

WORKSHOPS / ATELIERS

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Which support technology can give to mathematics formative assessment?

The FaSMEd project in Italy and France

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Abstract: This workshop is focused on the role technology may play in supporting the formative assessment process. Different examples from the case studies developed in France and Italy within the European Project FaSMEd will be analysed and discussed. In order to highlight the choices we made in relation to the aim of the project, before discussing the examples we will introduce the methodology adopted in each country and the theoretical frameworks to which we refer for the planning and analysis of the activities.

Resumé: Le but de cet atelier est d'examiner le rôle que la technologie peut jouer dans un processus d'évaluation formative. Des exemples provenant d'études de cas réalisées en France et en Italie dans le cadre du projet européen FaSMEd seront analysés et discutés. Pour mettre en évidence les choix que nous avons faits dans le cadre de ce projet, nous introduirons la méthodologie qui a été adoptée dans les deux pays et les cadres théoriques de référence aussi bien pour la construction que pour l'analyse de ces activités.

Introduction

The idea for this workshop was born from the collaboration of the French team and the Italian team engaged in the European project titled FaSMEd (Improving progress for lower achievers through Formative Assessment in Science and Mathematics Education). The aim of the project is to investigate the role of technologically enhanced formative assessment (FA) methods in raising the attainment levels of low-achieving students. Our hypothesis is that connectivity can support

- *teachers* in collecting data from the students, making timely formative interpretations, and informing their future teaching and, on the other side,
- *students* in exploiting the received feedback to improve their learning.

In line with this hypothesis, FaSMEd investigates: (a) students' use of FA data to inform their learning trajectories; (b) teachers' ways of processing FA data from students using a range of technologies; (c) teachers' ways of using these data to inform their future teaching; and (d) the role played by technology, as a learning tool, in enabling the teachers to become more informed about student understanding.

The research is based on successive cycles of design, observation, analysis and redesign of classroom sequences (Swan, 2014) in order to produce and feed into a set of curriculum materials and methods for teachers, that is called “toolkit”. The core of FaSMEd is constituted by the case studies involving different classrooms and feeding little by little the toolkit.

In this paper, after introducing the theoretical frame for the analysis of FA processes, we will present two examples, from our case studies. Both the examples are aimed at investigating the role played by technology in supporting FA processes, and focus in particular on how the teacher plans and implements a lesson, starting from the elaboration of different data from class activities, provided by the technological environment.

Our theoretical framework for Formative Assessment

According to the definition of FA to which the FaSMEd partners refer, FA is conceived as a method of teaching where

“[...] evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited” (Black & Wiliam, 2009, p. 7).

Such learning evidences can be collected, interpreted and exploited by the teacher in different moments of the learning process and with different purposes. In particular, we focus on three central processes in learning and teaching proposed by Wiliam and Thompson (2007):

- a) Establishing where learners are in their learning;
- b) Establishing where learners are going;
- c) Establishing how to get there.

Different agents are involved in these three processes: the teacher, the learners and their peers. Wiliam and Thompson (2007) conceptualise FA as consisting of five key strategies, that could be activated by these agents:

- 1) Clarifying and sharing learning intentions and criteria for success;
- 2) Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;
- 3) Providing feedback that moves learners forward;
- 4) Activating students as instructional resources for one another;
- 5) Activating students as the owners of their own learning.

The following table (from Wiliam and Thompson, 2007, as quoted in Black and Wiliam, 2009, p. 8) synthetizes how the key strategies could be activated by the three agents, within the three central processes in learning and teaching:

	Where the learner is going	Where the learner is right now	How to get there
Teacher	1 Clarifying learning intentions and criteria for success	2 Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding	3 Providing feedback that moves learners forward
Peer	Understanding and sharing learning intentions and criteria for success	4 Activating students as instructional resources for one another	
Learner	Understanding learning intentions and criteria for success	5 Activating students as the owners of their own learning	

Fig. 1: FA according to Wiliam and Thompson (2007)

Effective feedback from the different agents involved in these different processes plays a central role in FA. According to Hattie and Timperley (2007), there are four major levels of feedback, influencing its effectiveness. They are:

- (1) feedback about the task, which includes feedback about how well a task is being accomplished or performed;
- (2) feedback about the processing of the task, which concerns the processes underlying tasks or relating and extending tasks;
- (3) feedback about self-regulation, which addresses the way students monitor, direct, and regulate

actions toward the learning goal;

- (4) feedback about the self as a person, which expresses positive (and sometimes negative) evaluations and affect about the student.

Analysis of our examples: focus and research questions

We will present and analyse one example from the French case studies and one from the Italian case studies. For each of them, we will introduce the context, give information about the design of the activity, and analyse a brief excerpt from the videos collected in the classroom.

The focus of the analysis will be: (a) the teacher's ways of using technology to foster formative assessment and in particular of referring to feedback from technology to inform and modify her teaching; (b) the students' (in particular low achievers) exploitation of feedbacks given by technology, the teacher and also the classmates, in order to improve their mathematical understanding.

The main research questions that will guide our analyses are:

- 1) Which aspects of formative assessment can be highlighted?
- 2) Are there evidences of the teacher's use of feedback to inform and modify her teaching? Are there evidences of the students' exploitation of feedback to improve their understanding?
- 3) What is the role of technology in supporting the actors involved in these processes in providing feedback to each other?

An example from the French case studies

In France the project is held by the École Normale Supérieure de Lyon and different schools at different levels are involved, from upper primary school to the first year of upper secondary school.

In primary classes (grade 4-5), the focus is on mathematics. Three teachers are working on fractions, using calculators TI-Primaire Plus, an interactive whiteboard, a student response system and a micro document camera. In lower secondary school (grade 6-9) and at the first year of upper secondary school (grade 10) both mathematics and science are involved in a coordinated way. In particular, in one lower secondary school in Lyon, mathematics and science teachers are organising the work around a common theme, namely magnitudes and measure, testing a student response system. They are encouraged to share methodologies and, if possible, activities that could be approached from both perspectives.

In the grade 10 classroom, as well as in another grade 9 classroom out of Lyon, every student is equipped with a tablet. Mathematics and science teachers are using connected classroom technologies. They have the possibility to pose questions to students and collecting the answers, and to check the work done by each student on her tablet in real-time.

These technologies were sometimes already present in the classroom due to school local projects (this is the case for the tablet classrooms) or chosen by the teachers according to their needs at the beginning of the FaSMEd project.

All the classes engaged in the project are mixed ability classes, and some of them include identified lower achievers. In addition, the majority of them are situated in the suburbs where the social context is often source of difficulties.

We consider that formative assessment is a process that is observable over a long period of time. Therefore, our methodology is built in order to catch information over time: the observations as windows open on the classroom at key moments, accompanied by teachers' auto-reflections and description of the whole scenario, from the introduction to the institutionalization of knowledge at stake, in reference to the Theory of Didactic Situations (Brousseau, 2004). Hence, we ask the teachers to fill in a grid of description where the following points have to be considered.

- Before the lesson: the prerequisites, the objectives, the planned organisation of the classroom (which tools, which technologies, individual or collective work,...), in reference to the instrumental orchestration (Trouche, 2004), forecast difficulties of the students and forecast answers to cope with them.
- After the lesson: brief summary of what happened in the classroom, possible gap with the forecast plan for the lesson.

This information is used by the researchers for preparing the observation and by the teachers for enriching their data for the process of formative assessment.

From the different observations carried out in the FaSMEd project, we present a case study in mathematics. It is a grade 9 tablet classroom where each student has his/her own tablet and is responsible for it during school hours. For networking tablets, the teacher (Thomas) uses the NetSupport School software that allows classroom monitoring, management, orchestration and collaboration. As a mathematical platform, Thomas decides to use Maple TA that is an online testing and assessment system designed especially for courses involving mathematics. The classroom is also equipped with an IWB. All the digital equipment has been provided by the school, since the classroom takes part in a school project about the integration of technology in the classrooms. The teacher has to appropriate such technologies also from a technical point of view. Nevertheless, what is completely new for him, from a didactical point of view, is the use of such technologies for formative assessment in his classroom.

The case study leans on a sequence about linear functions, where the following competences are to be acquired, according to the different representations of functions.

- (a) Calculating and detecting images.
- (b) Calculating and detecting inverse images.
- (c) Recognising a linear function.
- (d) Shifting from the graphical frame to the algebraic frame and vice versa.

Thomas decides to create a sequence of questionnaires around these four competences, using Maple TA. Following a typical Thomas' teaching sequence with Maple TA, we propose to analyse three specific episodes taken from our observations and referred to the third quiz proposed by Thomas to the students during this learning sequence about linear functions. The first moment concerns a student taking the quiz and the teacher declaring his potential use of the class' results. In the second episode, the teacher comments the quiz results of a student and, during the third excerpt, the teacher comments the whole set of the class' results.

First episode

A student (Mathieu) is working on a question concerning the competence (a): calculating and detecting images. Formulated in the graphical register of representation, the question is: "The curve below represents a linear function. The image of 9 is -2. True/False." Since he is working alone on the mathematical task, Mathieu is *active as the owner of his own learning*. He is reading the task that he has received from the teacher on Maple TA. In this first phase of the work, technology is used as a communication mean for sending tasks to the students.

Mathieu faces the didactic situation devolved by the teacher. After a while, he copies the question, leaves Maple TA, and pastes the question on the interactive environment of his tablet (OneNote) in order to work on the given graphical representation using his previous experience of such an exercise. On his screen, indeed, we can see a previously solved exercise that is very similar to the new one (Fig. 2a). Mathieu starts using the same graphical technique (Fig. 2b), mobilising his knowledge as a reflexive student (Margolinas, 2004). He has transformed the didactic situation into an a-didactic situation where he acts on a reacting milieu.



Fig. 2 a and b: Mathieu working on OneNote.

The student starts acting on the technology at his disposal, using it as an interactive environment, and the fact that the teacher has devolved to him the responsibility for solving the mathematical situation (devolution) is at the base of this action. Finally, Mathieu submits his answer, sending it back to the teacher.

To go further in our analysis, we can move on to the teacher's level. This dynamics occurs when the teacher is confronted to the students' answers, and uses technology for analysing such data. In our case, talking to another student, the teacher declares his potential strategies depending on the students' responses.

Thomas : *"I don't know if I'm going to take it into account or not. The idea is that I would like to mark it. If I realise that it doesn't work... I don't know... I'm going to see what's going on... At least I'll know that you don't succeed here. You can skip it if you don't know what to do."*

The student's results are a feedback for the teacher, who will process and analyse these data. Depending on the student's performance, he may adapt his teaching, for example by choosing another FA strategy, and provide feedback to students. In his words, we detect also a 'feedback about the task' that Thomas gives to the student, by saying *"At least I'll know that you don't succeed here"*.

