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Chapitre 8

Forum of ideas / Forum des idées

8.1 Interdisciplinarity on teacher's training :childrens' literature and mathematics/ Interdisciplinarité dans la formation des enseignants :la littérature pour les enfants et les mathématiques

Maria Elisa Esteves Lopes Galvão, Angélica da Fontoura Garcia Silva, Tânia Maria Mendonça Campos, Ruy Cesar Pietropaolo

Anhanguera University of São Paulo - UNIAN - Brazil

Résumé :Le but de cette étude était d'analyser les réflexions d'un groupe d'enseignants dans un processus de formation continue, au cours d'un travail sur l'introduction de la notion de fraction par une situation interdisciplinaire. La méthodologie a été qualitative et interprétative. L'analyse des données nous a permis de constater que le travail interdisciplinaire et de réflexion ont permis des progrès dans la compréhension de l'objet mathématique et la pratique de l'enseignement.

Abstract :The purpose of this investigation was to analyse the reflections of a group of teachers in a process of continuous training, during a work on the introduction of the fraction concept through an interdisciplinary situation. The methodology was qualitative and interpretative. The data analysis allowed us to observe that the interdisciplinary work and reflection allowed progress in understanding the mathematical object and the teaching practice.

Introduction

This research is part of an investigation at the Education Observatory Project , a training and research project, funded by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) that aims the formation of collaborative groups, in order to promote the professional development of teachers who teach Mathematics in the early years of basic education in Brazil. We will present some results of the research about the teaching and learning process of fractions, which dealt with the introduction of fractional representation of rational numbers in the initial years, by an interdisciplinary situation. As input we used the registers collected during the training sessions and the recorded interview transcripts.

Theoretical foundation

Fazenda (1979), a Brazilian researcher, highlights that the introduction of interdisciplinarity requires profound changes in the learning and teaching processes. To the author : There is a change from a pedagogical relationship based on the transmission of knowledge of a discipline or field, established according to a hierarchical linear model to a dialogic pedagogical relationship in which the position of one is the position of all. In those terms, the professor becomes the active, the critic, the entertainer par excellence. (Fazenda, 1979, p. 48-49) Freire (1987), considers the interdisciplinarity as a methodological process of knowledge construction centered on relations between the subject, the context, the reality and the culture in which it is inserted. In the author's words : Reading is an intelligent, tough, demanding, but rewarding operation. Nobody reads or studies authentically if it is not assumed, on the text or object

of curiosity the critical form to be or being subject of curiosity, subject of reading, and subject of the process of meeting in who thinks. Read is is the importance of correct teaching of reading and writing. Teaching reading is engaging in a creative experience around the understanding. There is evidence of the inclusion of interdisciplinarity in the Brazilian legal guidelines (LDB) of 1971 and 1996. Legal indications reverberate in the guidelines in the national curriculum parameters-PCN of 1997. In the current curriculum proposed at São Paulo state, where this study was developed, there are no specific indications of interdisciplinary pedagogical proposals in the guidelines for teachers who teach mathematics for the early years. Despite the discussion on interdisciplinarity and the need of the contextualization of mathematics, the Brazilian teachers have difficulty to treat certain mathematical concepts across disciplines and even in varied contexts.

We also support ourselves in studies that discuss the issue of reflection about the practice. We know that reflective teacher, reflective teaching, reflective practitioner teacher became keywords in many global reforms. We consider the reflection not as an adjective, but as a concept. As an adjective, it would be an attribute of the teacher, which would lead to the closure of the discussion on the characteristics and principles related to reflective teaching, regardless of its importance as a theoretical movement of understanding of teaching work. Such issues were introduced between us since the studies of Schön (1983) and were associated with "emancipatory power" of teachers (Zeichner, 1993; Serrazina, 1998; among others).

We believe that the formation of a collaborative team can become an essential factor for the reflection on the practice actually occurs and encourages the establishment of more security and complicity between teachers.

