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**MATHEMATISATION: PROCESSUS SOCIAL  
& PRINCIPE DIDACTIQUE**



## Mathematisation as didactic principle seen through teachers' descriptions of mathematical modelling

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**Abstract.** Considering the 'popularity' of mathematical modelling as an arena for using mathematisation as didactic principle, teachers' conceptions of mathematical modelling in relation to how they view their main task as mathematics teachers, as well what use the mathematics students learn at school might be, was investigated drawing on interview data. While the teachers generally separated 'reality' and 'mathematics', the potential diversity of mathematical descriptions and the problematic nature of any "translation" were generally not discussed. Overall goals of teaching mathematics such as understanding, interest and usefulness were emphasised rather than relating modelling to mathematisation as didactic principle.

**Résumé.** Compte tenu de la «popularité» que la modélisation mathématique a gagnée en tant que domaine d'utilisation de la mathématisation comme principe didactique, les conceptions des enseignants de la modélisation mathématique par rapport à la façon dont ils considèrent leur tâche principale en tant que professeurs de mathématiques, ainsi que ce que les étudiants en mathématiques apprennent à l'école pourrait être, a été étudié en tirant parti des données d'entrevue. Bien que les enseignants se séparent généralement de la «réalité» et des «mathématiques», la diversité potentielle des descriptions mathématiques et la nature problématique de toute «traduction» n'ont généralement pas été discutées. Les objectifs généraux d'enseignement des mathématiques tels que la compréhension, l'intérêt et l'utilité ont été soulignés plutôt que de rapprocher la modélisation de la mathématisation en tant que principe didactique.

### a) Introduction

In mathematics education, mathematical modelling has become 'popular' as an arena for using mathematisation as a didactic principle making it possible to draw on real world contexts while not using standard word problems. As Jablonka and Gellert (2007), elaborating on mathematisation as a didactic principle, observe:

Since problems given in textbooks generally do not claim to mirror problematic situations authentically, the mathematisation required from the students is essentially an artificial activity (Jablonka & Gellert, 2007, pp. 2-3)

Mathematical modelling is often promoted with the claim to provide potentially more authentic and hence less "artificial" situations as well as the idea of potential adjustment of a model. Mathematisation as didactic principle in this view is contrasted with modelling, as the latter is driven by extra-mathematical interests (Jablonka, 1996; Skovsmose, 1990). Referring to Basil Bernstein's notions of horizontal and vertical discourse and Treffer's (1987) descriptions of horizontal and vertical mathematisation, Jablonka and Gellert (2007) further write:

The fiction is, that abstraction from extra-mathematical contexts to mathematical concepts and structures is possible and straightforward, but, actually, this process is a step from the contradictory world to a coherently organised esoteric sphere that has long since cut off its everyday roots. (p. 3)

Based on these observations, this paper looks at a group of teachers' conceptions of mathematical

modelling in relation to their ideas of their main task as mathematics teachers and of what use the mathematics students learn at school might be.

## b) Method

This paper draws on interview data collected by Frejd (2011), who investigated upper secondary mathematics teachers' conceptions of mathematical modelling, using a grounded theory approach<sup>1</sup>. From the re-analysis of the data presented here, linking the answers to the interview question A: *What is your interpretation of the notion of mathematical modelling?*, provided by the 18 teachers to their answers to the interview questions B: *How would you describe your main task as a teacher of mathematics?*, and C: *What use do you think your students will have of the mathematics they learn?*, this paper intends to illuminate these teachers' views of mathematisation, its relation to what they see as modelling, and/ or its importance as a didactic principle.

A thematic analysis of the available interview data seemed adequate for the rather open research interest of this paper. According to Braun and Clarke (2006), a thematic analysis is "a method for identifying, analysing and reporting patterns (themes) within data" (p. 78), giving it both theoretical freedom and flexibility. While assuming that what the teachers expressed during the interviews reflected their experiences and opinions on what was discussed, one also needs to point out that formulations of key notions and questions, as well as the interview settings themselves, are constituent elements of the interview discussions. As the overall contexts and social relationships that might explain the opinions put forward by the participants (latent themes) were not considered, this paper has tried to identify semantic themes that reflect the main patterns found in the data (Braun & Clarke, 2006, p. 86).

## c) Results of the thematic analysis

In tables 1, 2, and 3, the results of the thematic analysis are summarised, including the themes and some examples of codes constituting these themes. In Table 4, the coding of the answers of each individual teacher to questions A, B and C are shown.

On question A, five main themes were defined, as illustrated in Table 1.

**Table 1.** Themes/ examples of codes, question A (interpretation of 'mathematical modelling')

| A  | Themes  | Codes  |
|----|---|--|
| A1 | Describing or explaining something (in 'reality') in mathematical terms | Finding a mathematical function to describe an event<br>Investigating some kind of association<br>Using variables to simplify a relationship<br>Translating an everyday problem, or any problem, to a symbolic language<br>Making students formulate themselves mathematically<br>Describing reality in a mathematical way |
| A2 | Constructing a theory for some situation that you describe with math    | Creating a model for how reality works<br>Describing a system you do not know  |
| A3 | Making simulations  | One can make mathematical simulations  |
| A4 | Applied tasks   | Practical tasks<br>A kind of formula to be able to calculate in a situation  |
| A5 | Open tasks and problem solving  | Open tasks where students need to make own assumptions   |

The most common conception of mathematical modelling among the teachers has been identified as theme A1: *Describing or explaining something (in reality) in mathematical terms* (13 out of 18; cf. Table 4 below).

<sup>1</sup> See Frejd (2011) for a description of the selection of interviewees and the interviewing methods.

The reference to algebra, symbolic language, and mathematical function, suggest that this theme can be interpreted as referring to a mathematisation process involving some transition from a real-world context to the "esoteric sphere" of school mathematics. Notably, the teachers generally formulate this process as being unproblematic, as a kind of direct "translation" of a real-world situation into "symbolic language" (cf. Jablonka & Gellert, 2007, p. 5). The following excerpt, though, suggests that this process might not be seen as that 'simple' by all of these teachers:

Einstein writes in his equations that he cannot solve them but he is superb in expressing himself in the symbolic language that is mathematics. That's what I interpret as mathematical modelling. (T7)

Some teachers, however, do not explicitly link modelling to specific (school) mathematics, but refer to theory construction or simulation techniques or use terms like "open problems" or "practical tasks". The themes A2 to A4 suggest an array of functions of modelling, from theoretical understanding, simulation to use of standard techniques. A5 points to making assumptions and explicitly refers to the classroom setting.

On question B, *How would you describe your main task as a teacher of mathematics?*, there appeared to be more variation in the answers than on question A, coded in 8 themes (see Table 2). A main concern for several teachers seemed to be the promotion of students' understanding of the mathematics they are learning (theme B1, 7 out of 18):

The goal is to try to make everybody understand (T1)

Underlying this theme seems to be some teachers' concern about students finding mathematics "difficult", possibly related also to themes B2 and B3, as well as B6 (each 3 out of 18). While one teacher pointed to the more general "thinking models" that come with learning mathematics (B7), the three remaining themes are more utility oriented. None of the teachers discussed engaging in modelling and teaching related strategies as a goal.

