sign is in itself a specification of the particular, but can also be interpreted in terms of the general: «If a mathematician can perceive the general in the particular, this is, as Daval (1951, p.10) observes, 'because he has faith in the sign as an adequate representation of the meaning» (Radford, 2005).

Signs are, however, artefacts, linguistic objects (in the broad sense), terms which represent in order to indicate: «(…) objectivisation indicates a process the scope of which is to show something (an object) to someone. What are the means of showing the object? They are those which I call *semiotic means of objectivising*. They are objects, artefacts, linguistic terms, more generally signs used to render visible an intention and conclude an action» (Radford, 2005).
These means perform a multiple role concerning highly complex interrelationships between sign, culture and humanity: «(…) the entire culture can be seen as a system of systems of signs in which the meaning of a signifier becomes in turn a signifier of another meaning or indeed the signifier of its own meaning» (Eco, 1973, p. 156).
Moreover, the "cognitive role of the sign" is very important (Wertsch, 1991; Kozoulin, 1990; Zinchenko, 1985): I cannot examine closely this aspect in the present paper, although I consider it as a fundamental concept of General Semiotics: «all processes of signification between human beings (…) presuppose a system of signification as a necessary condition» (Eco, 1975, p. 20), so a cultural agreement to codify and interpret and thus produce knowledge.
The choice of signs, above all when composing languages, is neither neutral or independent , but rather preconstitutes the destiny of the thought expressed and of the communication realised. For example, «The language of algebra imposes a sobriety of thought and expression, a sobriety in ways of creating meaning unthinkable before the Renaissance. It imposes what I have elsewhere called a *semiotic contraction* and presupposes the loss of the *origo*» (Radford, 2005).
The loss of *origo* (origin, principle) has been widely studied by Radford (2000, 2002, 2003) and this loss constitutes the point of departure for the second part of this paper.

The objectives of the dissertation

The dissertation aims to demonstrate that:

- when a mathematical object is considered in a classrooom situation, various meanings come into play;
- every change of semiotic representation produces a different interpretation on the part of a given subect, student or teacher.

It has always been believed that conversion is the cause of the main change in interpretaton, whereas the dissertation demonstrates that this is not entirely true, since simple treatments can completely change the sense of the object represented.

This result is of considerable interest for the Mathematics Education in that it is often, ingenuously, believed that an object introduced during the process of teaching-learning remains unchanged in the eyes of the stuudents. We will see how the sense of the object can change continuously during a classroom activity.

Research methods used

The research has been conducted with numerous class a different school levels with learners at:

- Infants school
- Primary school
- Secondary school
- University
    - degree course in Mathematics
    - degree course in Education for Primary School teachers (in Italy and in Switzerland)
    - Secondary school Postgraduate Certificate in Education

As well as teachers:

- researching in their own classrooms
- attending in-service training courses

Given the variety of subjects and situations, the research has been conducted through:

- classroom observation
- activities proposed to participants
- interviews and discussions

As relevent facts emerged they were immediately used for discussion and clinical interviews.

Theoretical points of departure

The central theoretical point of departure is the definition of a Mathematical object and its variability in the classroom. In this respect we have chosen Godino's EOS theory as described below.

Object and mathematical object

The definition we propose of "mathematical object" derives from Blumer's (1969, pag. 8) description of an object as «*Mathematical object* (Godino, 2002): anything that can be indicated or to which one can refer" and can thus be expressed as "anything that can be indicated, referred to or named during mathematical construction, communication or learning.

We can distinguish different types of mathematical objects at various levels:

- "language" (terms, expressions, notations, graphs, …) in various registers (written, oral, gestural, ….)
- "situations" (problems, extramathematical applications, exercises, …)
- "actions" (operations, algorithms, techniques for calculating, procedures, …)
- "concepts" (introduced via definitions or descriptions) (line, point, number, mean, function, …)
- "properties or attributes of objects" (propositions concerning concepts, …)
- "argumentations" (for example, the validation or explanation of propositions, deductions, etc. …).