Methodology

This is a qualitative research, according to Bogdan and Biklen (1999). Attended the research, 16 teachers with over ten years of teaching work, in five sessions of 3 hours each : three sessions designed to the experience of methodology with focus on the use of children's literature, aiming to work with fractions; two sessions for interviews. The interviews have generated data on the reflections about the teachers' actions. Following the request of the teachers, we present the possible interdisciplinary methodological treatment for the development of selected subjects, more specially, finding strategies that favor students the use of partition, since the quotient meaning had been introduced previously. We choose a children's literature book named "Duck's lollipop" (anexo) written by Machado (2003). We believed that the use of the book would encourage the interdisciplinary work. The book would allow the contextualization of a situation, through the story of two little ducks that earn a lollipop from her mother and have to split it differently to the extent that their friends arrived. This contextualization would enable the student to think about the relationship between the number of pieces of the lollipop and the fraction denominator, and to understand the order relation of unit fractions. It could also broaden the understanding of the quotient meaning in the extent that the student understands the situation by changing the amount of lollipops and/or ducklings.

The formative process

After reading the book the participants observed that the story presented several situations involving fractions; the reading can be complemented with dramatizations, and the teacher may take this as an opportunity to introduce the mathematical language of fractions. We also discussed some students' difficulties in the learning process of fraction as well as the ruptures with ideas built on the initial work with natural numbers. Some of the teachers suggested the rewriting of the story by the students to stimulate creativity, evaluate the writing and understanding the idea of fraction. It was asked to the teachers who used the literature in the classroom to report the result to the group. We observed that the work developed had conquered them because most of the teachers had worked with the proposal in the classroom and they were unanimous in stating that it favored the understanding by the students. Such responsiveness was reinforced during the interviews.

The interviews

To detect Schön's "theories defended" (1983) we listened the teachers descriptions about their practice after the intervention. We wanted to observe in their speeches the called reflection-on-action. We noted, in all the teacher's answers, very positive experiences. We highlight the report of Professor Rosa :

(...) the book was very dynamic and meaningful because it was given a learning situation where the children could participate and make relationships about the use of fraction on a day-to-day basis. As an example, when the lollipop was divided in halves, the students related to a situation where they share a chocolate with their brothers.

The students drew the lollipops and painted the fractional representations. During this process, we discussed about the fractions representations. During comparisons and reflections the students realized that $\frac{1}{2}$, $\frac{2}{4}$ and $\frac{4}{8}$ represented the same although they were written in different ways. The discussions occurred from the children's reflections as : the painted part increased, decreased or remained the same. The children, in some moments, said the painted piece of the lollipop was the same in the three situations, but that in every division each one got a smaller piece. These discussions were fundamental for them to write their texts from the activity performed using daily life situations. They experienced situations that needed to make use of the concepts of fraction even unconsciously. The texts written by the students were read on a wheel of conversation and the fraction concepts learned with book Duck's Lollipop were related to situations exposed by them. (Professor Rosa)

Analyzing the teacher's report and other interviews, we noted that the positive evaluation was very connected to the content methodological approach and the active participation of the students. In our view, teachers valued the indication of handled materials and the use of literature as a backdrop. Regarding the use of children's literature, there are indications that found, in this alternative, as Machado (1991) calls "essentiality of impregnation". According to the author :

Between mathematics and the native language there is a relationship of mutual impregnation. Considering these two themes as curricular components, such impregnation is revealed through a parallelism in the roles they play a complementarity in the goals they pursue an overlap on the basic issues concerning the teaching of both. It is necessary to recognize the essentiality of this impregnation and have it as the basis for the proposition of actions aimed to overcoming difficulties with mathematics education (Machado, 1991, p. 10)

We realize the growing willingness expressed by professors to study in a collective way, possibly related to the exchange of experiences observed during the formative process. There is evidence that the subjects involved recognized that "the process of learning how to teach extends throughout the professor's career" (Zeichner, 1993, p. 17), pointed by the author as one more feature of the reflection.