Second episode

Teacher's feedback can be made on the spot, like in the second transcription that we propose to analyse. A student has completed his quiz, submitted his answers and got a 'feedback about the task' from Maple TA: "good answer" or "wrong answer". Then he calls Thomas in order to have further explanations.

Thomas: *"The first one is right, the second one is wrong, the third one is right, and the fourth one is wrong. Finally, I consider that you were right on the two that are easier to explain and you got false on the two that require more mathematical work. That's normal. I consider your result as normal."*

Both the teacher and the student benefit from the feedback in this episode. The student gets a 'feedback about the processing of the task' and also on his global performance according to the teacher's norm. The teacher, who analyses this quiz result on the spot and considers it as normal, gets information about the student's achievement.

Third episode

Sometimes the teacher's feedback is not given immediately, but during another lesson, as a result of a deep reflection, developed by the teacher, on the data at his disposal and this is the case of the third episode. When all students have completed the quiz, Thomas leads the correction of the questions with the whole classroom using NetSupport School. The lesson after, he proposes a

global lecture of the class' results at the three quiz and analyses the whole set of answers stored by Maple TA, by showing them at the IWB and commenting them with the students. In this way, he provides feedback for the whole class on the attained mathematical competences.

Thomas: “[Here are your results] on several trials. What we can see is that in calculating images you reached 0.778. What does it means? [...] about 8 successful students over 10, here. There we had 6 over 10, then 8 over 10. So we are good in calculating images. [...] I'm not going further. However, we'll come back on determining the expression of a linear function. 0.1, you see 0.1, 0.3, and here we went down at 0.2. [...] I would like to get to realise if I succeed in teaching you two or three things last time, so we are going to work again on these two questions. Open Maple TA, and answer the two questions of the day. Let's go.”

Thomas analyses the class' results and he *clarifies the learning intentions and criteria for success*. He has worked again with the students on the required competences during the correction phase, and now he wants to test again the competences revealed as not achieved by the analysis of the results, namely competences (b) and (d). Thus, he *engineers other learning tasks* on Maple TA. Two new questions are properly prepared and sent to students as a result of this dynamics. From Thomas' words, we can observe that, as he expected in the first episode, he has adapted his teaching depending on students' progressive achievement.

More generally, relatively to FA strategies, Thomas orchestrates the use of technology in direction of individual students, of the whole classroom or even of himself. Instrumental orchestration helps him in refining his FA strategies. Indeed, analysing students' data in order to share and discuss results in the classroom or to send new learning tasks to the students allows him to choose the most powerful FA strategy according to students' mastering of the competences at stake.

An example from the Italian case studies

In Italy the FaSMEd project involves 19 teachers, from three different clusters of schools located in the North-West of Italy. 12 of them work in primary school (grades 4-5) and the other 7 in lower secondary school (grades 6-7). Within the project, all the teachers work on the same mathematical topic: functions and their different representations (symbolic representations, tables, graphs).

Low-achievers are identified mainly through the teachers' assessment, and attend regular classes with the other students, because (as in France) schooling is based on mixed ability classes.

A research hypothesis of our team is that low achievement is linked not only to lack of basic competences, but also to affective and metacognitive factors. Furthermore, another important assumption is that argumentation can be exploited as a formative assessment tool in the interaction between the teacher and the students. As a consequence, we believe it is important that during class activities students should be guided to: (a) develop ongoing reflections on the teaching-learning processes; (b) make their thinking visible (Collins, Brown and Newmann 1989) and share it with the teacher and the classmates; (c) highlight their affective pathways (De Bellis & Goldin, 2006).

Starting from these assumptions, when we planned our work within the FaSMEd project, we looked for a technology that could support the teachers in the sharing of students' screens and of their ongoing and final written productions and in the collection of students' opinions and reflections both during and at the end of each activity. We chose connected classroom technologies, i.e. networked systems of personal computers or handheld devices specifically designed to be used in a classroom for interactive teaching and learning (Irving, 2006). They both enable to share the ongoing and final productions of the students, and to collect their opinions during the activities and at the end of them (Irving 2006, Roschelle et al. 2004, Shirley et al. 2011). Specifically, we chose the IDM-TClass classroom software, which allows the teacher to: (a) show, to one or more students, the teacher's screen and also other students' screens; (b) distribute documents to students and collect documents from the students' tablets; (c) create different kinds of tests and have a real-time visualization of the correct and the wrong answers; (d) create instant polls and immediately show their results to the whole class. Moreover, the students' written production can be displayed through

the data projector or the interactive whiteboard.

Each school has been provided with tablets for the students (who work in pairs), computers for the teachers and, where the interactive whiteboard was not available, a data projector. The students' tablets are connected with the teachers' laptop through the IDM-TCClass software. During the teaching-experiments, the teachers use this technology for the first time, and one researcher is present both to collect data and to help the teacher to carry out the activities.

The teaching experiments integrate the connected classroom technologies within activities coming from different sources. Among them, the ArAl Units, which are models of sequences of didactic paths developed within the project “ArAl – Arithmetic pathways towards favouring pre-algebraic thinking” (Cusi, Malara & Navarra 2011). In particular, for each lesson we prepared a set of different worksheets that can be sent by the teacher to the students' tablets. Each lesson is organized with the aim of (a) supporting the students in the verbalisation and the representation of the relations introduced within the lesson; (b) enabling them to compare and discuss their answers; (c) making them reflect at both the cognitive and metacognitive level.

In this paper we analyse an excerpt from a grade 5 class discussion referred to the following worksheet:

The activity: “L’Archeologo Giancarlo”

On the ArAl mountain, in the middle of the desert, the archaeologist Giancarlo has found some graffiti engraved on the rock. He reproduced the incisions on his notebook, writing their heights. This is the page where Giancarlo reproduced the incisions:


28 cm


21 cm


14 cm


7 cm

Giancarlo's collaborators discuss a lot on the relation hidden in the graffiti.
Nicola says: “You can find the height of an incision only if you multiply 7 by the number of the tips on its head”.
Battista concludes: “It is evident that, dividing the height of the incisions by 7, you can find the number of tips”.
And Paolo: “What are you saying? The number of tips is the result of the division of the height by 7!”.

What is the expression that correctly translates Battista's observation?

$7:h=p$

$k:7=n$

Fig. 3: The worksheet provided to students.

During the lesson reported in this example, the students, who work in pairs, are asked to answer to the question in this worksheet through a poll.

The IDM-TCClass software collects all the students' answers and processes them, displaying an analytical as well as a synthetic overview (bar chart) to the teacher. Using the software the teacher can choose to provide or not an immediate automatic correction of students' answers (right/wrong). We (the teacher and the researchers) decided not to provide this correction. The software enables also to choose the time given to students before completing the poll. In this case, students had 6 minutes at disposal.

During the lesson, when all the students answer to the question, the teacher (Monica) shares with them her screen, where the bar chart and the list of students' answers are displayed:

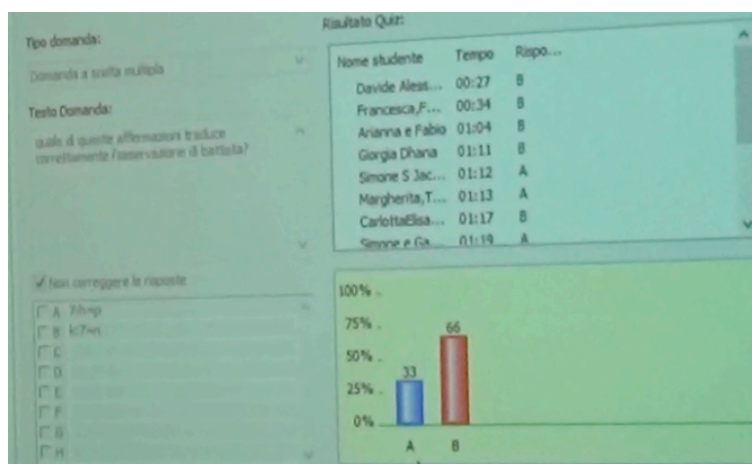


Fig. 4: The teacher's screen shared with the students.

The worksheet is also projected on the interactive whiteboard, next to the poll.

The software's processing of the poll's data enable to highlight that the 33% of the students chose the answer $7:h=p$, while the 66% of the students chose the answer $k:7=n$. The names of the students and the corresponding answers are also displayed.

The teacher chooses not to tell to the students what the right answer is, and asks to the different pairs to explain why they chose a specific answer. The class discusses on the possible strategies that could be used to identify the correct expression, in case the only reading of Battista's observation is not enough. The students are invited to check if the number of tips and the height of every incision verify the two expressions. Some students are asked to substitute, in the two expressions, the different values connected to each incision (4,28; 3,21; 2,14; 1,7). One of them observes that she discarded expression A because the result of the division $7:28$ is not 4. Alice, softly, says that $7:7=1$. Monica asks her to explain what she means. We report the related excerpt:

Transcript from the class discussion focused on the results of the poll

1. Teacher (to Alice): "What were you saying?"

2. Alice: "I was saying that, for example, the figure, the one on the bottom right, is 7cm, so $7:7$ is 1, therefore the result is not a decimal number, while with the others (*the other figures*) it is (*the result is a decimal number*). ...

Monica focuses on Alice's observation and states that the chosen expression should represent all the incisions, not only the first one. Lisa and Nicolò ask if they can change their mind.

9. Teacher: "Have you changed your mind? That is, Lisa, you chose answer A, but now you have changed your mind. Why?"

10. Lisa: "Ahem ... 7 is only that figure. While, if you divide the height by 7, you mean all the figures." ...

Another student, Jack, declares that, although h in Italy always stands for the height, in the expression " $7:h=p$ " h does not refer to the height.

14. Teacher: "It does not refer to the height. Is it right, Lisa?"

15. Nicolò (*raising his hand*): "Monica, because h refers only to one (*height*), while k..."

16. Lisa: "Both (*the letters*) ... (*Nicolò is speaking*)...no, wait! (*to Nicolò*)"

17. Teacher: “One at a time” 18. Lisa: “Both the letters are always the height, but h is only for one (<i>height</i>) ... only for this one (<i>Lisa goes near the interactive whiteboard to indicate the incision 7 cm height</i>), while k is valid for all (<i>the incisions</i>).” 19. Teacher: “k is valid for every incision. (<i>Stefano is raising his hand</i>) Stefano?” 20. Stefano: “The first expression ... No, I mean: the second expression is more correct than the first. Battista says ... where is it? (<i>Stefano is trying to find Battista’s statement</i>) ‘It is evident that dividing by 7’. It is ‘Dividing by 7’, not ‘dividing the height’... that is ... 21. Teacher: “Dividing 7 by ...the height” ...
<i>Dialogue between Monica and Amalia, who observes that Lisa’s interpretation of the two expressions is right and declares that, after having listened what Lisa and Nicolò said, she realised that the expression could be interpreted in different ways. Nicolò asks to intervene.</i>
36. Nicolò: “In the first statement (<i>he is referring to the first expression</i>) 7 is divided by the height. Instead, in the second (<i>expression</i>) the height is divided by 7!” 37. Teacher: “Very good! So ... Many times, I realised that many times it is not the same thing. It is necessary to pay attention. It is necessary to think very carefully to what is written. Exchanging, inverting the numbers is not the same thing.” ...