**Table 2.** Themes and examples of codes, question B (main task as math teacher)

| B  | Themes  | Codes  |
|----|---|--|
| B1 | Promoting students' understanding                     | Understanding different contexts<br>Killing the myth that math is difficult<br>Making it easy for the students |
| B2 | Teaching the students as much mathematics as possible | Understanding their thinking to be able to teach what they are supposed to learn                               |
| B3 | Stimulating the students                              | Making students interested<br>Making students like mathematics   |
| B4 | Firm basis for further studies                        | That they get a good ground to build on  |
| B5 | Managing life in society                              | Mathematics useful outside school  |
| B6 | Fostering self confidence                             | What makes students not learn mathematics is the lack of self confidence                                       |
| B7 | Developing thinking models                            | A good way to train the brain thinking models  |
| B8 | Mathematics is necessary                              | We simply need mathematics   |

On question C, though, obviously with some overlaps with question B, several teachers pointed to mathematics as a subject promoting a way of thinking involving symbols, patterns, and logic (theme C5, 8 out of 18; see Table 3), a theme partly linked to theme C3 (only 1 out of 18). The most common answers, however, were coded within theme C2 (9 out of 18), emphasising mathematics as a tool in science and "everywhere", possibly linked to themes C1 and C6 (3 and 2, respectively, out of 18). Self confidence in

mathematics was seen as useful in terms of what could perhaps be interpreted as a type of mathematical literacy (theme C4, 3 out of 18).

**Table 3.** Themes and examples of codes, question C (usefulness of math for students)

| C  | Themes  | Codes  |
|----|---|--|
| C1 | Practical use in society                          | Mathematics is necessary<br>Everybody has use of math in society                             |
| C2 | A tool for use in other (school) subjects         | Used in all sciences<br>For use everywhere   |
| C3 | A general way of formulating and solving problems | This way of formulating problems one always needs in life                                    |
| C4 | Having mathematical self confidence               | Check calculations they encounter in society<br>Self confidence to use the tools when needed |
| C5 | A way of thinking                                 | Logical thinking; using a symbolic language; training thinking; thinking patterns            |
| C6 | Generally useful                                  | Usefulness generally, in mathematics, science, societal                                      |
| C7 | Much of it is not useful                          | Most of it one would not need to manage one's life   |

From Table 4 it becomes clear that the answers to question C by some teachers were more comprehensive than the answers to A and B, with C2 and C5 often coming together. One can also observe that some teachers are coded differently on all questions (T5 and T6, T7 and T8, and others), suggesting very different conceptions on teaching goals and modelling.

**Table 4.** Distribution of teacher (T1, etc.) answers over the themes (A, B, C); for example, the answers by teacher T5 were coded as A1 and A3, B1, and C2.

| Theme | T1 | T2   | T3   | T4 | T5   | T6 | T7 | T8 | T9 |
|-------|----|------|------|----|------|----|----|----|----|
| A     | 1  | 1    | 1    | 1  | 1, 3 | 5  | 1  | 4  | 2  |
| B     | 1  | 4, 5 | 2    | 2  | 1    | 3  | 6  | 1  | 1  |
| C     | 1  | -    | 1, 2 | 4  | 2    | 5  | 5  | 4  | 2  |

| Theme | T10  | T11     | T12 | T13  | T14 | T15        | T16  | T17     | T18 |
|-------|------|---------|-----|------|-----|------------|------|---------|-----|
| A     | 1    | 1       | 1   | 2    | 1   | 1          | 1    | 1       | 5   |
| B     | 1    | 3       | 3   | 6    | 6   | 1, 5       | 2, 7 | 1       | 8   |
| C     | 2, 5 | 2, 5, 7 | 2   | 2, 6 | 5   | 1, 3, 5, 7 | 2, 5 | 2, 4, 5 | 6   |

#### d) Discussion

While it is certainly the case that many teachers in this study did not elaborate a conception of mathematical modelling (Frejd, 2011), there is some considerable variety in their views on the function of mathematical descriptions or techniques with respect to 'reality'. What they have in common, however, is a sharp distinction between this 'reality' on the one hand, and mathematics on the other. Based on this, one could allege that they hold an epistemological view that is based on this duality. Mathematical modelling is seen as

a providing some mirror of reality or, for some, mathematical techniques are directly useful for solving practical problems. The potential diversity of mathematical descriptions and the problematic nature of any "translation" were generally not discussed. Motivation of students might be one major reason for engaging with some forms of applied tasks. Mathematisation as didactic principle, with the goal of developing mathematical structures and methods, or mathematical modelling, as driven by particular interests and so producing a range of models for the same problem, were not explicitly separated. As a didactic principle, however, mathematisation was also interpreted in a formal sense, for developing generic thinking tools. Describing 'reality' (A1) was mostly linked to the theme promoting understanding (of mathematics) (B1). That also themes B2, B3, B5, as well as C2, were associated with A1 (see Table 4) indicate that for these teachers, drawing on real world contexts in the teaching of mathematics is seen to promote their overall goals of teaching mathematics such as understanding, interest and usefulness. In his study, from which these data were drawn, Frejd (2011) also asked the question *Why do you use modelling in your teaching?* to those (9) of the teachers reporting they do use it. Of the answers, one was coded as *To practice the transfer between different discourses*. To problematize this "transfer" is absent in the teachers' discourse.

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## Investigative tasks: possibilities to develop teachers' technological pedagogical content knowledge

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**Abstract.** The aim of this paper is to discuss the research undertaken in a continued mathematics teacher education focusing on investigative tasks to teach spatial position geometry, using Cabri 3D software. The theoretical basis about the formative process came from Imbernón's ideas about teacher education in and to change, collective reflection from Zeichner ideas and from development of TPACK defined by Mishra and Koehler. The qualitative research, in the sense of Bogdan & Biklen, was part of a broader project of the Education Observatory Brazilian Program. The subjects were nine teachers and the investigative tasks focused here occurred in six four-hour meeting with the intention of providing reconstruction of teaching practice. Data collection was done by direct observation, audio and video recordings of the meetings and the materials produced by the subjects. Data analysis was interpretative. In this paper, the episode related to "Skew quadrilateral" task was analysed and established conclusions. We found evidence of reflection and construction of new references to the practice by teaching strategies with investigative tasks and use of digital technology.