Final considerations

The interdisciplinary work and the reflections contributed to understand the mathematical object and the analysis of issues related to teaching and learning which had not been discussed so far. The interviews indicated that the reflection during the intervention favored important changes regarding the pedagogical practice. The teachers emphasized the use of children's literature as one of the possible means to contextualize a situation and to conduct an interdisciplinary work. The collaborative work provided an experience of a new approach to the topic, a better understanding of the issues related to teaching and learning fractions and a reflection about practicing interdisciplinary work.

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8.2 Mathematical modeling : Secondary teacher preparation in the United States

Cynthia Oropesa Anhalt

The University of Arizona, Department of Mathematics
Tucson, Arizona, United States of America

Abstract : This study examines the evolution of 11 prospective secondary teachers' (PTs) understanding of mathematical modeling through the implementation of a modeling module in a curriculum course. The module consisted of readings, carefully designed modeling activities, individual and group work, discussion, presentations, and reflections. The results show that while most PTs had misconceived definitions of mathematical modeling prior to the module, they developed the correct understanding of modeling as an iterative process involving making assumptions and validating conclusions connected to everyday situations. The study also reveals how the PTs translated the modeling cycle into practice in the context of a given problem.

MSC : 97M06

Introduction

Mathematical modeling requires that students experience the use of mathematics to interpret physical, social or scientific phenomena. However, mathematical modeling problems present unique challenges for teachers that stem from working with open-ended problems, making and validating assumptions, and interpreting the mathematical results in the context of the situation given (Blum & Niss 1991, Blum & Ferri 2009). Teaching mathematical modeling requires teachers to fully understand the practice of modeling as a process.

Theoretical perspectives

Establishing a mindset for consideration of mathematical modeling should in essence distinguish models from the process of modeling. In other words, we can consider models as products, while modeling as a process that involves the construction of a model as one of its elements. In particular, building a model is not the same as experiencing the modeling process. Anytime that modeling is used to explain an everyday situation or problem, the goal(s) of the modeling problem should be considered and made explicit; that is, there should be a purpose for creating models. There are various perspectives on modeling that suggest different purposes (Blomhøj 2009). English (2007) writes that “modeling problems are realistically complex situations where the problem solver engages in mathematical thinking beyond the usual school experience and where the products to be generated often include complex artifacts or conceptual tools that are needed for some purpose, or to accomplish some goal” (p. 121).

The study

This research was conducted with secondary pre-service teachers (PTs). The goals of the study were to increase the understanding of the demands of teacher preparation for teaching modeling, the PTs' conceptions of mathematical modeling and how they evolve over the course of the study, and how the elements of the modeling cycle are reflected in the PTs' solutions of a modeling problem. The research questions that guided the study were : (1) how do PTs translate the modeling cycle into practice in the context of a given problem? and (2) how did the PTs' conception of mathematical modeling evolve throughout the implementation of a mathematical modeling module? The PTs explored the mathematical modeling problem “the lost cell phone” (Anhalt & Cortez, publication pending) that lends itself to the modeling cycle and whose context is relatable to students and motivates mathematical content and practices.

The participants were eleven secondary pre-service teachers in a senior-level curriculum and assessment course two semesters prior to their student teaching internship. The course is part of a secondary teacher preparation program in a mathematics department at a large public university. Their task was to create a model for the problems posed, write explanations of their reasoning and their solution, and then present a team poster of their findings to their peers in addition to turning in individual reports. Before an introduction to mathematical modeling as curriculum in the secondary levels, the instructor administered a pre-questionnaire to capture the PTs conceptions about mathematical modeling, and then a post-questionnaire at the end of the module to capture their conceptions after engaging in

mathematical modeling tasks. Class discussions consisted of the elements of modeling, such as making assumptions, creating a model, computing, interpreting the results, and validating the conclusions in the context of less involved type of modeling problems that could be solved in one class session in small groups. Three teams were formed and given a complex modeling task, “the lost cell phone” to create a model for locating the lost cell phone :

A lost cell phone needs to be found. Fortunately, three cell phone towers detect the cell phone signal. Tower 1 detects the signal at a distance of 1072.7 meters. Tower 2 detects the signal at a distance of 1213.7 meters. Tower 3 detects the signal at a distance of 576.6 meters. Based on a coordinate system used by the city, the cell towers are located at (x,y) coordinates as follows : Cell Tower 1 is at position (1200, 200) measured in meters from the center of town. Cell Tower 2 is at position (800, -450) measured in meters from the center of town. Cell tower 3 is at position (-100, 230) measured in meters from the center of town. Create a model for finding the location of the cell phone. Explain your reasoning.