These objects are then organised within more complex entities such as conceptual systems, theories, …» (D'Amore, Godino, 2006).

A related notion is that of *semantic function*, in which a relationship is established between two (ostensible or non ostensible) mathematical objects based upon a representational or instrumental dependence, whereby one can be used in place of the other and vice versa (D'Amore, Godino, 2006). Furthermore, «(…) the mathematical objects referred to in mathematical practices and their developments, on the basis of the linguistic practices of which they are a part, can be considered in terms of the following dual aspects or dimensions: (Godino, 2002):

- personal – institutional: as we have already seen, shared systems of practices within an institution give rise to "institutional objects", while systems used by a single individual can be considered as "personal objects";
- ostensible (graphs, symbols, …) - non ostensible (which evoke "doing" Mathematics, represented in texts, oral, graphic, gestural, …);
- extensive – intensive: the relationship established between an object introduced in a linguistic practice as a specific, concrete example (for example, the function $y=2x+1$) and a more general, abstract class (for example, the family of functions $y= mx+n$);
- elementary – systemic: in some circumstances mathematical objects function as unitary entities (presumably already known) while in others they function as systems which can be broken down for analysis;
- expression – content: prior and subsequent to any semiotic function.

These aspects are presented in complementary pairs which exist in a dual and dialectic relationship and are considered as attributes applicable to distinct primary and secondary objects, thereby giving rise to distinct 'versions' of such objects» (D'Amore, Godino, 2006).
If, however, we consider the linguistic practice of representation: «I think that we must distinguish between two types of objects within the development of mathematical competence (mathematical learning): the mathematical object itself and the linguistic object that expresses it» (D'Amore, in D'Amore, Radford, Bagni, 2006).
I shall turn back to the representation soon, in order to investigate its roles more specifically.

Preparation and conduct of the experiment

The preparatory stage of the experiment consisted principally in creating a series of suitable situations. I had already had occasion to witness how, in different situations, students tended to change the sense of the object in consideration when it was subject to semiotic transformations. In the research it was necessary to identify this change whenever it appeared. This is not always easy, since the change is often only hinted at and not rendered explicit. Thus in the preparation of the experiment I prepared a grid containing possible changes of sense on the basis of the argoment and the mathematical objects presented during the lesson.

The experiment required a close observation of the semiotic representations of the objects considered, often not clear and explicit, at times only hinted at with gestures or body language signs, examples of non-formalised writing, oral and written affermations.

During the experiment a new fact emerged immediately: that the teachers themselves at times changed the sense attributed to the objects considered, when a semiotic transformation through treatment or conversion was necessary.

This fact caused a change on my part during the experiment, in that I began to observe and pose questions through interviews with the teachers, for example by proposing different semiotic perspectives for the students and monitoring the attitudes and the reasoning of the teachers.

As a result of this, Imoved to interviewing teachers on in-service training courses and collected much interesting data.

Conclusion of the results of the experiments (a posteriori analysis)

What I would like to emphasize here is how the sense of a mathematical object is more complex than it is considered within the usual pair (object and its representations). There are semantic links between pairs of this kind:

(object, its representation) – (object, its other representation)

These links are due to semiotic transformations between the representations of the same object, but then cause the loss of sense of the initial object. Although both object and semiotic transformations are the result of shared practices, the outcomes of the transformations can require *other* attributions of sense through *other* shared practices. This is highly suggestive for all studies of ontology and knowledge.

The phenomenon described can be used to complete the picture proposed by Duval of the role of the multiple representations of an object in understanding it and also to break the vicious circle of the paradox. Every representation carries with it a *different* "subsystem of practices", from which emerge *different* objects (previously called O_1, O_2, O_3 y O_4). But the articulation of these objects within a more general system requires a change of perspective, a movement into another context in which the search for a *common structure* is a part of the system of global practices in which distinct "partial objects" play a role.