**Keywords:** Continuing Education, TPACK, Reconstruction of Practice, Spatial Geometry, *Cabri 3D*.

**Résumé.** Le but de cet article est de discuter les résultats d'une enquête menée dans un processus de formation continuée des enseignants de mathématiques, axés sur les tâches d'investigation pour l'enseignement de la géométrie de position à l'aide du logiciel Cabri 3D. L'intention a été de fournir aux enseignants l'expérience avec des situations didactiques qui faisaient appel aux technologies, afin de les proposer une réflexion sur les processus d'enseignement et d'apprentissage de la géométrie. La base théorique du processus de formation se concentre sur les idées d'Imbernón en vue de former les enseignants « dans » et « pour » le changement (c'est pour instruire ces enseignants en ces temps d'incertitude et de changement social rapide); sur la réflexion collective, centrée dans les idées de Zeichner; et sur le développement de TPACK (technological pedagogical content knowledge – connaissance du contenu pédagogique et technologique), défini par Mishra et Khoeler. La recherche est qualitative, au sens donné par Bogdan & Biklen, et représente une partie d'un projet plus vaste appelé "Programa Educacional Brasileiro Observatório da Educação". Neuf enseignants ont participé de la recherche et les tâches d'enquête ont été développées pendant six réunions de quatre heures chacune, avec l'intention de rendre favorable la (re)construction de la pratique d'enseignement. La récolte des données a été réalisée par: l'observation directe, par des enregistrements audio et vidéo des réunions et par la collecte des documents produits par les neuf enseignants. Pour l'analyse de ces données, nous avons utilisé la méthode interprétative. Dans cet article, l'épisode qui nous avons choisi pour présenter c'est la tâche « Quadrilatère Reverse », lequel a été décrit, analysé et nous a permis d'établir des conclusions. Nous avons rencontré des évidences de réflexion et de construction de nouvelles références pour la pratique d'enseignement à partir de l'utilisation en tant que stratégies des tâches d'investigation et des technologies numériques.

**Mots-clés:** Formation Continue, TPACK, Reconstruction de la Pratique, La Géométrie Spatiale, *Cabri 3D*.

## 1. Introduction

The vertiginous social and technological changes of the late twentieth century and the early years of the twenty-first century have introduced a new paradigm of living in society. The different ways for people to communicate and relate, as well as the speed and ease of access to information have set up a new *modus operandi*. It is in this reality that young people are immersed and must deal with the technological artefacts in their future professional and social life. These changes have affected students' expectations regarding school and teachers' role.

The mathematics teaching practice needs to be reviewed considering this current scenario and the various technological possibilities that are available in the students' daily lives. In our schools, the use of technologies for teaching often serves for curriculum content illustration and appearance "modernization" without, however, modifying the teaching essence. We understand that the pedagogical use of technology involves the integration of different semiotic representations of the concepts, in a way that the manipulation and the exploration of objects help the establishment of relations, as well as the modelling and investigation of problem situations. (LOBO DA COSTA & PRADO, 2015). Thus, technology can be an ally to *think-with*, in the sense given by Papert (1994), favouring the construction of meaning.

A relevant pedagogical strategy in teaching is to use investigative tasks, which can be developed integrating technological resources. The experimentations with these resources are favoured by the manipulation/application of mathematical concepts, rising hypothesis and creating representations on the software interface. For example, the students as they investigate the conjectures posed, explore several mathematical possibilities, may find possible solutions, in addition, these investigations could also subsidize the validation of results found.

[...] to bring to the classroom the spirit of genuine mathematical activity, thus constituting a powerful educational metaphor. The student is called to act as a mathematician, not only in the formulation of questions and conjectures and in the performance of tests and refutations, but also in the presentation of results and in the discussion and argumentation with his colleagues and the teacher (PONTE, BROCADO & OLIVEIRA, 2003, p.23).

In this sense, it is necessary that the formative actions, supported by Imbernón's (2006) and Zeichner's (1993) ideas, give the teachers the opportunity of *learning-by-doing*. That is, to experience investigative tasks using the technological resources and reflect on this learning experience, seeking to relate it to the possibilities of reconstructing their own teaching practice, from the construction of pedagogical, technological and mathematical content knowledge.

From this problematic, the continued education addressed in this study was designed to promote the discussion of investigative tasks involving technology. In this paper, we report on one of these tasks for the Space Geometry teaching.

## 2. The Research

The qualitative research, according to Bogdan & Biklen (1994), aimed to understand how investigative tasks can boost the construction of technological pedagogical knowledge of content - TPACK (Mishra & Khoeler, 2006). The tasks, developed in a process of continuing mathematics teacher education, focused on the teaching of Spatial Position Geometry, using *Cabri 3D software*. This education process was part of a broader Project of the Education Observatory Brazilian Program, financed by CAPES. The subjects were nine teachers and the investigative tasks focused here occurred in six four-hour meeting with the intention of providing construction of teaching practice. The instruments of data collection were: direct observation, protocols of the activities and audio and video recordings of the formative meetings. The analysis was interpretive.

In this paper, we discuss the development of an investigative task called "Skew Quadrilateral". This term is used here to indicate a quadrilateral with non-coplanar vertices and no-coplanar sides. We also use "skew lines" meaning nonparallel lines in space that do not intersect.

Initially we proposed that the teachers investigated with the *Cabri 3D software* the conditions for four points in the space being vertices of a quadrilateral. The group constructed several quadrilaterals and in the

collective discussion the established conclusion was that the four vertices should be non-collinear points three-to-three. Teachers of the group stated that this may be a useful activity to discuss with their students the conditions of the vertices for the determination of a quadrilateral. Similarly, an investigative task could also be proposed to discuss the positioning conditions of points for the existence of a triangle with vertices at these points. However, all the quadrilaterals constructed by the group of teachers had the four vertices in the same plan. Using the software, the participants moved the vertices of the constructed quadrilaterals and investigated the different possibilities regarding the types of quadrilaterals and erroneously concluded, at that moment, that a quadrilateral is always a 2D figure. To allow the group in confronting the conclusion to a situation in which there is no plan to which all the vertices of the constructed quadrilateral belong. After that, we proposed a task aimed to lead them in investigating the existence of quadrilaterals that are not 2D figure. The task and an example of skew quadrilateral construction (AEGF) are in figure1.

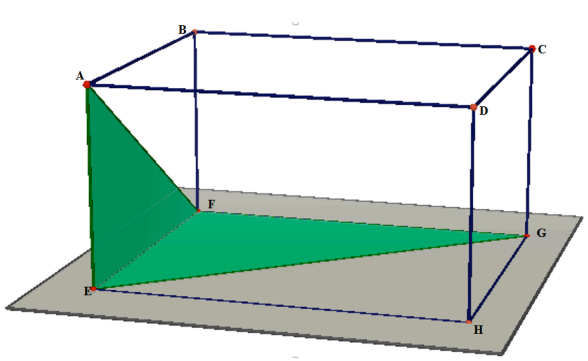
| <b>Skew Quadrilateral</b>   |   |
|---|---|
| <p>1) Construct a rectangular parallelepiped ABCDEFGH, using the <i>Cabri 3D</i>.</p> <p>2) Consider 4 distinct and non-coplanar vertices of the parallelepiped and construct a quadrilateral.</p> <p>3) Investigate the relative positions of the straight-line support on the sides of this quadrilateral, which is called the skew quadrilateral.</p> <p>List the positions found.</p> <p>4) Investigate and answer:</p> <ul style="list-style-type: none"> <li>• Are the diagonals of a skew quadrilateral always <b>in skew lines</b>?</li> <li>• Are two opposite sides of a skew quadrilateral always <b>in skew lines</b>?</li> </ul> |  |

Figure 1. Skew Quadrilateral Investigative Task

Source: Adapted from Muraca (2011).