The FA process ‘*establishing where the learners are in their learning*’ is central in this lesson: the discussion is planned in order to support the students in making the motivations of their choices explicit. This enables to highlight erroneous ways of reasoning and incomplete explanations, but also to highlight the evolution of students’ reasoning, together with the way in which it is influenced by the other students’ interventions. For example, it is evident how Lisa and Nicolò, two low-achiever students, are *activated as owners of their own learning* during the discussion: they ask to correct their initial answers, effectively motivating their new choice (from line 10). Moreover, it is possible to highlight examples of the *activation of students as instructional resources for one another*. Lisa (lines 10 and 18), for example, refers to Alice’s intervention (line 2) and elaborates it to start developing her own argumentation. Also Nicolò (line 36) refers to Stefano’s intervention (line 20) and elaborates it.

Another FA process that is central in this lesson is ‘*establishing what needs to be done to get them there*’: the teacher intervenes to highlight the most effective ways of reading symbolic expressions and of identifying the one that better represents the involved relations, providing also guidance on how to read the tasks and the texts of the problems (line 37).

Different kinds of feedback are given during this discussion. In particular, it is possible to highlight feedback related to two of the four categories proposed by Hattie and Timperley (2007): feedback about the task and feedback about the processing of the task. Students’ explanations of the reasoning on which their choice was based represent an example of *feedback about the task*, which is given among peers, because of the different levels of effectiveness of these explanations. For example, Stefano’s intervention (line 20), which highlights that the expression $7:h=p$ does not represent Battista’s sentence because, in the symbolic expression, 7 is divided by the height and not vice-versa, represents a feedback for Nicolò, who refers to Stefano’s statement, clarifying it in an effective way (line 36).

The teacher’s meta-level intervention in line 37 aims at sharing criteria to correctly identify the expressions that better represent specific relations among quantities: it can be interpreted as *feedback about the processing of the task*. This is also an example of the teacher’s exploitation of feedback from the students, because Nicolò’s statement (line 36) provides the teacher the opportunity to discuss the importance of a careful interpretation of symbolic expressions (line 37).

Another example of this kind of feedback is Alice's intervention (line 2), which introduces the special case of the 7cm figure, enabling Lisa to understand her mistake and ask to change her answer, proposing motivations (line 10, line 18) that clearly refer to Alice's observation.

As already stressed, starting from the poll, the teacher has organized a rich discussion, which enables the activation of different FA strategies by the different agents. The technology plays an important role in supporting the agents involved in these processes, in particular in providing feedback to each other. First of all, the software elaboration of the data and the graphical representation of the results of the poll give the teacher the chance to ask for the interpretation of these results and to plan the order of students' interventions during the discussion (Monica decides to start the discussion involving first those who have given the wrong answer).

The teacher's choice of not providing, to students, an immediate automatic correction of their answers may represent a support for students at different levels: (a) it enables to focus on the explanations of the answers, more than on the identification of the correct answer; (b) it pushes the students to motivate their answers; (c) at the affective level, the lack of a written evaluation ensures that the students do not feel worried when they comment upon their choices.

Finally, the long time given (6 minutes) to students to choose their answer enables them to reflect, in pairs, on the motivations on which their choice is based. The moment that precedes the answer to the poll is, therefore, preparatory to the subsequent discussion.

Conclusion

The observations in the classrooms show clearly the contribution of FA in the teaching and learning processes. Moreover, the technology appears as a medium facilitating the different FA strategies but also the dynamics between these strategies. It is possible to *'clarify learning intentions and criteria for success'* also without technology but technology allows to display these intentions and to share with students as a class or with the student as an individual. *'Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding'* is also facilitated by giving the opportunity of *'providing feedback that moves learners forward'* on the spot as well as after reflection. The possibility given by technology to store data and the ease to come back to these data is an important functionality that teachers can use to enhance their teaching strategies. As recognised also by the teachers in the interviews, technology is not at the base of FA, but appears as an essential tool to improve the effects of FA for students and for teachers as well.

Concerning students, it appears that the strategies of *'activating students as the owners of their own learning'* and *'activating students as instructional resources for one another'* constitute the core of FA, since they enable the active involvement of all the participants (teacher, students, peer/group) within the FA process. These strategies are also facilitated by the instrumental orchestration and the possibility given to students to use technology regarding the particular moment of FA at stake.

Our analysis brings to the fore the crucial role of the teacher as a guide in FA lessons with technology. When we began the project, most of the involved teachers stated that FA was present in their practices. However, most of the time, FA was not developed over time and appeared occasionally in the classroom more as a reassuring method than as a teaching strategy. Professional development is surely a big issue of the next years in order to consider technology as a tool enabling the enhancement of teaching strategies including FA.

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The Tangram Chinese Puzzle in Context: Using Language as a Resource to Develop Geometric Reasoning in a Collaborative Environment

Le tangram casse-tête chinois en contexte : utiliser le langage comme ressource pour l'élaboration de raisonnements géométriques dans un environnement collaboratif

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Abstract: Geometric thinking and geometric measurement is a large part of the curricula in most elementary, middle, and high school systems across the globe. The aim of this workshop is to help teachers and teacher educators consider the role of language in the classroom as a tool for mediating meaning through word choice for meaningful mathematical communication. In this workshop we focus on language as a central resource for negotiating meaning in mathematics as we consider the teachers' language choices that influence the ways that mathematics is presented to learners, in particular, how language influences student understanding of geometric thinking and measurement. The workshop will incorporate hands-on activities and discussions about language as a resource for mathematical meaning. The workshop will span across several of the conference themes, including mathematical content and curriculum, teacher education, classroom practices, and consideration of students' first language(s) as a resource for communicating, understanding, and learning mathematics.

Résumé : La pensée géométrique et la mesure géométrique constituent une grande partie des programmes d'enseignement dans la plupart des écoles primaires, des collèges et des lycées dans le monde entier. Le but de cet atelier est d'aider les enseignants et les formateurs à considérer le rôle du langage dans la salle de classe comme outil pour médier le sens à travers le choix des mots pour une communication mathématique riche de signification. Dans cet atelier, nous nous concentrons sur le langage comme une ressource centrale pour la négociation de signification en mathématiques puisque nous considérons que les choix langagiers des enseignants influencent les façons dont les mathématiques sont présentées aux élèves, en particulier, et la façon dont le langage influence la compréhension de la pensée et de la mesure géométriques des élèves. L'atelier comprendra des activités pratiques et des discussions sur le langage comme ressource pour la signification en mathématiques. L'atelier s'inscrit dans plusieurs thèmes de la conférence, y compris le contenu mathématique et les programmes, la formation des enseignants, les pratiques en salle de classe, et la prise en compte de la langue maternelle des élèves comme une ressource pour la communication, la compréhension et l'apprentissage des mathématiques.

Workshop Activities and Methods of Delivery

Creation of the Tangram Puzzle

This workshop on consideration for language as a resource for developing geometric thinking and measurement will be interactive and hands-on in that participants will create the ancient Chinese Tangram puzzle using plain paper (with no pre-printed pattern) and scissors. Participants will work in small collaborative teams with individual and team accountability to create the Tangram puzzle pieces (see Figure 1). The goal is to ensure that all team members successfully create the puzzle. The puzzle pieces include, relative to one another in size, two large isosceles right triangles, one medium isosceles right triangle, two small isosceles right triangles, one small square, and a medium size parallelogram. Directions for creating the puzzle will be provided orally

and in writing.

As we consider language as a resource for the activities with which we will engage, the workshop will provide opportunities for participants to discuss the language(s) that students bring to the classroom as a resource for working collaboratively with other students to accomplish team mathematical goals. Participants of this workshop will take on two roles: one that will allow them to engage with the mathematical content and consider the curricular influence of the activities, and one that will create a space for discussion about the mathematical activities for teacher education that will focus on student participation, collaborative communication, and mathematical language development.

Creation of Polygons

Once the puzzle creation process is completed, participants will collaborate in teams to create various polygons including re-creating the square, and newly creating a trapezoid, parallelogram, triangle, and rectangle. During this process, special attention will be given to the role of language in communication of the position and orientations of the puzzle pieces in the formation of the various polygons. The teams will be encouraged to have each team member create a different polygon so that the team creates all five of the polygons. When the five polygons are created, we will move forward with the next activity using the five polygons to explore concepts of area and perimeter.

Exploration of Area and Perimeter Concepts

Area and perimeter are typically a focus in curriculum as students are going into the upper grades of elementary years and early secondary years in the study of geometry. Questions to guide exploration of area and perimeter concepts will be considered using the Tangram puzzle pieces within each large polygon shape. The teams will work collaboratively to compare and contrast the five large polygons' (square, trapezoid, parallelogram, triangle, and rectangle) areas and perimeters.

Initially, participants will predict which perimeter measurements are greater – the perimeter of each of the large polygons or the sum of the perimeter measurements of each of the smaller pieces that make up the puzzle. The teams will need to find the perimeters of the large polygons and will be asked to reason to make a conclusion about the various perimeter measurements regarding the various polygons. These conclusions may be made by discussing specific measurements of the polygons and then asking the participants to consider the same conclusion through generalized equations.

Following the perimeter measurements and generalizations, participants will explore area of each of the puzzle pieces by comparing the pieces through relative fractional sizes. For example, one of the two larger pieces of the puzzle is $\frac{1}{4}$ of the area of the whole puzzle, while the five smaller pieces vary from $\frac{1}{8}$ to $\frac{1}{16}$ of the total area (see Figure 2). Given the total area of the Tangram Puzzle (64 inches), the area of each puzzle piece will be discussed among the teams; this will be followed by discussions about student exploration and learning about areas and fractional parts of areas with respect to knowing the full area of the original area formed by all seven Tangram puzzle pieces. Additionally, exploration and discussions about the formula equations for each of the various small puzzle polygon pieces will bring synthesis and closure to the activities on the concept of area and perimeter specific to the Tangram puzzle.

Creativity in Designing Images

The participants will be given the opportunity to create an image of their choice using all seven pieces from the Tangram puzzle. Ideas will be provided as an inspiration point for personal creativity (see Figure 3). Some images we will share are from literature books, such as Grandfather

Tang’s Story by Ann Tompert (1990), in addition to images made from Tangram puzzle pieces from websites, such as Tangram Puzzle Patterns (<http://patterns2.othermyall1.net/tangram-puzzle-patterns>) and Activity Village (<http://www.activityvillage.co.uk/tangrams>).