The instructions for the teams, in addition to creating a model, were to write a description of the team’s problem-solving strategies and approaches as they navigated their way through the modeling cycle. One class session was dedicated to team poster presentations of team solutions and followed with class questions and discussions of solutions and approaches, thus allowing for comparisons and contrasts of the various approaches and solutions.

Data sources and analysis

On the same day of the presentations, individual reports were submitted describing their thought process during the teamwork and a reflection of the activity. The reports included their assumptions, solutions, reasoning, any revised assumptions, and justification of their results. This study used mixed methods in analyzing data from multiple sources. The data sources aforementioned included team posters, oral presentations, instructor field notes from class discussions, PTs’ individual written reports with reflections, and pre- and post-questionnaires. The data were systematically coded by modeling elements and by themes that emerged in the data relevant to the evolution of their conceptualization of mathematical modeling. In the analysis of the pre- and post-questionnaires, principles of grounded theory method (Strauss & Corbin 1990) were used allowing us to code the data through the lens of emerging themes, which was particularly critical since the open-ended nature of the problems allowed for a wide range of narrative responses. The conceptual themes that formed provided insight into the participants’ thinking and learning of mathematical modeling.

Results

The teams all approached the problem using analytic geometry by plotting three circles with centers at the tower locations and radii given by the distances to the cell phone recorded by the towers. The teams worked toward finding an intersection point of the three circles on a two-dimensional plane, and the three teams reached the same conclusion : the circles do not intersect at a single point. After realizing that there was no point where all three circles intersected, all teams identified a roughly triangular region whose vertices are the intersection of pairs of circles. The area of the triangular region was about 2,600 square meters. The PTs revealed that it was at this precise moment that they realized the difference between a more familiar word problem and a modeling problem. Initially, the PTs expected this approach to yield the location of the cell phone and be finished. The thinking that followed this discovery made the modeling cycle come to life, engaged them at a higher level and motivated them to persevere in finding a better model.

Team 3 decided that they were satisfied with their outcome of 2,600 m². This team seemed uncomfortable introducing new justifiable assumptions because they felt that by doing so, they would somehow change the problem. This seems to point to a conflict between having to introduce assumptions in modeling problems and using only information provided, as in typical word problems.

Team 2 took the triangular region formed by the three circles centered at the towers and argued that the cell phone would most likely be within a circle through the three vertices of the triangular region. They drew the new circle and gave its center, with coordinates (244, 642), as the likely location of the cell phone. Interestingly, the location they found does not lie on any of the three original circles surrounding the towers, therefore their answer is not a solution to any one of the equations of the circles, yet it made sense to them to conclude that this must be the location of the lost cell phone since it is a point “near” all the circles.

Team 1 surmised that the lack of a unique intersection point may be attributed to error in the distances to the cell phone measured by the towers. They then made the new assumption that the errors were up to $\pm 5\%$ of the distance recorded by each tower and obtained a region where the cell phone is located rather than providing a single

point. The annuli created by the $\pm 5\%$ margin of error in the radius of each tower intersect in a new trapezoidal region. Importantly, all points in the trapezoidal area formed satisfy the assumption of the $\pm 5\%$ error margin and are therefore acceptable.

Three main findings regarding the PT's conception of modeling are largely based on the pre- and post-questionnaires and the PTs' reflections on their experience with the "lost cell phone" problem. First, without any previous exposure to mathematical modeling, half of the PTs initially misunderstood it as a teacher demonstration model, a visual model or manipulatives as models. However, at the end of the study, none of the PTs confused mathematical modeling with other interpretations of the word "model." Second, three PTs initially had an accurate idea of mathematical modeling; however, by the end of the study they were able to articulate more clearly that modeling is more than problem solving, involving assumptions and validation. Third, four PTs went from not associating real-life contexts with mathematical modeling to understanding that the motivation for modeling often comes from real-life settings.