We observed that by the instructions of the task, the four vertices of the quadrilateral are non-coplanar points, which made it impossible to construct a 2D quadrilateral. At the time of the collective discussion, the participating teachers stated that they did not know this type of quadrilateral and some of them asked if this figure could still be considered as such a quadrilateral. It was necessary to return to the definition of quadrilateral for the establishment of consensus. Therefore, this investigative task helped to develop teacher's content knowledge. With regards to the requested investigation about the relative positions between the diagonals as well as opposite sides of the skew quadrilateral, the representation with the software and the movement that allowed for modifications of positions, both were essentials to promote changes in participating teachers' ideas and for the establishment of new conclusions. That is, the diagonals are always skew lines, the same occurring with two opposite sides. The following figure shows the diagonals representation of the skew quadrilateral and the diagonals support lines, which are skew lines.

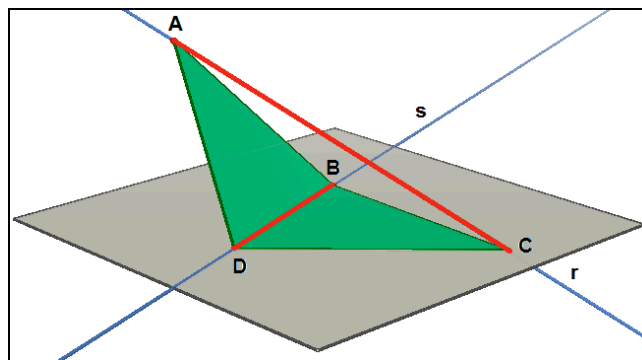


Figure 2. Diagonals' supports lines of a **skew** quadrilateral representation

*Source: Muraca (2011 p. 129).*

In the collective discussion, one of the teachers emphasized the possibilities of using this task as a didactic strategy to help his students to assign meaning to the skew lines. As for the sides of the skew quadrilateral, when asked about the positions found they were unanimous in saying that any two sides of the skew quadrilateral are either concurrent or are skew lines. Hence, the discussion focused on how to prove this statement. We reached the conclusion that this investigative task aided the process of concept construction, the representation on the software was able to provide a new vision related to the figure.

Continuing the investigations, we proposed the task described in Table 1.

**Table 1.** Task of analysis and rewriting of statements about the Skew Quadrilateral

| <b>Skew Quadrilateral Task</b>   |
|--|
| Read the sentences below; rewrite them to express true statements - that is, valid for all cases.  |
| <u>Affirmation 1</u> : Given the distinct coplanar points A, B, C, D, it is possible to construct a skew quadrilateral whose vertices are those points.        |
| <u>Affirmation 2</u> : Given the non-coplanar distinct points A, B, C and D, it is possible to construct a skew quadrilateral whose vertices are those points. |
| <u>Statement 3</u> : It is possible to construct a skew triangle.  |

*Source: Adapted from Muraca (2011).*

The teachers were asked to use the software to investigate and analyse the didactic possibilities of this task before rewriting the sentences, listing the possible difficulties of their students in a similar task.

We notice that Affirmation 1 is false, since if the points are coplanar the quadrilateral will be contained in a single plane, so it is not skew. As for Affirmation 2, it is true, because if the four vertices are not in the same plane, there is no plane containing the quadrilateral, so it is a skew one. Finally, in relation to Affirmation 3, it is false, because the vertices of a triangle are always three non-collinear points, that is, they are always coplanar points, so the triangle is always a 2D figure, from which it is possible to conclude that there is no skew triangle.

The last of the three statements caused the most controversy. Seven of the nine teachers conjectured that their students would claim that if there is a skew quadrilateral should exist a skew triangle. Some of the teachers' observations were as follows:

**Teacher A and Teacher B:** *It is possible to construct a skew triangle, if the three points are not coplanar.*

**Teacher C:** *If you have the distinct points A, B and C not coplanar, you can construct a skew triangle whose vertices are those points.*

The analysis of the rewriting proposals of the participating teachers and the previous analysis before the application of this type of task to the students, allows us to conclude that, in the perception of these teachers, the explorations and investigations with the software are not enough to lead to the correct conclusion. Since in this case, the postulate of determining the plan needs to be accessed. However, they consider that the investigations and explorations allowed to pose and test conjectures, but it is necessary that such conjectures be validated in some way and the software alone does not play that role. At that moment they understand that it is the moment of pedagogical mediation.

The participating teachers, when analysing the potential of the tasks experienced, were unanimous in recognizing that they could potentiate the gathering of conjectures, exploration and research on the validity of statements made about relative positions, both between lines and between planes, and they can students to be better prepared to face demonstrations. Particularly, participant teachers considered the activity on the skew quadrilateral as great potential for the discussion with the students about positions between the lines in the space as well as plans. We analyse that the participant teachers mobilized technological, content and pedagogical knowledge during the meetings.

### 3. Final Considerations

This study evidenced possibilities for continued education in order to boost the construction / reconstruction of knowledge by the participants. The educational process provided experience of learning situations by investigative tasks, integrating the technological resources, helped not always to deepen the conceptual

understanding but also to rethink teaching. Discussions throughout the tasks showed that as teachers gave meaning to their own learning using the tools of the software, they came to recognize that this could be a new way of learning and that this kind of learning may occur with their students. We found that during this formative period there were some indications of the construction of technological, pedagogical and mathematical knowledge by the participants as well as reflection on the practice, but in a limited way. This can be understood by the fact that the teacher's experience in the investigative task was centred on the learning itself (software, type of activity, content exploration, etc.). However, we consider that this experience and the sharing of the learning process itself with its peers and the trainer were fundamental to lead them to reflect on the possibilities for the teaching of Spatial Position Geometry envisaged by the investigative approach using technological resources.

This situation showed us that the process of continued education in the TPACK perspective, aimed at facilitating the reconstruction of the practice is not a simple process, nor does it occur immediately. It is necessary to contemplate continuity actions in the formative process giving the teacher an opportunity to construct new references for their practice and also to concretize them and to discuss the actions undertaken and the strategies used in practical situations, with their peers. However, the concretization requires new constructions of knowledge that the continued education should provide so that an investigative approach with the use of technology was put into practice with the students and the application discussed and shared in the group, so that, there is reflection on the practice towards its reconstruction.

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## Problem-based learning: an investigative approach to teach optimization problems

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**Abstract.** This paper presents a research that aimed to identify possibilities of an investigative approach to teach functions from geometric problems. The proposal was carried out as part of a broader Brazilian project of the Programa Observatório da Educação and developed with a group of nine high school teachers from public schools of São Paulo city, throughout two four-hour meetings. The theoretical support related to problem-based learning (PBL) came from Savery's summary, investigative tasks from Ponte's studies and jeux des cadres from Douady definition. The activities were posed in the geometric framework and implemented using concrete materials and technological resources. The research methodology was qualitative; the data was comprised of teachers' productions (digital files and paper and pencil registers). The interpretative analysis highlighted that, in the participant teachers' view, these investigative activities may help the students to understand better the aspects related with the study of functions, to integrate several mathematics field contents and to develop research capacity and perseverance in the search for results, particularly regarding to the use of different strategies, validation techniques and results control.