The Role of Informal Language in the Development of Formal Mathematical Language

Initially, participants will be shown various designs and images created with the Tangram pieces, some of which come from children’s literature, Grandfather Tang’s Story by Tompert (1990). From these, participants will be given the opportunity to create their own image with their Tangram puzzle pieces in secrecy. Participants will be asked to discuss the role of informal language for teaching and learning of mathematical concepts that lead to formal mathematical language. In this part of the workshop, participants will utilize a combination of informal and formal language to describe specific orientation and position of specific puzzle pieces to their teams in a game format. The activity is explained below.

Mathematically Speaking is an activity in which participants take turns in taking a leadership role in creating a design with their puzzle pieces in secrecy by concealing their design behind a visual barrier. The design can be created from an image taken from ideas provided (on cards) or created by the individual. Then, the participants will listen for descriptions of the position and orientation of the individual puzzle pieces as described by the leader. The goal is for the participants to re-create the same image by following the description as the leader verbalizes. A list of formal mathematical terms will be provided to both the leader and the participants as a reference. The list of terms will include the terms such as right angle, parallel, adjacent, triangle, rectangle, square, parallelogram, 90 degrees, etc. Once the descriptions of the various pieces are finished, the leader’s visual barrier will be removed to reveal the image intended for everyone to create for comparison to that of each of the team members’ images.

This activity is designed to give the participants the opportunity to reflect and discuss the purpose for a focus on the deliberate use of mathematical terms in addition to informal language descriptions and gestures to illustrate how the informal language assists the development of formal mathematical language in a particular target language. This activity will emphasize meaning through mathematical communication, which incorporates gestures and first or second languages for students.

We draw from a theoretical framework (Moschkovich 2012) that takes into consideration language as a socio-cultural-historical activity and resource that allows us to communicate mathematical ideas. The literature on the language of specific disciplines provides a more complex view of mathematical language as extended discourse that includes syntax and organization (Crowhurst 1994), the mathematics register (Halliday 1978), and discourse practices (Moschkovich 2007). We aim to focus on language as a resource that allows communicative competence for participation in mathematical discourse practices by all students, including those who are second language learners. These communication forms may take place through learners’ first or secondary languages and the use of gestures (Fernandez & McLeman 2012) in which meaning is a central focus.

Engaging students in hands-on activities (such as constructing and manipulating the Tangram puzzle pieces) can provide a platform for students to engage in both informal and formal mathematical language. Expanding talk to include gestures (i.e. pointing, drawing shapes, mimic concepts that have a spatial dimension which cannot be as easily described with speech) can communicate meaning in non-verbal ways. Fernandez & McLeman (2012) suggest that using gestures to make references to features in drawings or activity materials can allow students to create a meaningful argument without using precise academic language.

Expected Outcomes

Expected outcomes for this workshop are for participants to consider the content and curricular connections of this workshop to their own context in their countries for teacher education in upper elementary to middle grades. The particular mathematical content and curricular areas in geometry, geometric measurement, the number system, and algebraic equations involving application of the Pythagorean Theorem in geometric measurement are of particular focus in the Tangram puzzle. Participants will take the opportunity to discuss teacher education in their contexts with respect to supporting teachers in developing suitable knowledge and competencies in specific domains of mathematics in addition to challenges and resources embedded in social dimensions of language development and mathematics learning. We will draw from Brenner’s (1998) framework for equitable classroom practices that bring together cultural relevancy (mathematical activities that are relevant to students’ lives), social organization (socially productive student participation), and cognitive resources (students’ experiences and language), to guide a discussion on how the activities of the workshop can be useful and considered effective practices for diverse student populations.

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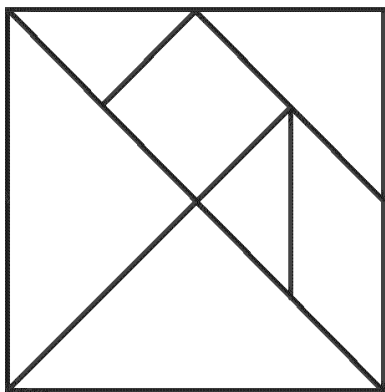


Figure 1. Tangram Puzzle

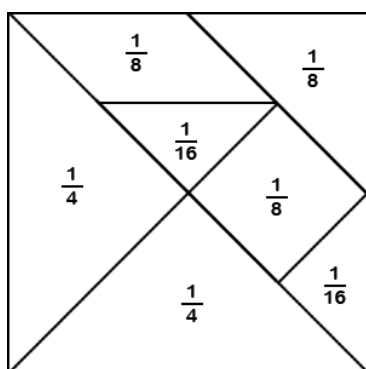


Figure 2. Fractional Areas of Tangram Puzzle

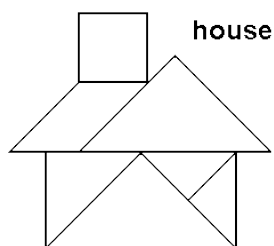
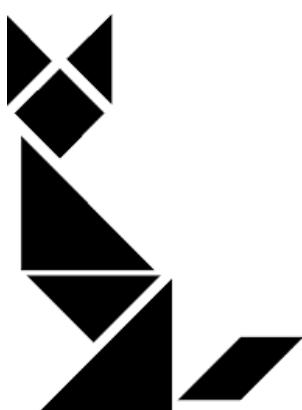


Figure 3. Tangram Images

Teaching and learning with MERLO: a new challenge for teachers and an opportunity for students

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Abstract: The aim of this paper is to present an Italian research experience developed at the University of Turin and involving both researchers and teachers from secondary school. The group combined different background and experience and shared a common interest: working together in order to improve the quality of mathematics teaching and learning. The focus of the work is the refinement and application of an innovative didactical and pedagogical tool, called Meaning Equivalence Reusable Learning Object (MERLO), that is based on shared meaning of semiotic representations in different sign systems.

Résumé: Le but de cet article est de présenter une expérience de recherche italienne développée près de l'Université de Turin et qui entraîne soit de chercheurs soit des professeurs de l'école secondaire supérieure. Ce groupe représente une variété d'expérience et de disciplines académiques différentes avec un intérêt commun : travailler ensemble pour améliorer la qualité de l'enseignement et de l'apprentissage des maths. Le but principal de l'étude est un instrument didactique et pédagogique innovatif appelé Meaning Equivalence Reusable Learning Object (MERLO) qui est fondé sur la signification partagée des représentations sémiotiques dans plusieurs groupes de signes.

Introduction

The application of Meaning Equivalence Reusable Learning Object (MERLO) involves different countries: Canada, Israel, Russia, Italy (Shafrir et al., 2015). In this paper, we focus on the Italian experience, which engages researchers from the University of Turin and teachers of secondary school, coming from Piedmont and Lombardy, who follow a Master program for prospective mathematics teachers' educators at the University of Turin, Department of Mathematics.

As starting point, we will describe the MERLO theoretical framework, which provides a general approach, suitable for different subjects. The MERLO approach was born in a general context and our aim here is to apply it in the field of mathematics education, for the teaching and learning of mathematics at the lower and upper secondary school. The Italian institutional dimension is very important and for this reason some references to Italian national guidelines and to the national assessment tests will be emphasized.

The researchers and the teachers involved in the research experience found in MERLO activities a very useful didactical tool that could be in line with these theoretical directives in Italian schools:

- The coordination of multiple representations of the same object in more than one semiotic register is fundamental for the understanding and learning of the underlying mathematical meaning (Duval, 2006);
- The social aspects are important in human learning processes, because social learning precedes the development of individual competences (Vygotskij, 1934).

Some research results, directed at the design and implementation of MERLO activities in class, will be provided in order to give information to teachers who would like to experience the challenge of using MERLO in their teaching practice.

The paper ends with some final remarks and proposals for the related workshop.

The MERLO theoretical framework

As mentioned, MERLO is an acronym standing for Meaning Equivalence Reusable Learning Objects. It is an innovative didactical and pedagogical tool developed and tested since the 1990s by Uri Shafrir and Masha Etkind at Ontario Institute for Studies in Education (OISE) of University of Toronto, and Ryerson University in Toronto, Canada (Etkind et al, 2010, Shafrir & Etkind, 2014).

They combined in their research the main results related to:

- Cognitive, meta-cognitive and affective aspects in learning processes, also in difficulty contexts;
- Concept science and conceptual thinking;
- Peer cooperation in class (for more about these points see Etkind & Shafrir, 2013).

MERLO is a very adaptable tool and, for this reason, it was applied in different contexts and countries, for several uses and subjects: mathematics, physics, biology, architecture, medicine...

The research (identified here by the name of “MERLO project”) involved also Ron Kenett, an expert in the field of statistics and Ferdinando Arzarello, Ornella Robutti and their research group in mathematics education, from the University of Turin in Italy.

MERLO (Arzarello, Kenett, Robutti and Shafrir, to be submitted; Etkind, Kenett, Shafrir, 2010) is a database that allows the sorting and mapping of important concepts through exemplary target statements of particular conceptual situations, and relevant statements of shared meaning.

Each MERLO activity is structured with:

- A target statement TS that encodes different features of an important concept;
- Four other statements from different types - Q2, Q3 or Q4. As shown in the template in the Figure 1, these different types of statements are linked to the target statement by two criteria: *Meaning Equivalence* and *Surface Similarity*. The term *Meaning Equivalence* designates a commonality of meaning across several representations (e.g. the equation $y = x^2$ and the graph of a parabola in the Cartesian representation); while the term *Surface Similarity* means that representations “look similar”, sharing the same sign system and being similar only in appearance, but not in the meaning (e.g. $y = x^2$ and $x = x^2$).

TARGET STATEMENT TS		Meaning Equivalence	
		YES	NO
Surface Similarity	YES	Q1	Q3
	NO	Q2	Q4

Figure 1: template for constructing a MERLO activity

Experience shows that inclusion of Q1 statements makes the activity too easy (Etkind, Kenett. and Shafrir, 2010); for this reason, we use it only in case of students with great difficulties.

In the MERLO activity for students the type of each statement is not revealed. The students are required to recognize the statements in multiple representations that share the meaning (TS and Q2) and to write the reasons for their decisions. In this way, MERLO activity combines multiple-choice (recognition) and short answers (production).

Application in mathematics education: the Italian research experience

MERLO appears to be a very suitable tool for mathematics education and for the teaching and learning of mathematics in the Italian school context. For this reason, the idea of developing a research experience was born inside the research group in mathematics education of the University of Turin. The Italian experience at the University of Turin involves a Master programme for prospective mathematics teachers' educators. Among the 29 students enrolled in 2015, all are active secondary school teachers. In addition, there is a small group (7 of them) who are also working with researchers on the design, refinement and adaptation of MERLO activities in Italian upper and lower secondary schools. This activity is developed according to the philosophy of a teacher education programme, m@t.abel (see Arzarello et al., 2015), promoted by the Italian Ministry of Education, where the competencies scrutinized by MERLO are deployed.