Discussion

The variety in the solutions exemplifies the subjective determination of what makes a modeling solution acceptable in a given context. In the classroom, a teacher may find it valuable to begin a modeling activity with a discussion about what type of accuracy might be reasonable in the solution so that there is a target precision in the desired solution. In terms of precision in communication, regular participation in mathematical discourse allows the opportunity for students to develop proficiency in communicating mathematical ideas with more accuracy, especially when asked for clarification by peers.

With regard to the evolution of the PTs' conception of mathematical modeling, there was an initial misinterpretation of the words "model" and "modeling" in the context of "mathematical modeling." It is unfortunate that the words "model" and "modeling" are used with different meaning in a variety of settings in mathematics education. For example, manipulative models are physical materials and concrete objects; visual area model or fraction model are figures that illustrate areas or fractions; model as a representation refers to a graph, an expression, or a table of values; and modeling also refers to demonstrations by the teacher which should then be reproduced by the students. Mathematical modeling is none of the above alone, and without previous exposure, half of the PTs had misconceptions.

Building knowledge of the process of mathematical modeling for PTs in teacher preparation programs will take careful consideration of their background knowledge in mathematical modeling. This raises issues for close examination of the curriculum for the mathematical preparation of teachers. A critical finding of this present study is that of uncovering these particular PTs' conception of mathematical modeling which informs the preparation of mathematics teachers.

Final Remarks

In conclusion, the combination of modeling activities dispelled misconceptions the PTs had about what mathematical modeling was and provided valuable experiential knowledge that became evident in the evolution of their conceptions of modeling. The "lost cell phone" problem was carefully chosen to lead the PTs naturally through the modeling cycle while allowing multiple approaches, different sets of assumptions, and ultimately different acceptable models. Teacher preparation work in courses pertaining to mathematics curriculum is a multi-layer endeavor. The modeling problems explored should provide prospective teachers with the experiences of doing mathematics as learners of mathematics, and then extend them further to a professional level to examine the mathematics in the context of the curriculum and student expectations.

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8.3 Problem solving : Analyzing narrative genre aspects of prospective mathematics teachers’ discourse

Janet M. Liston

The University of Arizona, Department of Mathematics Tucson, Arizona, United States of America

Abstract : This study examines the language and word choices of prospective secondary teachers (PTs) as they facilitated mathematical discussions with elementary PTs while working through problem-solving tasks. This paper showcases one particular task that involved writing a word problem for which $314 = 8$ could make sense in the context of the written problem. The results show that as the PTs experienced challenges to accomplishing the task, their discussion led to a natural progression of discourse through various stages of a narrative genre structure.

MSC : 97M06

Introduction

Language is the principal resource for making meaning through mathematical discourse in the classroom. However, prospective teachers (PTs) can find using the specialized language of the mathematical register challenging. One reason may be because PTs lack classroom experiences and have not yet had opportunities to converse with students (Chandler, Shepherd, & Smith, 1998). Additionally, PTs may not yet see a need for care in their expressions and word choices (Chandler et al., 1998). This study examines the mathematical discourse of problem-solving and draws out relationships that follow from PTs’ particular language and word choices.

Theoretical perspectives

Halliday (1993) argued that “the distinctive characteristic of human learning is that it is a process of making meaning –a semiotic process; and the prototypical form of human semiotic is language” (p. 93). Therefore, language can be described as a semiotic system because it involves sets of meaningful choices. According to Halliday and Matthiessen (2004), word choices emerge from contexts such as those found in the classroom; and as speakers interact with their listeners, their word choices construe the context of that classroom in particular ways.