**Resumé.** Cet article présente une recherche visant à identifier les possibilités d'une approche d'investigation pour enseigner les fonctions à partir de problèmes géométriques. La proposition a été réalisée dans le cadre d'un projet plus large au Brésil, le Programme Observatoire de l'éducation et a été développé avec un groupe de neuf enseignants du secondaire des écoles publiques de São Paulo, lors de deux réunions de quatre heures. Le soutien théorique lié à l'apprentissage par problème (PBL) provient du résumé de Savery, des tâches d'enquête des études de Ponte et des jeux de cadres de la définition de Douady. Les activités ont été posées dans le cadre géométrique et mises en œuvre avec des matériaux concrets et des ressources technologiques. La méthodologie de la recherche était qualitative, les registres composées de productions d'enseignants (fichiers numériques et registres en papier et crayon). L'analyse a souligné que, dans l'opinion des enseignants participants, ces activités d'investigation peuvent aider les élèves à mieux comprendre les aspects liés à l'étude des fonctions, à intégrer plusieurs contenus de mathématiques et à développer la capacité de recherche et la persévérance. Dans la recherche de résultats, en particulier en ce qui concerne l'utilisation de différentes stratégies, techniques de validation et contrôle des résultats.

### 1. Introduction

A challenge for teachers while teaching mathematics is to adopt an approach that allows the student to assume an active and conductive attitude towards learning. In this sense, leading the student to adopt an investigative posture in relation to mathematics, to raise conjectures, test them, explore proposed situations, act autonomously, discover ways and validate found solutions provide a better understanding of the mathematization processes.

Problem-Based Learning (PBL) uses problems related to everyday life to stimulate research by and

among students. Specialists elaborated these problems with the aim of developing skills foreseen in the school curriculum. This approach to teaching is particularly challenging, but it depends on the problems selected and the activities in which students engage, that should encourage them and allow independent and autonomous actions. Thus, we present a PBL proposal for the study of the variations in functions coming from geometric problems, which we developed in the framework of a broader project of the Observatory Program of Education of CAPES.

## **2- Theoretical foundations**

The theoretical foundation was built upon ideas summarized by Savery (2006) on Problem-Based Learning (PBL), Ponte (2003) on investigative tasks and Douady (1992) and Douady & Perrin-Glorian (1989) on "*jeux des cadres*".

For Savery (2006), PBL is an approach focused on student work and driven from real-world situations. For that reason, choose mathematical resources suitable for resolution providing opportunities like those that will be faced in life in society. Students must participate and collaborate with the group, which seeks to develop skills such as critical thinking. The activities should be finalized with a discussion of the concepts and principles used, to highlight the relevant general aspects.

Investigative tasks are open-ended and designed to be applied for teaching and learning purposes. We follow Ponte (2003, p. 94), for whom "investigating means working from issues that interest us and are not quite clear initially, but which we can clarify and study in an organized way." When developing investigative tasks, students should have moments of exploration, conjecture, hypothesis testing, justification and validation, with the accompaniment, help and guidance of the teacher. The implementation of such tasks can boost the exploration of mathematical ideas related to it, as well as mobilize diverse strategies for its solution and validation. Serrazina, Vale, Fonseca & Pimentel (2002) consider an investigative task as close to that of problem solving, as both seek to engage students in complex processes of thought, provide participants with an opportunity to develop both cognitive abilities and attitudes. Unlike the problem-solving process that is driven by a specific question to answer, the approach in an investigative process takes in account some other questions related to the situation.

The activities we propose for research involve problems dealt with in the scope of Geometry, more especially, area of plane figures, a context that leads to paths in which consider the use of Algebra, promoting a change of frames (*jeux des cadres*). For Douady and Perrin-Glorian (1989), the construction of mathematical knowledge is associated with the explicit use and adaptability of tools in a process of formulating and solving problems. In this process, the mathematical concepts intervene interactively, as object or as tool, characterizing the tool-object dialect. The changes of frames are the different formulations of a problem that may allow new access and different ways of coping with the difficulties identified in the resolution process through alternative tools or techniques not available in the previous frame. Given a problem formulated in a frame, the learner's experiences and knowledge may lead him to translate it into another frame and thus reinterpret the initial questions in this new context, establishing possible correspondences. For Douady (1992) an important characteristic of Mathematics is the ability to translate a problem into several frames: algebraic, numerical, geometric, analytical, etc ..., causing us to have several resolution tools. At the initiative of the teacher, a problem must be suitably chosen to allow the approach from different perspectives, thus increasing the possibilities of establishing relations and strategies of resolution. It is noteworthy that these are the main aspects explored in the activity here reported and analysed, which was extracted from the sequence of problems discussed in the continued education.

## **3. The Research**

The research of qualitative methodology according to Bogdan & Biklen (1994), is characterized by being focused on the understanding of processes and meanings, with a detailed record of the object of study, being essentially descriptive. The research participants were comprised of nine mathematics teachers active at public high schools in São Paulo City. The data were collected through video recordings of the meetings and collection of the records produced by the teachers. The analysis was interpretive. Specifically, for the accomplishment of the investigative tasks, focus of this paper, two meetings of four hours each took place, in the scope of the other activities of the project. In the meetings, research tasks involving knowledge about functions and the calculation of areas were proposed, starting from problems stated in the geometric framework, which were first explored, discussed and analysed by teachers using both concrete material and *Geogebra* software. Then the teachers were invited to apply them in the classroom. The methodological



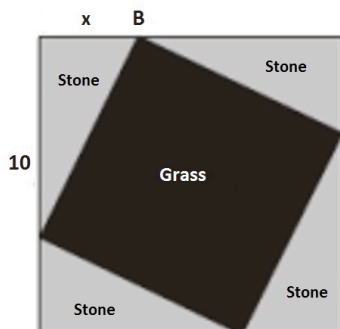
procedures used by the researchers at the meetings were: (1) to propose problems that allowed the exploration and investigation of the situation posed, with the use of concrete materials and the use of *Geogebra*; (2) developing investigative activities that would lead to change of frames; (3) identify possibilities of an investigative approach to teach optimization geometric problems.

In this article, we discuss and analyse one of the research activities.

### 3.1 Description and Analysis

The optimization problem that subsidized the investigative activity is presented below.

A mayor wants to build a squared square of 10 m on the side, which will have four triangular stone garden beds and a square grass garden bed, as in figure.



The mayor has not yet decided what the grass area will be, so the length of the AB segment is indicated by  $x$  in the figure

A) Calculate the area of the grass garden bed to  $x = 2$ .

B) Write the expression from the grass area as a function of  $x$  and sketch its graph.

C) It is known that the grass garden bed costs R \$ 4.00 per square meter and the stone garden beds cost R \$ 3.00 per square meter. Use this information to answer the following two items.

What is the smallest amount of money the mayor must have to build the five garden beds?

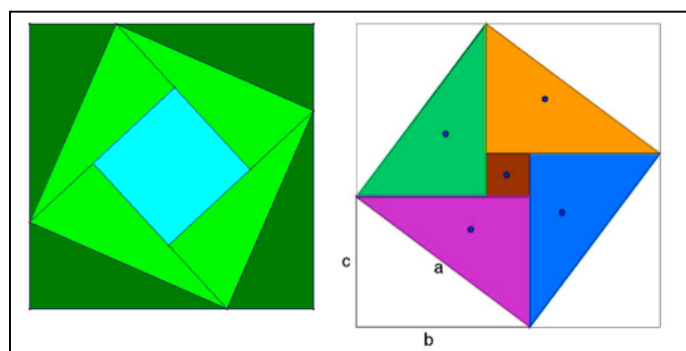
If the mayor has only \$ 358.00 to spend on the five garden beds, what is the area of the largest grass garden bed that the square can have?