The progressive development of the use of different representations undoubtedly enriches the meaning, the knowledge and the understanding of the object, but also its complexity. In one sense the mathematical object presents itself as unique, in another as multiple.

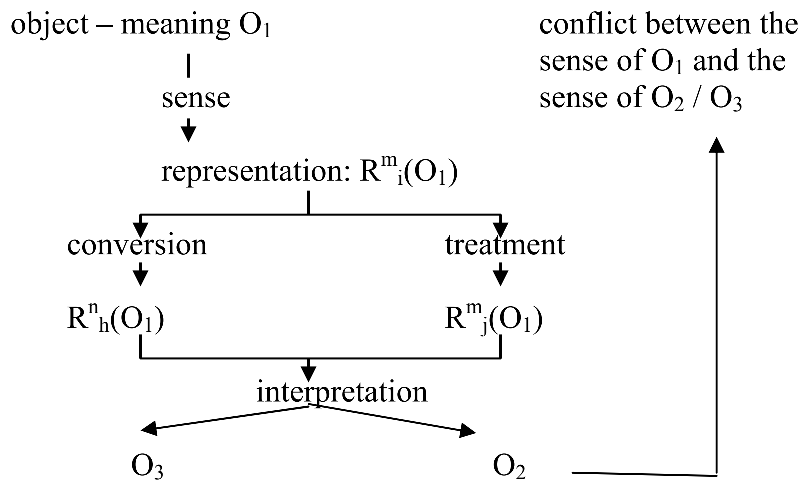
What is then the nature of the mathematical object? The only reply would seem to be "structural, formal, grammatical" (in the epistemological sense) together with "global, mental, structural" (in the psychological sense) which we as subjects construct within our brains as our experience is progressively enriched.

Clearly these considerations lead to potential future developments in which ideas, apparently diverse, will work together to search for explanations for phenomena concerning the attribution of sense.

Conclusions of the dissertation (for teaching practice)

In order to be effective, teaching practice must be based on an awareness that in the classrooom every time a semiotic representation is changed then a consequence may well be that students change the sense they attriibute to the mathematical object considered.

The following diagram summarises the complexity of what has happened in the classroom, in order to highlight the connection between objects, meanings, semiotic representations and sense:



For example:

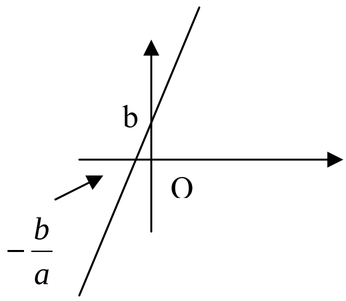
Secondary school pupils

«A point is a geometric entity which has *zero dimension*; it's small and round; if you change its form it isn't a point any more».

«$y=x^2-2x+1$ is a parabola; (after explicit treatments, $x^2-2x+y+1=0$ is obtained); $x^2-2x+y+1=0$ is *almost* a circumference».

«(…) $(x-1)(x+2)=0$ isn't an equation, (while) $x^2+x-2=0$ is».

Total cost of *y* € for the rent of a party location for *x* hours at *a* € per hour, plus the fixed cost of *b* €; the students and the teacher produce the semiotic representation: $y=ax+b$; a transformation is effected via the treatment to $x-\frac{y}{a}+\frac{b}{a}=0$ which is represented as:



and universally interpreted as a "straight line". The semiotic representation obtained from the initial representation via treatment and conversion is no longer recognised as the same mathematical object and assumes a different *sense*.