A semiotic system, however, does not only represent a system of choices; it also triggers different resulting behaviors –based on the particular choices made (Eggs, 2004). These different behaviors are referred to as functions of language; and in a functional approach the concern is to describe two dimensions of language use. First, what are the possible language choices people can make? Second, what is the function of the choices made? As Eggs (2004) has suggested, this approach affords a view of word choice not as ‘right’ or ‘wrong’, but rather as ‘appropriate’ or ‘inappropriate’ to a particular context. For example, the red light on a traffic light not only means here is a red light –it also means stop now. The color of each light-choice encodes an appropriate action from a set of possible behaviors at traffic lights (Eggs, 2004).

Extending these ideas, Moschkovich (2007) has described two features of discourse practices : meaning for utterances and focus of attention. While language functions, word choices, and their meanings are not mathematical in and of themselves, they are embedded in mathematical practices; and “mathematical practices are not simply about using a particular meaning for an utterance, but rather using the language in the service of goals while coordinating the meaning of an utterance with a particular focus of attention” (Moschkovich, 2007, p. 25).

The study

This study was conducted with four secondary and 24 elementary PTs in the context of a university mathematics content course. The elementary PTs participated in the role of learners of mathematics for teaching, while the secondary PTs facilitated group discussions with the elementary PTs regarding problem-solving tasks. As instructor of the class, I chose tasks specific to the course’s curriculum –specifically, topics of the conceptual understandings of whole and rational-number operations. The primary goal of this study was to better understand the nature of the PTs’ mathematical discourse and to examine the ways in which the secondary PTs used language as a semiotic system, while coordinating the meaning of chosen words towards a particular focus of attention. The main research question

guiding the study was : How does the structure of language choices made by secondary PTs impact how others in the discourse are able to make sense of (and attach meaning to) problem-solving tasks?

Throughout the course tasks were given that asked the elementary PTs to write word problems for specific operational expressions. For this paper I have chosen to showcase the discourse of Brice (secondary PT in the role of discourse facilitator) and Marisa and Rachel (two elementary PTs in the course) regarding one particular problem-writing task :

Write a word problem for $31 \div 4$ so that each of the three answer values (7, 8, and $7\frac{3}{4}$) could each make sense in the context of the problem you write :

the answer to $31 \div 4$ is 7

the answer to $31 \div 4$ is 8

the answer to $31 \div 4$ is $7\frac{3}{4}$

For this paper, I have chosen to focus only on the PTs' discourse regarding how to write a word problem for which an answer of 8 could make sense and be considered reasonable.

Data sources and analysis

The primary source of data was audio-recorded mathematical dialogues between the elementary PTs and the discourse facilitators (secondary PTs) as they focused on specific tasks. Below is a partial transcription of an audio recording of a class session (50 minutes). Brice (secondary PT) served as the facilitator in conversation with elementary PTs Marisa and Rachel. I had not given Brice any instruction regarding how to facilitate the discussion with Marisa and Rachel; however, Brice did have access to the task one week prior to presenting it in class. The purpose of analysis was to document evidence of the semiotic nature of their mathematical discourse. A discourse sample coding is shown in Table 8.2.

Results

In examining the above discourse I did not analyze Brice's word choices as either "right" or "wrong" –but rather, how his word choices furthered understanding and reflected particular ways to focus attention on the task. Several times, Brice chose his words to orient the Marisa and Rachel to the task (So, your job on this question is to write a story problem that will get, where the answer is going to be 8; lines 1-2, code O). Additionally, Brice re-directed attention back to the main goal of the task (There's no circumstance that you could think of that with an answer of 7 groups and 3 left over, you could get 8 as an answer? lines 21-22, code R). The function of the words in Brice's question (is there a real life situation? line 22, code G) aimed to direct the PTs' attention towards thinking about a context they might have experienced outside of the classroom.

Marisa's words (If there's 3 left over [the remainder], you can't round; you round up when it's over 5; lines 3-4, code C) focused the group's attention on her mathematical uncertainty regarding rounding 7 R3 up to 8. Rachel offered a resolution to this by suggesting that Marisa think of the remainder of 3 as a fractional part, or $\frac{3}{4}$ (lines 5-6, code D). Following this, Brice again encouraged them to continue focusing their attention on rounding up (Okay, so can you think of any circumstance when you'd round up? line 7, code R).