**Figure 1.** Garden beds' problem

Source: OBMEP 2005 Collection - N3 - Phase 2

In order to guide the research activity, we proposed that teachers made a model of the square on paper and used this model to explore the problem.

Teachers have doubled the triangles to "inside" the square. In this way they could observe that, with the folds, a new square appears in the interior, as in the models of 'figure 2'.

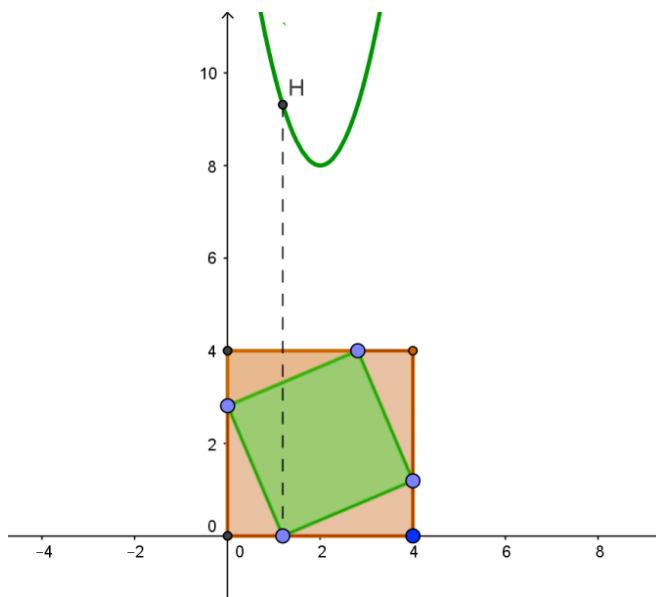


**Figure 2.** Paper foldings built for research

Source: Project "OBEDUC Practices" collection

Next, a model was elaborated using *Geogebra*; exploring this model the participants observed that the area of the inner square varied from an initial value to a minimum value and grew again. The teachers found the function that describes the area and prepared, in pairs, the resolutions to collectively discuss their conclusions.

The model (as in figure 3) allowed to investigate what happens with the area of the new square when we vary the segment chosen as the leg of the right triangle (cathetus) in the original square.



The research allowed us to investigate some aspects of the problem and to answer the question: "What is the function that expresses the area of the new square (the grass garden bed)? and also relate the variation of the garden bed area to the variation of the function found.

**Figure 3.** Imagem no Geogebra (Source: Project "OBEDUC Practices" collection)

It is noteworthy that the properties observed in the construction of the paper model supported the construction of the model in *Geogebra*, which in turn allowed new explorations and consequently added new conclusions. These manipulations guided the change of the geometric frame to the algebraic, to obtain the solution of the problem. There were three separate referrals for the calculations of the areas and the costs that were discussed, compared and validated in the whole group.

The analysis of the statements of the participating teachers evidenced the conviction about this approach in helping the students understand aspects related to the functions, in the establishment of connections between them and the areas of the geometric figures studied. In addition, it facilitated the integration of content in different fields of mathematics and the development of research capacity and perseverance in the search for results, especially regarding the use of different strategies and results control.

Here are some of the statements made by teachers:

The student today has great difficulty to abstract and this proposal that he realize that by decomposing it is possible to transform one figure into another is interesting. (Prof. Al)

The most difficult for them is to associate and analyse the mathematics involved. (Prof. Cel)

However, participating teachers warned that:

For these investigations, the student must have constructed the concept of area, perimeter .... Knowledge that needs to be used in this situation. (Prof. Cel)

It takes a job before to teach Geogebra, because in it you can move the point and hence maintain the properties of the figure [rectangle] ... (Prof. Al)

#### 4. Final considerations

The analysis of the data allowed us to conclude that in this Problem-Based Learning approach we identify as possibilities for teaching and learning: the use of experiments and investigations with passages through concrete and *Geogebra*, which can attract Students' interest in learning, especially with the pre-construction work. The intradisciplinary work, since the activities enable to deal with several associated contents, such as area; functions, which helps to establish connections between such contents and between the geometric and algebraic frames.

Finally, it is worth mentioning that the participants in the teacher continuous education process emphasized that they realized the need for the teacher to prepare, to master the subject, to be open to the students' suggestions and to be aware that in solving the problems many unexpected strategies can emerge, as in this case, to calculate the area and estimate the costs. In this sense, the preparation, planning and performance of the teacher are fundamental for conducting a class from the perspective of the PBL and, on

the other hand, it is expected that the student adopts an investigative stance, compatible with society increasingly mathematically which is inserted.

### **Acknowledgment**

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## La modélisation mathématique et processus de mathématisation dans la formation des enseignants

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**Résumé.** Le curriculum de Québec a une approche par compétences où la première à développer chez les élèves du secondaire dans la discipline des mathématiques est la *résolution de situations- problème*. Les enseignants et les auteurs des manuels scolaires étaient habitués à suivre un programme d'études axées sur la résolution de problèmes (en général problèmes fermés) et cela a impliqué une résistance au changement pour des raisons naturelles. Une situation problème, en principe, est un problème en contexte et de type ouvert, impliquant les processus de modélisation et de mathématisation pour sa résolution. Dans ce document, nous proposons un modèle qui impliquerait que les enseignants de mathématiques en formation puissent construire une structure structurante dans le sens de Bourdieu (dans un apprentissage socioculturel), qui permettrait la résolution de situations-problème et la construction du concept de covariation entre variables et les fonctions.

**Abstract.** In Quebec, the curriculum is based on competences. Firstly, the secondary-school-curriculum of mathematics entails situation-based problems. In the past, teachers and authors of school textbooks followed a programme of studies that focused on the resolution of problems (in general closed problems). This habit is hard to change. A situation-based problem, in principle, is an open-type-problembased on a specific context. In order to resolve situation-based problems, students need to apply processes of modelling and of mathématisation. In this paper, we offer a model which implicates that the teacher-trainersof mathematics construct a founding structure in the sense of Bourdieu (in a socio-cultural training) which would advance the resolution of situation-based problems and the conceptualizing of co-variation between variables and functions.

### 1. Introduction

Au Québec nous suivons un programme par compétences qui stipule que la première compétence est la résolution de situations-problème, dont les caractéristiques, sont : 1<sup>er</sup> compétence : La situation n'a pas été présentée antérieurement en cours d'apprentissage; L'obtention d'une solution satisfaisante exige le recours à une combinaison non apprise de règles ou de principes dont l'élève a fait ou non l'apprentissage; Le produit ou sa forme attendu n'a pas été présenté antérieurement.. La 2<sup>e</sup> compétence est de déployer un raisonnement mathématique et la 3<sup>e</sup> doit promouvoir la communication avec un langage mathématique (MELS 2007, p. 22). Aussi, le ministère de l'Éducation stipule que l'enseignement doit suivre des hypothèses d'apprentissage socioconstructivistes.