As an example of MERLO activity consider the following (Figure 2), which was inspired by a question asked in a test of INVALSI, the Italian National Evaluation Institute for the School System (INVALSI, 2012). The test is about recognition of relations and functions in different semiotic systems. It is linked with a real life context and shows:

- A natural language description of two tariff plans, chosen as target statement TS;
- The same tariff plans represented in a different way (Cartesian graph, table and formal language) as Q2 statements, that share meaning, but do not share surface similarity with TS;
- Another Cartesian graph, chosen as Q4 statement, which does not share neither meaning, nor surface similarity with TS.

Mario is going on holiday at a ski area. To take advantage of ski lifts (chair lifts, cable cars, ...), he can choose between two offers, A and B, both valid for the entire winter season.

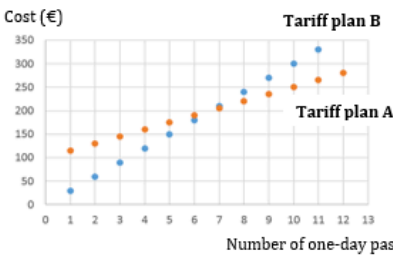
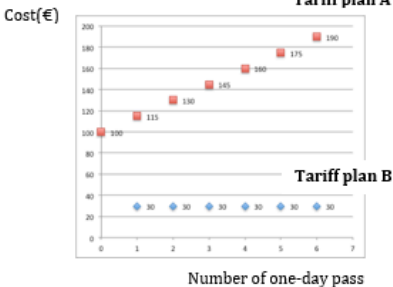
1. Mark the statements (at least two) that share the same mathematical meaning. 2. Write the reasons that guided you in the choice.	TS A[] Tariff plan A: initial fixed cost 100 € plus 15 € per day (for every day you use ski lifts). Tariff plan B: 30 € per day, with no initial cost.	Q2 B[] 																					
Q2 C[] <table border="1" data-bbox="178 1480 550 1727"> <thead> <tr> <th>Number of days in which you use ski lifts</th> <th>Tariff plan A (cost in €)</th> <th>Tariff plan B (cost in €)</th> </tr> </thead> <tbody> <tr><td>1</td><td>115</td><td>30</td></tr> <tr><td>2</td><td>130</td><td>60</td></tr> <tr><td>3</td><td>145</td><td>90</td></tr> <tr><td>4</td><td>160</td><td>120</td></tr> <tr><td>5</td><td>175</td><td>150</td></tr> <tr><td>6</td><td>190</td><td>180</td></tr> </tbody> </table>	Number of days in which you use ski lifts	Tariff plan A (cost in €)	Tariff plan B (cost in €)	1	115	30	2	130	60	3	145	90	4	160	120	5	175	150	6	190	180	Q2 D[] Tariff plan A: $c = 100 + 15g$ Tariff plan B: $c = 30g$ g number of one-day pass c cost (€)	Q4 E[] 
Number of days in which you use ski lifts	Tariff plan A (cost in €)	Tariff plan B (cost in €)																					
1	115	30																					
2	130	60																					
3	145	90																					
4	160	120																					
5	175	150																					
6	190	180																					

Figure 2: an example of MERLO activity

MERLO activities, such as the one just presented, are consistent with national and international guidelines.

Indeed, we can read from Italian national guidelines (MIUR, 2010):

Lo studente studierà le funzioni del tipo $f(x) = ax + b$, $f(x) = |x|$, $f(x) = a/x$, $f(x) = x^2$ sia in termini strettamente matematici sia in funzione della descrizione e soluzione di problemi applicativi. [...] Sarà in grado di passare agevolmente da un registro di rappresentazione a un altro (numerico, grafico,

funzionale), anche utilizzando strumenti informatici per la rappresentazione dei dati.

The student will study functions as $f(x) = ax + b$, $f(x) = |x|$, $f(x) = a/x$, $f(x) = x^2$ both in strictly mathematical terms and in function of the description and solution of applied problems. [...] He/She will be able to shift easily from one register of representation to another one (numerical, graphical, functional), also using computer tools for data representation. (Our translation)

And from PISA 2012 mathematics framework (OECD, 2013):

Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizen.

In addition, the typical MERLO task for students based on the recognition of a shared meaning in different sign systems is in line with national (INVALSI, in Italy) and international (PISA, TIMSS) assessment tests, where the ability of shifting between representations of the same object into different registers is widely evaluated.

We believe that the use of MERLO activities is particularly appropriate in the institutional context of Italian schools, where nowadays the challenge is to work on students' skills and their assessment at the end of the first two years of upper secondary school.

The use of MERLO could give a contribution in this direction, providing one more tool for this type of evaluation. Furthermore, it may compete, with other types of assessment, in bringing out the best aspects of each student (multiple intelligence) by teachers. In a next section we will show and explain some possible ways for implementing and using MERLO in class.

Results from the UNITO experience

In the following sections we present some results based on the research experience at the University of Turin (UNITO), from September 2014 until May 2015. The results concern in particular some methodological choices about the design and the implementation in class of MERLO activities. The methodological choices were born from the joined work of researchers and teachers, who share some practices after exchanging knowledge and experience.

The design of MERLO activities

Our aim in discussing the design of MERLO activities is to describe the process for creating the final product and its evolution in time. We argue that it is useful to the reader to understand the process and not only the presentation of some examples in their final version. With this objective in mind we start describing the general process that teachers can follow to design a MERLO activity and then we present a specific example.

The first step in creating a new MERLO activity is clarifying the teacher's choice of the mathematical concept or the mathematical knowledge on which to work with the class. Once it has been chosen, the teacher-author creates around it a set of representations that share the same meaning. This set is included within a BoM - Boundary of Meaning, which establishes the boundary of the shared meaning by the representations. Outside the BoM, in a disjoint set, there are representations that have no common meaning with the previous ones and with the chosen mathematical concept. However, some representations that are outside the BoM may present an exterior similarity with those inside the BoM, due to similar words or kind of representation.

A MERLO item is designed with five statements, with at least two of them internal to the BoM and the remaining ones being external to the BoM. The students are required to identify only the internal ones and justify their choice. External elements act as distractors, which can attract a student who does not know well the underlying mathematical concept. A delicate task for the

teacher is to produce not too obvious distractors. The teachers' experience in teaching in class with students can get useful information about common errors and spread misconceptions.

An important aspect to highlight about the design of a MERLO activity is the following: all statements, both internal and external to the BoM, are true from the mathematical point of view. This is a particular choice, shared by our group of researchers and teachers, consequent of an extensive internal debate. At the beginning of the experience some MERLO activities were designed taking into account also statements that were mathematically false, following the traditional style of national and international tests. However a reflection on MERLO methodology, based on recognition of shared meaning between different representations, eventually led the group to agree that is most formative to put the student in front of the comparison of items that have true (although different) mathematical meanings.

The members of the group discussed a lot each other about the Italian formulation of the task. This is the original MERLO task for students, formulated in English:

At least two out of these five statements – but possibly more than two – share equivalence-of-meaning.

1. Mark all statements – but only those – that share equivalence-of-meaning.
2. Write down briefly the reasons that guided you in making these decisions.

It inevitably required a rethinking for adapting to a mathematical context inside an Italian culture and not only a simple literal translation. After various changes, deriving from comparison also with other mathematicians, the task for students assumes the following Italian formulation:

1. Segnare le rappresentazioni (almeno due) che condividono lo stesso significato matematico.
2. Indicare le ragioni che guidano nella scelta.

- [1. Mark the statements (at least two) that share the same mathematical meaning.
2. Write the reasons that guided you in the choice]

Finally, teachers have to consider another important aspect during the design process of a MERLO activity and in particular during the design of the representations that are external to the BoM: there is the possibility of some shared meaning between these statements. We want to avoid every possible link among statements outside the BoM, otherwise there might be two different ways to complete the task. We prefer to design MERLO activities with a single answer, aspect we ought to highlight to students, saying them that each MERLO sheet has a single answer.

The methodological choices just described and shared inside the group of researchers and teachers, do not arise by chance, but they are the result of the experience in the design of new MERLO activities inside an Italian mathematical context. These two aspects, that are the design process and the growth of methodological choices, are closely interrelated. As a result, the MERLO activities produced by the group follow various stages, evolving over time (design – revision – re-design).

As an example consider the data and forecast concept and in particular the notion of percentage frequencies. The inspiration for this example is an INVALSI question (INVALSI, 2012), which was transposed into a MERLO activity. The first version was designed at the beginning of the initiative and it is heavily influenced by traditional tasks such as INVALSI tests. This can be seen from the Figure 3 and in particular from square B.

From the first to the second version several changes are observed, not only referred to the square B but also in E and in the formulation of the task for students.

In the third version (Figure 5) there is a change in a graphical representation.

The Figure 6 represents the final version for students, where the type of each statement is not revealed and the teacher can change the position of statements.

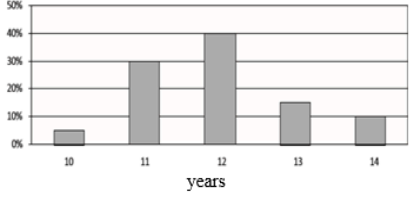
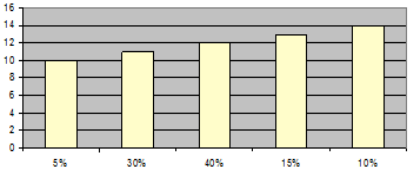
	TS A [] Children by age (in percentage) 	Q2 B [] Read a data distribution, extracts the information, perform operations from the data, given from the exercise.
Q2 C [] The percentage of children with 10 or 13 years old is the half of those with 12 years old, and is equal to the difference between the percentage of children with 11 years old and 14 years old.	Q3 D [] Percentage of the children's age 	Q4 E [] Every age has on average 20% of the children and the average age of the children is 12 years old.

Figure 3: first version of MERLO activity about percentage frequencies

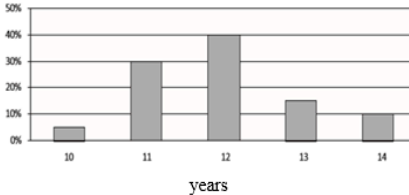
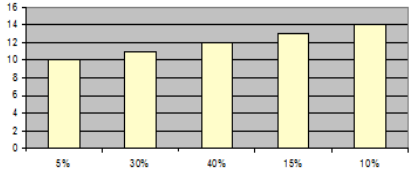
	TS A [] Children by age (in percentage) 	Q2 B [] A gym is attended by 200 children: 10 of them are 10 years old 60 are 11 years old 80 are 12 years old 20 are 14 years old and the remaining are 13 years old
Q2 C [] The percentage of children with 10 or 13 years old is the half of those with 12 years old, and is equal to the difference between the percentage of children with 11 years old and 14 years old.	Q3 D [] Percentage of the children's age 	Q4 E [] For calculating the arithmetic average of age of a group of children, you have to add up all their age and then divide the result by the total number of children.