University students

$x^2+y^2+2xy-1=0$ TREATMENT ⇒ $x+y=\frac{1}{x+y}$

sense: from «A circumference» to «A sum which has the same value as its reciprocal»; Researcher: «Is it or isn't it a circumference?»; student A: «Absolutely not. A circumference must have x^2+y^2»; student B: «If it is simplified, yes» [i.e it is the semiotic transformation of treatment which gives or not a certain sense: the inverse treatments would lead back to a circumference];

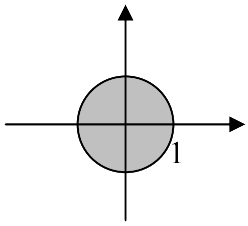
(n-1)+n+(n+1) TREATMENT ⇒ 3n

sense: from «The sum of three consecutive whole numbers» to «The triple of a natural number»; Researcher: «Is it possible to consider it the sum of three consecutive whole numbers?»; student C: «No, like that, no, like that it's the sum of three equal numbers, n".

The sum of the first 100 natural positive numbers (according to Gauss) is considered. The final semiotic result of successive changes effected via some treatments and conversions 101×50; this representation is not recognised as being a representation of the initial object; the presence of the multiplication sign forces all the students to search for a certain sense in mathematical objects in which the term "multiplication" (or a similar term) appears.

Postdoctoral students/Trainee teachers

Mathematical object: The sum of two square numbers is less than 1; semiotic representation universally shared: $x^2+y^2<1$; after changes in semiotic representation via treatments: $(x+iy)(x-iy)<1$ and conversion:



arriving at: $\rho^2+i^2<0$. In spite of that fact that the transformations are clearly and explicitly carried out, discussing each change of semiotic register, nobody is willing to admit the unique nature of the mathematical object in question. The final representation is considered a "parametric inequality in C"; the *sense* has been modified.

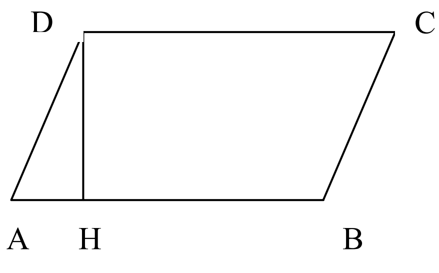
Postdoctoral students/Trainee secondary school teachers

A) Mathematical object: Series of triangular numbers; interpretation and conversion: 1, 3, 6, 10, …; change of representation via treatment: 1, 1+2, 1+2+3, 1+2+3+4,….; this representation is seen as «Sequence of the partial sums of the succeeding natural numbers».

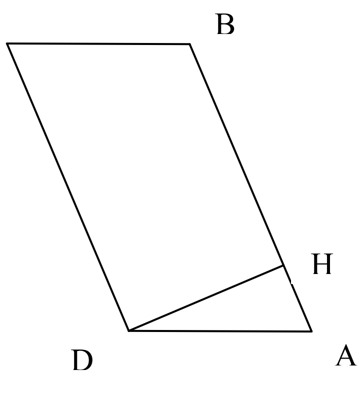
B) Mathematical object: Sequence of square numbers; interpretation and conversion: 0, 1, 4, 9, …; change of representation via treatment: 0, (0)+1, (0+1)+3, (0+1+3)+5,….; this representation is seen as «Sum of the partial sums of the succeeding odd numbers».

In none of these examples did the students accept that the *sense* of the final semantic representation obtained via the semantic transformations illustrated coincided with the *sense* of the initial mathematical object. Such a result clearly indicates a path for future analysis.

Primary school teachers

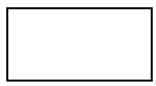


«DH is the height»

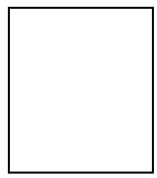


«DH isn't the height»

Middle school teachers
From the text: «The height of a rectangle is 2/3 of the base, knowing…»;



«This figure represents the situation…»



«… but this doesn't»;
why? «Because here the base is shorter».

Secondary school teachers
«I can make a *bijective mapping of* N with Z, but Z has more elements than N».

From these examples it is easy to understand how teachers CANNOT introduce mathematical objects and then transform them semiotically, believing that a stability in their identification has been achieved, but rather how students constantly change the *sense* of objects on the basis of the representation adopted.
A study of this kind would seem essential for the conduct of a good teaching-learning practice.

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