Cette nouvelle approche pour les enseignants, les auteurs de manuels scolaires et les étudiants a donné lieu à une longue période d'adaptation (13 ans) dans le milieu éducatif. Le ministère de l'Éducation a décidé de suivre un cadre théorique de Perrenoud (1999/2000):

Une compétence est une capacité d'action efficace face à une famille de situations, qu'on arrive à maîtriser parce qu'on dispose à la fois des connaissances nécessaires et de la capacité de les mobiliser à bon escient, en temps opportun, pour identifier et résoudre de vrais problèmes. (MELS, 2007 p. 16)

Le ministère de l'Éducation a pris en compte le cadre théorique de Duval (1995) avec celui de Perrenoud.

L'approche théorique de Duval porte sur le renforcement des connaissances à partir d'une perspective constructiviste (centrée sur l'individu) ; un objet ou concept mathématique est construit chez l'individu à travers l'articulation entre différentes représentations de l'objet mathématique ou concept. Toutefois, en vertu de la promulgation par le ministère utilisant l'approche théorique de Duval, cette construction est liée à des représentations institutionnelles, et donc, est loin de faire référence à une compétence liée à la résolution de situations problème. Cette dernière affirmation est basée sur le fait que le cadre théorique ci-dessus, n'a pas pris en compte dans sa structure la construction des concepts via des représentations non institutionnelles qui peuvent jouer un rôle important dans les concepts de construction, de modélisation mathématique et processus liés à la mathématisation.

Nous nous demandons pourquoi le ministère de l'Éducation n'a pas privilégié la modélisation mathématique et les processus de mathématisation correspondants comme objet d'étude pour le développement de la 1<sup>re</sup> compétence dans la discipline des mathématiques.

## **2. La modélisation mathématique et les processus de mathématisation**

Le paradigme de la modélisation mathématique a été étudié pendant des décennies, mais c'est dans ce siècle qu'il est considéré comme le plus important. Selon Lerman (2014) :

Research results indicated that the identification of problem-solving strategies and the process of modeling their use in instruction was not sufficient for students to foster their comprehension of mathematical knowledge and problem-solving approaches. (p. 498)

Ainsi, les chercheurs se sont intéressés de plus en plus à ce problème (Blum et al. 2007, Quiroz et al. 2015; Quiroz 2016). La modélisation mathématique considère les processus de mathématisation lors de l'exécution d'une tâche mathématique. Par exemple Lerman (2014) mentionne que:

Mathematization provides a particular challenge for mathematics education as it becomes important to develop a critical position to mathematical rationality as well as new approaches to the construction of meaning. (p. 442).

La recherche montre que les élèves en difficulté d'apprentissage ne parviennent pas à développer des éléments de contrôle dans la résolution qui leur permet d'atteindre un résultat cohérent exempt de contradictions (Saboya et al. 2015).

Notre modèle cherche à promouvoir la construction d'une structure structurante dans le sens de Bourdieu (1980) qui se traduirait par la construction des concepts et des structures de contrôle dans la résolution de situations problème en contexte ; la structure développée sous une approche socioculturelle de l'apprentissage. Pour ce faire, nous devons suivre une méthodologie dans la construction d'activités (voir Hitt, Saboya & Cortés 2017).

Notre modèle (Hitt, 2013) est immergé dans une perspective de la théorie de l'activité de Leontiev (1978). Compte tenu de la méthode d'enseignement ACODESA (Hitt-Gonzalez Martin, 2015, Hitt-Savoie-Cortés 2016, Hitt-Saboya-Cortés 2017) des activités *ad hoc* on été élaborées, permettant de guider l'enseignant(e) en formation et des étudiant(e)s dans les processus de mathématisation.

En 2007 (voir Hitt-Passaro), nous avons présenté les résultats des élèves de secondaire 2 et un groupe d'enseignant(e)s en formation en utilisant la même activité d'un randonneur autour d'une piste. Dans le cas des enseignants, en fonction de la piste proposée par eux-mêmes (situation du type ouvert), la difficulté algébrique peut être plus dans un cas que l'autre. Si les représentations spontanées ont été produites par une équipe faible, cette équipe n'avancera pas dans le processus de mathématisation. Voici les questions proposées aux futurs enseignants, questionnaire réduit (avec les élèves nous avons demandé aussi les processus inverses que consiste du passage d'une représentation graphique à un dessin qui détermine l'emplacement du mât, voir Hitt et González-Martín 2015).

|  |  |
|--|--|
| <b>Page 1</b> Renseignements personnels et de l'équipe |  |
|--|--|

|               |
|---------------|
| <b>Page 2</b> |
|---------------|

Un randonneur entreprend une longue randonnée en forêt. Il suit une **piste** qui lui permet de revenir à son point de départ à la fin de la randonnée. Durant sa promenade, il ne repasse **jamais au même endroit** et il ne fait qu'un seul tour de piste.

Un **poste de secours** est situé à l'intérieur de la région délimitée par la piste. Un grand mât avec un drapeau permet au randonneur de repérer l'emplacement du poste de secours quel que soit l'endroit où il se trouve sur la piste.

**Trace une piste de ton choix et place le poste de secours à l'intérieur en respectant l'énoncé.**

Ma piste de randonnée :

### Page 3

La distance entre le randonneur et le poste de secours varie selon l'endroit où se trouve le randonneur sur la piste. Décris cette variation

### Page 4

Trouve une nouvelle manière de présenter le phénomène décrit à la page 3 sans que le dessin de la piste y apparaisse.

### Page 4 continuation

En utilisant la réponse à la question précédente, décris en mots de quelle façon la distance entre le randonneur et le poste de secours varie selon les endroits où se trouve le randonneur sur la piste.

Cependant, un exemple d'une proposition de covariation entre variables impliquant une relation non fonctionnelle avait été donné par une équipe qui est tombée dans des contradictions sans fournir une approche cohérente. Une équipe de futur enseignant(e)s a proposé la troisième figure (voir plus bas), demandant des processus algébriques plus complexes, la construction d'une fonction en parties (Variables : distance parcourue et distance de la personne au mât).