Figure 4: second version of MERLO activity about percentage frequencies

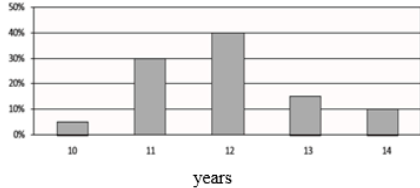
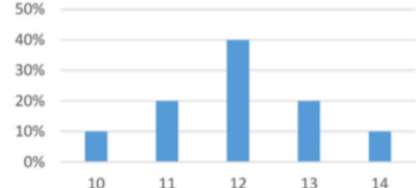
	TS	Q2
1. Mark the statements (at least two) that share the same mathematical meaning. 2. Write the reasons that guided you in the choice.	A [] Children by age (in percentage) 	B [] A gym is attended by 200 children: 10 of them are 10 years old 60 are 11 years old 80 are 12 years old 20 are 14 years old and the remaining are 13 years old
Q2	Q3	Q4
C [] The percentage of children with 10 or 13 years old is the half of those with 12 years old, and is equal to the difference between the percentage of children with 11 years old and 14 years old.	D [] Children by age (in percentage) 	E [] For calculating the arithmetic average of age of a group of children, you have to add up all their age and then divide the result by the total number of children.

Figure 5: third version of MERLO activity about percentage frequencies

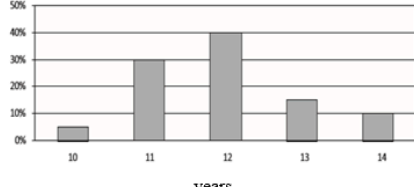
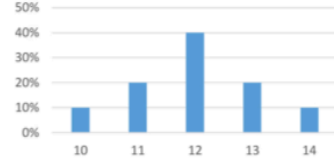
	A []	B []
1. Mark the statements (at least two) that share the same mathematical meaning. 2. Write the reasons that guided you in the choice.	Children by age (in percentage) 	A gym is attended by 200 children: 10 of them are 10 years old 60 are 11 years old 80 are 12 years old 20 are 14 years old and the remaining are 13 years old
C []	D []	E []
For calculating the arithmetic average of age of a group of children, you have to add up all their age and then divide the result by the total number of children.	The percentage of children with 10 or 13 years old is the half of those with 12 years old, and is equal to the difference between the percentage of children with 11 years old and 14 years old.	Children by age (in percentage) 

Figure 6: student version of MERLO activity about percentage frequencies

The implementation of MERLO in class: activities

In this section we present some methodological suggestions about the implementation of MERLO activities in class, in order to help teachers in their work at school. The starting point is the description of the way in which a MERLO item is used in a Canadian context where it was born and in particular in a Department of Architecture. The paper by Etkind, Kenett, Shafrir (2010) describes a MERLO interactive quiz as an in-class procedure that provides learners with opportunities to discuss a PowerPoint display of a MERLO item in small groups, and send their individual responses to the instructor's computer via mobile text messaging, or by using a clicker (CRS – Classroom Response System).

See <https://docs.google.com/file/d/0BxJwogdRc6UHYTVaRVN0RWdvQms/edit>. The authors write that such a quiz takes 20-30 minutes, and includes the following 4 steps (Etkind, Kenett, Shafrir, 2010):

1. Small group discussion - approximately 5 minutes - following PowerPoint projection of a MERLO item, students are asked to form small discussion groups of 3-5.
2. Individual response - approximately 3 minutes - each student enters the *recognition response* on her clicker (CRS - Classroom Response System) or cell phone, marking at least 2 out of 5 statements in the MERLO item that – in his/her opinion – share equivalence of meaning; then writes down his/her *production response*, briefly describing the concept he/she had in mind while making these decisions, and turns the page upside down on her desk.
3. Feedback on *production response* and class discussion - approximately 5 minutes - PowerPoint projection of the MERLO item, including the teacher’s description of the conceptual situation; followed by students’ discussion and comparison of their individual production responses.
4. Feedback on *recognition response* and class discussion - approximately 5 minutes - PowerPoint projection of the MERLO item, showing the correct recognition feedback (i.e., correctly marked/unmarked statements); followed by students’ discussion and comparison of their individual recognition responses.

The application of MERLO pedagogy in the Italian context of secondary school required a refinement of the methodology just described because most Italian secondary schools do not have technological devices like clickers and sometimes there is not even the possibility of projecting a Power Point slide in class. In addition, some changes were needed for a cultural adaptation and also for an adaptation to secondary school level, that is obviously different from University level.

During our research experience, researchers and teachers were involved in discussions and debates on a theoretical level and in the following experimentation in class, in order to reach a suitable methodology for implementing the MERLO pedagogy in our context. At the end the rethinking leads to a reorganization of the methodology, which remains still divided into various phases, even if times are not strictly defined.

For involving students in a classroom activity that lasts about an hour, the teacher can give two or three MERLO sheets (depending on the level of their difficulty in relation also with the preparation of the class). The printed sheets are delivered to each student who will be involved in the following phases:

1. Individual phase (15-20 minutes): each student, after receiving the MERLO sheets with the activities, is required to identify which boxes are linked by a mathematical concept and to write the reasons for his/her choice. We would like to stress the importance to write the reasons for the choice, in order to develop students’ argumentative skills. At the end of this phase the sheets with individual answers are collected by the teacher.
2. Phase in groups of three or four students (15-20 minutes): the teacher divides the students into groups (each group should be composed with pupils of the same level, in order to promote discussion). A blank copy of the MERLO activities that had been solved in the previous individual phase, is returned to each group: the pupils have to compare their personal choices with those of the classmates, discussing for arriving at the ultimate goal of a shared answer.
3. Class discussion (15-20 minutes): the final discussion moderated by the teacher collects the views shared within the groups or the views of those individuals who did not arrive at an agreement with their group. The next metacognition phase is aimed at the clarification and reflection on the personal process of construction of knowledge.

Hence, as described, teachers can use MERLO as a tool to set up activities based on students’ working groups and on classroom discussions. The students are also requested to account for their answers and so the discussion among students and between them and their teacher is promoted. The

social aspects are important for MERLO activities resolution and then they may foster learning processes. In this perspective, we think MERLO pedagogy is in line with Vygotsky's thought that human learning presupposes a specific social nature (Vygotsky, 1934).

The implementation of MERLO in class: assessment

MERLO can be used for formative assessment in class: teachers can use MERLO activities to check what students really understood and to receive a feedback about the level of comprehension of a mathematical concept in class. The information coming from a MERLO activity and in particular from different kinds of mistakes, gives useful suggestions both to teacher and to student about the kind of deficit in comprehension. Indeed, if a student does not mark a Q2 statement, then it means that he/she has an incomplete understanding because he/she does not recognize the same concept represented in a different way with another sign system; if a student marks a Q3 statement, then it means that his/her understanding is superficial and influenced by similar representations; while the identification of Q4 as connected to the conceptual node is significant when the Q4 is a good distractor, that is "close" in the meaning, although not sharing it with the target statement.

For now we see the MERLO activity just as formative assessment, to improve and promote the connected processes of teaching and learning mathematics at secondary school.

We are also experimenting MERLO for oral questions in class: we propose a MERLO activity and the student is asked to identify statements that share a mathematical meaning, and to say orally which it is. Which may be the differences in oral performances with respect to written? Often in the work only on the paper some students, mainly those who are in greater difficulty, match the right answer but do not write reasons for their choices (even if they have some reasons). During an oral discussion, instead, the teacher can investigate the reasons of the choices, can guide and help these students in making explicit the concepts they have in mind. The teacher has a role of mediator in this case. He/She also can ask which is the meaning of the other boxes (those out of the boundary of shared meaning) and why the students did not chose them.

How could we assess this performance? If a student marks only the statements that share the same meaning and is not able to give the reasons that guided him/her in the choice, then we can say he has a basic level. Instead, if he/she can argue and say the reasons for the choices, then he/she has reached a medium level. Finally, if a student is able to explain exhaustively the meaning of each statement and the relationships among them and gives also the reasons for not a choice, then he/she has reached an advanced level.

Final remarks and proposals for workshop

As final remarks we have the pleasure to quote some sentences (said by the teachers and by the students involved in the research experience) to highlight the didactical potentiality of MERLO.

Teacher A:

We think it is a useful tool for teachers and students, because it helps and stimulates the arguing, starting from an object to think about and discuss.

Teacher B:

Using MERLO in oral questions in class, it is easier for me to know students mental processes. Because some of them make a choice but do not write anything about arguing, for several reasons...

Interesting observations emerged from students of the degree course in mathematics (education address, future teachers). They were involved in the resolution and analysis of some MERLO activities and these are the meaningful words that came out in the final discussion:

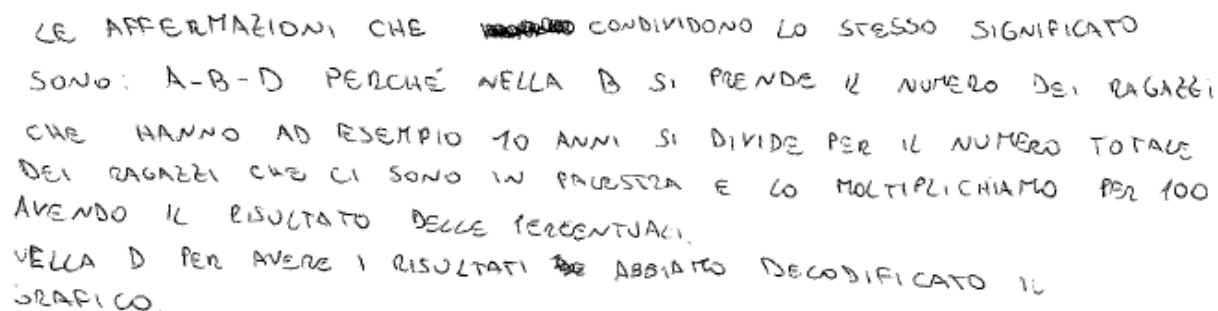
I think MERLO has a big usefulness, because it allows to really understand. It takes away the rigidity of mathematics, that the school tends to give (with a textbook or a traditional lecture). Even now at

University, when ideas are already clear, it makes you see the same thing in different ways.

About the idea of seeing the same mathematical thing in different ways, we can quote the words of a student, who gives these explanations:

Oh, this is a graphical representation of that definition!

Here we see that the student can recognize the same concept in different registers. In general, MERLO provides activities that should develop this expertise in students, even in the case of pupils with particular difficulties. Here are the observations came out in a problematic class, as answer to the MERLO shown in Figure 6:



LE AFFERMAZIONI CHE ~~NON~~ CONDIVIDONO LO STESSO SIGNIFICATO SONO: A-B-D PERCHÉ NELLA B SI PRENDE IL NUMERO DEI RAGAZZI CHE HANNO AD ESEMPIO 10 ANNI SI DIVIDE PER IL NUMERO TOTALE DEI RAGAZZI CHE CI SONO IN PALESTRA E LO MOLTIPLICHIAMO PER 100 AVENDO IL RISULTATO DELLE PERCENTUALI. NELLA D PER AVERE I RISULTATI ~~DE~~ ABBIAMO DECODIFICATO IL GRAFICO.