Discussion

In my analysis of this transcript, I determined that the PTs' discourse functioned in a semiotic way towards making meaning and finding a context in which an answer of 8 could make sense for 314. Brice's language choices were appropriate to the role of facilitating Marisa and Rachel's understanding of the task, as he afforded them opportunities to voice their ideas and concerns. Instead of giving away too much of a hint, he asked questions and directed Marisa and Rachel to think about why they might want to round up in a real-life experience.

While Rachel viewed her resolution as sounding totally crazy, she did argue that her idea would work (line 24). Rachel maintained that even if a robot-dog had only 3 legs, it could still be considered as one whole dog –thus justifying rounding up to 8. While more 'typical' solutions do exist (e.g., needing 8 vans to transport 31 children to the zoo so as not to leave any children behind), Rachel was able to think of a circumstance for which she wanted to round up; and I sense that Brice's repeated prompting helped encourage Rachel towards finding her solution.

1. Brice- So, your job on this question is to write a story problem that will get, where
2. the answer is going to be 8, okay?
3. Marisa- I want to round up. If there's 3 left over, you can't round, you round up when
4. it's over 5.
5. Rachel- I feel like you'd have to make it a fraction, because if it's $7\frac{3}{4}$, you could round
6. up to 8.
7. Brice- Okay, so can you think of any circumstance when you'd round up?
8. Marisa- But don't you usually round up when it is like a number 5 or over, not 3?
9. Brice- Yeah, you'd round up to the nearest tenths, right? So, fraction-wise... when
10. would you round up with a fraction? You can round up at $3/4$'s, where else?
11. Where's the cut off?
12. Marisa- Oh, oh, half.
13. Brice- If we're talking about rounding, we usually round up. But is there a time you
14. can think of when you have a part, you automatically have to have the whole?
15. Even if you only had $\frac{1}{4}$, you'd still need a whole; you'd have to round up?
16. Rachel- Couldn't you use people because you can't have like part of a person?
[pause in conversation while Rachel reflects on what she had just said]
17. Rachel- ... no wait. I'm doing this totally wrong, backwards. I need to think this through
18. before I begin talking.
19. Brice- That's all right. Sometimes talking helps you think it through. So do you have an idea?
20. Rachel- How can you round up something?
21. Brice- There's no circumstance that you could think of that with an answer of 7 groups
22. and 3 left over, you could get 8 as an answer?...is there a real life situation?
23. Rachel- So... you're building robot dogs. Okay, this sounds totally crazy... , but put like
24. 4 legs with each dog, but then the last one only has 3 legs, but it still works.
25. Brice- So it's a 3-legged dog?
26. Rachel- Yeah, but then there's still 8 of them [dogs], right?
27. Brice- That makes sense.

TABLE 8.1 – Discourse

Closing thoughts

Upon reflection, I began to see how the above discourse transcript resembled the structure of a narrative genre. In any narrative (or story) there is an interest in 'how it will turn out'; while reading the story, one begins to put together possible scenarios –not unlike the many solution paths one might envision while problem solving. But there is a structure to stories, and the 'story' for this problem-solving discussion (in which $314 = 8$) led to a natural progression of discourse through stages of a narrative structure : e.g., orientation, complication, evaluation, resolution, and re-statement of the problem (Labov & Waletzky, 1967). Further research on the narrative structure of problem-solving discourse may lead to a linguistic methodology of narrative analysis that could be used to increase PTs' awareness of their own language choices.

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Code	Function of Language Used	Explanations
O	Orientation to the task	PT gives preliminary information needed to understand the task
G	Attention to the task	PT centers talk on the goals of the task
C	Complication to the task	PT gives a statement of uncertainty involving problems that need to be solved
D	Determination made regarding task	PT offers a determination or resolution to a complication
R	Re-launch or restatement of the task	PT brings talk back to the goals or specific elements of the task

TABLE 8.2 – Example of mathematical discourse codes.

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