Figure 7: student's answer

The statements that share the same meaning are: A-B-D because in B we take the number of children with 10 years old, for example, we divide it for the total number of children who are in the gym and multiply by 100, having the result in percentages. In D, to have the results, we have decoded the graphic.

In conclusion, we think that the spread of MERLO pedagogy at schools could improve the quality of the teaching and learning of mathematics. Thinking that an active involvement can be most effective to approach this new kind of pedagogy, we are planning to engage the participants in a workshop. We will propose some MERLO activities about different conceptual nodes (numbers, geometry, relations and functions, data and forecasts), directed to different scholastic levels: the participants will required to solve and analyze them in the perspective of teacher, working and discussing in groups. The workshop will end with a final discussion, which will collect the various points of view and will offer the possibility of a comparison among researchers and teachers. Since it is a workshop, we chose to focus the paper on operational aspects: from the design process of a MERLO activity, to the implementation in class. However if the reader is interested in some more theoretical aspects, he/she can read the plenary in this conference by Robutti: "Mathematics teacher education in the institutions: new frontiers and challenges from research" (Robutti, 2015), where the research experience is analyzed with theoretical lens.

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Origami: an important resource for the teaching of Geometry

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Abstract: During our workshop, through simple materials manipulation, as paper folding, we introduced several geometrical concepts. Combining the pleasure of make nice objects, we discovered and analysed geometrical properties of polygons. We have found out that even the demonstration of theorems could be simplified using origami.

Résumé: Pendant l'atelier nous avons introduit plusieurs concepts générales de géométrie en utilisant la manipulation de matériels simples, comme le plissage du papier. Le plaisir de créer de petit objets s'est mélangé avec la découverte des propriétés géométriques des polygones. La démonstration des théorèmes aussi, comme nous avons expérimenté, a été simplifiée en utilisant les origamis.

Main goals

The main aim of this kind of innovative approach to the teaching of Geometry is to improve student's knowledge of the geometric topics selected for the class and to develop learning skills by exploring and studying the topic while folding the model. (Golan & Jackson, 2009)

Students have a greater involvement in the learning process: from a spectator role to a protagonist role. This deepens their knowledge and motivates them to learn more.

Methodology

Origametry, the approach to Geometry concepts through origami, is a discovery experience. The teacher guides students in this process, stimulating both observation and reflection. Starting from a visualization level, students are brought to reflect on the properties of the geometrical figure (analysis level) and to find relationships between figures and properties (abstraction level) (Van Hiele, 1986), developing logical and sequential thinking. Throughout folding the final object is never revealed: on one side it helps students focusing on geometric properties and on the other side nourishes curiosity and pleasure of discovery.

Observed conclusion

Testing origami approach to different geometrical topics in several schools, through workshops, we had the opportunity to observe that geometric terminology, linked to paper folding experiences, becomes really part of student's own knowledge.

Furthermore paper folding activity helps to involve weaker students too, enhancing their self-esteem thanks to a different approach to Geometry that involves different skills and works on emotional aspects, like curiosity, pleasure and joy. Through a manual activity they can discover geometrical properties themselves, which will improve their self-confidence and the learners become more aware of their mathematical skills.

Both accuracy and the keeping of the rules are picked up in a natural way. Furthermore, this approach increases collaborative behaviour and cooperation among pairs producing a serene working atmosphere.

Theoretical background: Van Hiele level of geometric understanding

Pierre van Hiele theory involves levels of thinking in geometry that students pass through as they progress from merely recognizing a figure to being able to write a formal geometric proof (Mason, 2009).

There are five levels, which are sequential and hierarchical.

Level 1 (Visualization): Students recognize figures by appearance alone, often by comparing them to a known prototype. The properties of a figure are not perceived. At this level, students make decisions based on perception, not reasoning.

Level 2 (Analysis): Students see figures as collections of properties. They can recognize and name properties of geometric figures, but they do not see relationships between these properties. When describing an object, a student operating at this level might list all the properties he knows, but not discern which properties are necessary and which are sufficient to describe the object.

Level 3 (Abstraction): Students perceive relationships between properties and between figures. At this level, students can create meaningful definitions and give informal arguments to justify their reasoning. Logical implications and class inclusions, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood.

Level 4 (Deduction): Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. At this level, students should be able to construct proofs such as those typically found in a high school geometry class.

Level 5 (Rigor): Students at this level understand the formal aspects of deduction, such as establishing and comparing mathematical systems. They can understand the use of indirect proof and proof by contrapositive.

According to van Hiele theory, progress from one level to the next level is more dependent on educational experiences than on age or maturation. Some experiences can facilitate (or impede) progress within a level or to a higher level.

A student progresses through each level of geometric understanding as a result of instruction that is organized into five phases of teaching.

Information: Through discussion, the teacher identifies what students already know about a topic and the students become oriented to the new topic.

Guided orientation: Students explore the objects of instruction in carefully structured tasks such as folding, measuring, or constructing. The teacher ensures that students explore specific concepts.

Explicitation: Students describe what they have learned about the topic in their own words. The teacher introduces relevant mathematical terms.

Free Orientation: Students apply the relationships they are learning to solve problems. They can investigate more open-ended tasks.

Integration: Students summarize and integrate what they have learned, developing a new network of objects and relations by reflection

Origametria and the Folding together project

The name *Origametria* is made from the words *origami* and *geometry*. The term was created by the Israeli Origami Centre (IOC), founded by Miri Golan in 1992, to describe its innovative program to teach curriculum geometry through origami. Since 2008, Israeli Ministry of Education has formally approved this program that has become part of the elementary schools geometry curriculum.

The IOC began to teach an origami program with the purpose of developing learning skills. The program was designed to enhance self-esteem and a sense of accomplishment, while developing learning skills such as motor skills, spatial perception, logical and sequential thinking, hand-eye coordination, focusing and concentration, aesthetics and 3D perception.

One of the projects developed by the centre, the Folding together project, brings together Israeli and Palestinian children to make origami. Origami were chosen because of several reasons.

From a relational point of view, it promotes collaboration and co-operation among children. Furthermore, it does not require a special talent, so any child can be involved in the project. Then origami is a relatively inexpensive activity to run, only paper is required.

From a didactical point of view, origami has many educational benefits such as helping to understand Mathematics and Geometry, increasing spatial awareness and fine motor control, as well as concentration. It helps students to understand and interpret verbal and written instructions.

The children are proud of what they make. They enjoy showing their models to friends and family, thus extending the positive message of the project into many social environments beyond it.

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Reflective activities upon teaching practices reflexes: grades and errors

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Abstract: In this multi-levelled (the individual, the small sub-group, the whole group of the sub-groups) Reflective Laboratory we focus on the numeric grade as the result of implicit diverse mathematics grading processes that co-exist within and across the levels of the educational system in the teaching and learning mathematics. In order to reveal the sedimented grading processes, the participants are invited to successively assume diverse roles at three levels: class (teacher), school unit (principal), educational system (minister/secretary/inspector of education). From the perspective of each level, we shall reflect upon the systemic interactions amongst and across the students' frequent mathematics errors, the assigned numeric grades, the interpretations, the pedagogic and broader educational actions. The exchange of the successive views is expected to reveal the noematic convergences and divergences of practice in the mathematics teaching and learning that exist in the complex teaching-learning space emerging amongst the classroom, the school unit and the broader educational system.

Résumé: Dans ce réfléchissant laboratoire à plusieurs niveaux (l'individu, le petit sous-groupe, l'ensemble du groupe des sous-groupes), nous nous concentrons sur la note numérique comme le résultat de diverses processus implicites de classement d'élèves en mathématiques qui coexistent au sein et entre les niveaux du système éducatif. Afin de révéler l'aspect interactive de ces classements, les participants sont invités à assumer successivement différents rôles à trois niveaux: la classe (enseignants), l'unité de l'école (directeur de l'école), le système éducatif (ministre / secrétaire / inspecteur de l'enseignement). Du point de vue de chaque niveau et à travers de fréquentes erreurs mathématiques des élèves, nous allons réfléchir sur les interactions systémiques entre les notes numériques attribuées à ces erreurs, leurs interprétations et les initiatives éducatives en vue. Notre objectif est de faire révéler les convergences et divergences des pratiques et/ou des représentations noématiques qui coexistent dans cet espace complexe d'enseignement-apprentissage au sein de la salle de classe, l'unité de l'école et le système éducatif en général.

Errors, grading(s) and grades

The students' errors lie at the heart of the educational processes gathering the interest of mathematics education researchers, practitioners and policy makers (Ruthven, 2000). Mathematics education researchers have focussed on the nature of the students' errors and on the cognitive processes that are linked with these errors (for example, the theme of the CIEAEM 39 meeting in Sherbrooke was "The role of errors in the learning and teaching of mathematics"), identifying a multiplicity of sources that may cause a specific error, thus identifying a multiplicity of 'misconceptions' (or alternative conceptions; Fujii, 2014). Furthermore, the students' errors are present in everyday practice, being an indispensable part of the teaching-learning process. They help in revealing the divergences of the constructed meanings, thus constituting 'sign-posts' indicating 'off-course' (or alternative) learning paths.

By grading these errors, the protagonists of the educational process (including the teachers, the learners, the principals, the families, the policy makers) obtain a measure of the quality of the educational outcome. Considering the multifaceted teaching-learning phenomena and that in most cases this measure is condensed to a simple number, in this workshop we attempt to reveal the complexity hidden within each grade, thus revealing the noematic divergences that may lie within the communications amongst the protagonists. For example, the principals may analyse the grades to obtain a measure of the teachers' quality of their teaching and/or to identify the high (or low)

attaining classes or students. This information may be communicated to the Ministry of Education or to the students' families and to the broader community. Furthermore, the policy-makers utilise errors to filter-out the students who have not reached the required level of understanding of the subject-matter: the exams grades are the gate-keepers of the educational system, including the access to higher education degrees. Moreover, international organisations classify countries according to the students' performance in tests (for example, PISA, TIMSS), which in turns affects the educational policies of the classified countries.

Drawing upon a training instrument developed in the Laboratory of Learning Technologies and Didactical Engineering of the University of the Aegean (LTDE; Kalavassis, Kafoussi & Skoumpourdi, 2005), a series of activities are proposed to un-settle the established teaching practices reflexes that someone may hold with the purpose to construct bridges between two constructions of the meaning of a grade:

- a) the *top-down approach* (the grade, explicitly or implicitly, is assigned according to external to the class criteria; such as the National Exams), and
- b) the *bottom-up approach* (the grade is assigned according to context-situated, teacher-sensitive, class-specific criteria).

Hence, in this workshop, the semiotic polysemy that a grade entails is highlighted to reveal the plethora of pedagogical implications and choices linked with the students' grades.

Systemic interactions: school class, school unit, educational system

The polysemy of a grade is evident in the educational process. For example, during the course “Didactics of Mathematics and Science: Interdisciplinary approach” of the newly founded master's programme *Didactics of Mathematics, Science and I.C.T.: Interdisciplinary Approach* of our Department, the students were asked to grade hypothetical students' responses and to discuss their rationales and the pedagogical implications. It was revealed that the same grade or different grades could be linked with the same or diverse rationales, including a variety of aspects such as the epistemological views of the respondents, the context and the pragmatic implications of the grade (for example, entering university).

This situation can be modelled as a multivariate function with each respondent assigning different weights to each variable, thus defining qualitatively different functions. Hence, the grade acts as a single *semiotic accumulation point* to which different *noematic functions* may converge. The diversity of the converging functions is nevertheless sedimented to a simple sign, thus condensing (even conveniently masking) the complex relationships held by each protagonist about the corpus of the epistemic knowledge, the school unit and the other protagonists.

Moreover, we posit that these functions may act and/or have implications on three distinct, yet interacting, organisational levels:

- on the *micro-level* (for example, the individuals' life or a class),
- on the *meso-level* (for example, a school-unit or a district), and
- on the *macro-level* (for example, a country or networks of countries such as UNESCO).

In order to gain deeper understanding about the process of grading students' errors and the interactions that occur amongst the organisational levels, we propose a *soft systemic approach* according to which we consider the overarching educational system within which we identify the sub-systems of the school unit and the school class. A *system* (Bertalanfy, 1968) can be viewed as an integrated whole, with specific goals, clearly differentiated from its environment, whilst structurally and functionally supersedes its parts and their properties.

Following this systemic approach, the school unit may be viewed as an open system, interacting

with the broader systems of the society and the educational system, as well as with the narrower sub-system of the school class (Moutsios-Rentzos & Kalavasis, 2012). Though the educational system sets the broad educational goals, each school unit and school class constitute sub-systems with their own special characteristics that interpret and re-define the broader goals to fit their own goals, which are affected by the protagonists of each sub-system and the specific social context within each school unit is settled.

The workshop: bridging top-down and bottom-up grading approaches

In this workshop, simulate the grading process considering the inter-systemic and intra-systemic interactions sedimented to a single grade. The conceptual structure of the workshop is diagrammatically outlined in Figure 1. Drawing upon the aforementioned training instrument developed in LTDE, upon the micro, meso, macro level differentiation and upon the top-down and bottom-up contrast, the workshop is organised two parts. The structure of the instruments utilised in the workshop is diagrammatically outlined in Figure 2.

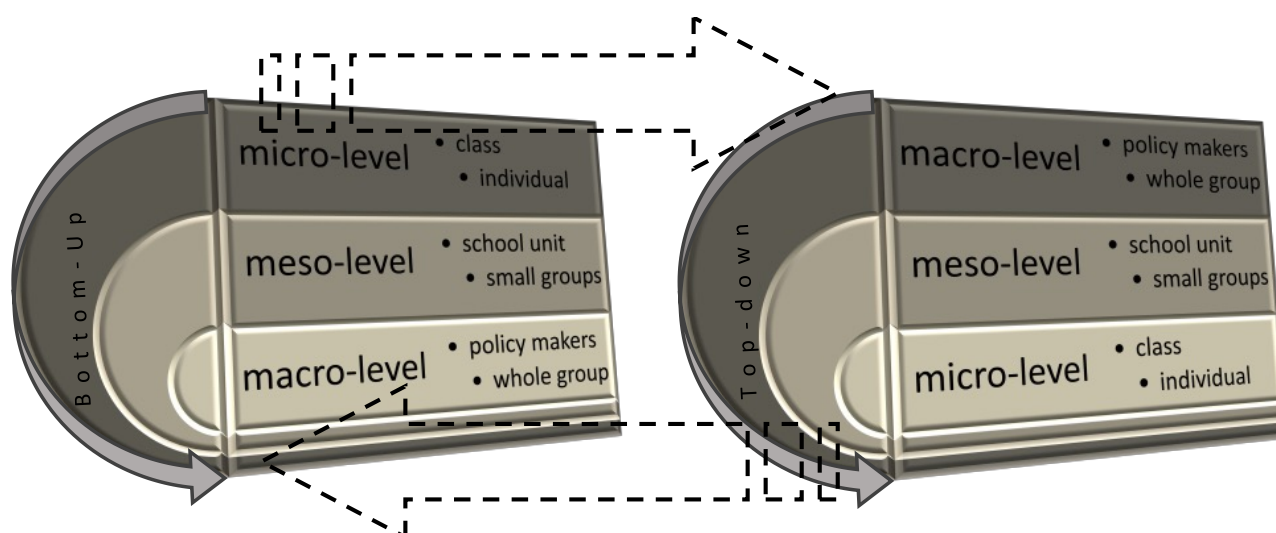


Figure 1. Diagrammatic outline of the structure of the workshop.

Following these, in the first part, simulating a bottom-up grading approach, the participants are presented with sets of students' responses and they are asked to grade them. Each set may contain responses that are all correct, or all erroneous, or half correct and half erroneous. The participants are asked to first work *individually* (micro-level; simulating the teacher in a single class), subsequently in *small groups* (meso-level; simulating the teachers and the principals in a school unit) and finally as a *whole group* (macro-level; simulating the decisions made by policy makers). During each phase, the participants are asked to consider and to reflect upon: the meaning of the assigned grade; their rationale backing their choice of grade (including teaching-learning experience, epistemology, mathematics, psychology); the appropriate pedagogical actions linked with the assigned grade. The results of each phase will be compared and contrasted in order to unearth the implicit stereotypes that may affect the grading process. We posit that the bottom-up approach to giving meaning to a grade combined with the reflections upon the meaning transpositions that may occur as the participants assume the expected roles in the different levels-systems allow for our gaining deeper understanding in the complexity that a grade entails.

In the second part, following a top-down grading approach, the participants are asked to discuss a grading scheme provided by an official organisation (for example, the grading scheme provided by the Ministry of Education about a National Exams task) regarding one of the tasks discussed in the

first part of the workshop and to consider the implications in the three levels (systems/subsystems): the educational system (macro-level), their school (meso-level) and their class (micro-level).

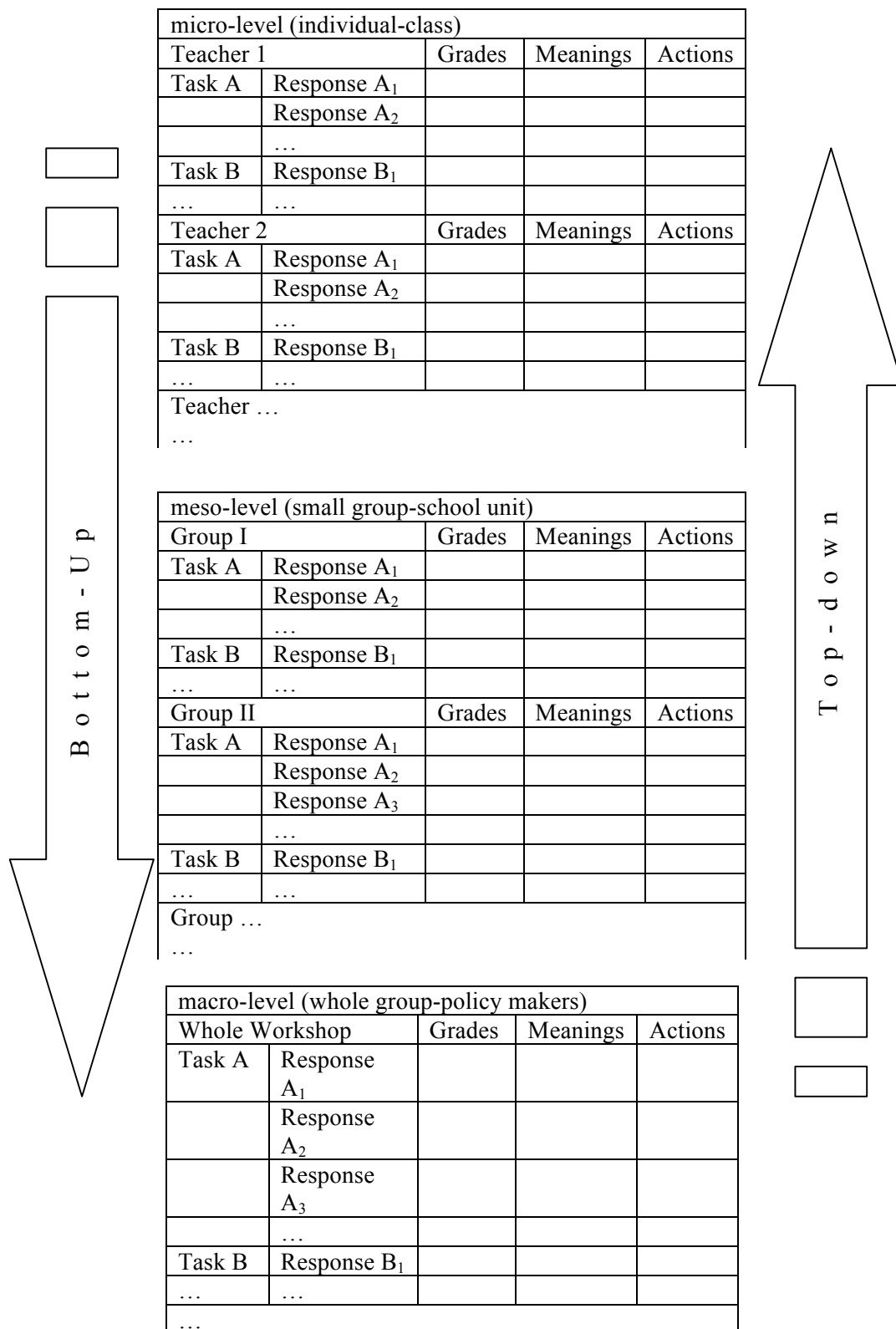


Figure 2. The structure of the instruments utilised in the workshop.

The workshop will conclude with a discussion about the noematic convergences/divergences within/amongst the two grading processes and the three levels, in order to identify aspects that may be meaningfully bridged and aspects that are inherently incongruent noematic constructions. Furthermore, we shall reflect upon the condensed complexity of the space of educational interaction and upon the fragility and unpredictability of its consequences in the environment accentuated by the stresses that the market and the digital era impose on mathematical education.

Overall, in this workshop we expect the participants to meaningfully re-position themselves with respect to the grading processes and the assigned numeric grades. Drawing upon successive reflections upon the interpretations, the constructed meanings and the corresponding actions in terms of the diverse roles and perspectives, we aim to reveal the plethora of realities, the multiplicity of meanings and pedagogical actions and consequences in the students' learning mathematics that co-exist within (and sometimes conveniently masked by) a simple number: the assigned numeric grade to a student's response.

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