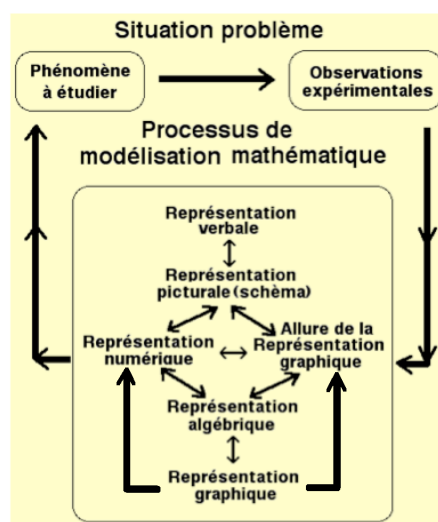


Figure 1. Modèle suivi dans l'expérimentation



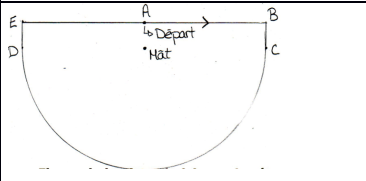
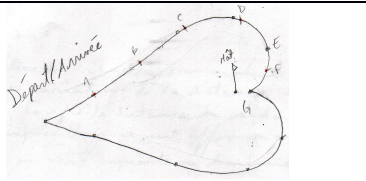
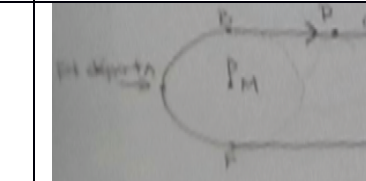
| Niveau élémentaire  | Niveau intermédiaire<br>relation non fonctionnelle                                | Niveau avancé  |
|---|---|--|
|  |  |  |

Figure 2. Différents productions des étudiants

### 3. Discussion

Notre proposition est immergée dans une perspective socioculturelle, que favorise l'utilisation d'objets physiques (par exemple la corde dans la covariation entre les variables : distance parcourue et distance de la personne au mât, avec des pistes irrégulières), favorise l'émergence de représentations spontanées dans un processus de communication dans la salle de classe, et cette communication permettra son évolution vers les représentations institutionnelles.

Compte tenu d'un programme de mathématiques axé sur le développement des compétences mathématiques, et de résolution de situations-problème, nous proposons la construction d'une structure structurante qui permettrait le développement des compétences dans les processus de modélisation mathématique et processus de mathématisation.

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## Student-teachers' re-inventing mathematisation as a didactic principle

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**Abstract.** As part of their field experiences in a mathematics education masters program, student-teachers conduct mini teaching experiments with individual students. In this paper, we reflect on the experiences we had with three student-teachers who worked with a sixth grade student on the multiplication of fractions. Their goal was to support the student's development of understanding the multiplication algorithm that she had already learned. Overall, initial results indicate that these student-teachers came to be involved in practices that supported the development of a new way of designing their instruction as they strove to assist the student in mathematising her activity. On the other hand, the analysis of this case study brings to the fore some of the challenges involved in fostering mathematisation as a didactic principle.

**Résumé.** Les étudiants-enseignants d'un programme de maîtrise en mathématiques effectuent de petites expériences d'enseignement avec des étudiants individuels, dans le cadre de leurs expériences sur le terrain. Dans cet article, nous réfléchissons aux expériences que nous avons eues avec trois étudiants-enseignants qui ont travaillé avec une élève de la sixième année de l'école primaire sur la multiplication des fractions. Leur objectif était de soutenir le développement de l'élève pour comprendre l'algorithme de multiplication qu'elle avait déjà appris. Dans l'ensemble, les résultats initiaux indiquent que ces étudiants-enseignants sont devenus impliqués dans des pratiques qui ont soutenu le développement d'une nouvelle façon de concevoir leurs instructions alors qu'ils s'efforçaient d'aider l'élève à mathématiser son activité. D'autre part, l'analyse de cette étude de cas met en évidence certains des défis impliqués dans la promotion de la mathématisation en tant que principe didactique.

### 1. Introduction

Two years ago (starting in 2015) we embarked on a research project that seeks to understand how to support teachers in developing an inquiry-based and student-centered approach in teaching mathematics. This project is conducted with a group of teachers (two in-service and 12 pre-service) in the context of a masters program. In the previous year, our primary goal was to support them in starting to develop a vision of instruction in which building on students' reasoning would be necessary for supporting progressive mathematisation of their ideas. Activities in which teachers were engaged gave them opportunities: (1) to enhance their understanding of fractions, (2) to learn how to interpret students' ideas, (3) to rehearse aspects of the practices they were supposed to develop, and (4) to sensitize themselves to the constraints of the institutional setting of schooling in our country. A preliminary analysis of our data shows that teachers had already started to re-organize their knowledge and beliefs. This year (2016 – present) we organized a design experiment concerning their field experiences in public schools of Athens. Our aim was to investigate how to support teachers in starting to reorganize their practices, as they are involved in a cycle of creating, implementing, and revising their plans for teaching.

One of our primary considerations in organizing our student teachers' field experiences was the institutional setting of the schools (Cobb, McClain, Lamberg, & Dean, 2003). In the centralized educational system of our country, the materials (textbooks and teacher guides) that teachers use in their classrooms are guiding them towards instruction that is mainly focused on procedural fluency, rather than on conceptual understanding. We judged that student teachers should interact with students over an extended period of time. Otherwise, they would only come across students' typically procedural reasoning. By engaging the student-teachers in conducting mini teaching experiments (8-10 half-hour meetings) with individual students, we hoped that they could familiarize themselves with the challenges of supporting the students' mathematical growth. Importantly, their experiences might be extremely useful, as most of their future students would be coming from classrooms with a procedural orientation.

In this paper, we focus on a group of three student-teachers that worked with a sixth-grade student on the multiplication of fractions. Though the child knew how to multiply two fractions, her conceptual understanding of the algorithm was absent. Thus, the group decided to focus their work on supporting her development of an increasingly sophisticated understanding of the procedure. The fact that the end product of mathematisation was already known makes the analysis of this case particularly interesting. Our goal of analysis was to identify some of the key challenges involved in student-teachers' coming to use progressive mathematisation in teaching, as well as in our attempts to support them.

## **2. Conceptual framework**

Student-teachers' field experiences are typically organized on the basis of a particular assumption concerning the relation between theory and instructional practice (cf. Cobb & Bowers, 1999). Student teachers are called to apply the theoretical principles they already know from their university courses to their instructional practice. Within this perspective, the principles of a theory, such as Realistic Mathematics Education instructional theory (Gravemeijer, 1994; Van den Heuvel-Panhuizen, & Drijvers, 2014), could be directly transferred to practice. In contrast to this assumption, we viewed student teachers' activity in conducting their teaching experiments as a source for re-inventing these principles. Thus, based on our ongoing assessments of their activity, we identified some of these principles as goals of their learning. On the other hand, research in practice-based teacher education was a basis for designing supports for student teachers' development of their practices (Ball, Sleep, Boerst, & Bass, 2009; Borko, 2004; Grossman, Hammerness, & McDonald, 2009; Lampert et al., 2013). In particular, this research literature indicates that teacher education should include opportunities: (1) to enact the targeted practices in practical settings of graduated difficulty with the support of skilled others, (2) to analyze and critique representations of the targeted practices, (3) to implement instructional materials that can be conducive to the desired practices, and (4) to work as a community of learners with the guidance of skilled others.

## **3. Methodology**

In approaching our work as a professional design study (Cobb, Jackson, & Dunlap, 2014), we set three interrelated goals for student-teachers' learning<sup>2</sup>. These goals were oriented towards the progressive mathematisation of students' activity and included: (1) setting clear goals for students' learning, (2) selecting and sequencing tasks in relation to each other, and (3) facilitating students' learning by encouraging them to express their reasoning and building on it.

To support the student-teachers' realizing these goals, we asked each group of them to complete specially designed forms concerning their plans for each of their sessions with the children as well as their evaluation of each meeting. Apart from giving them written feedback on these forms, we engaged them in discussing their plans and evaluations during our weekly meetings at the university. In these meetings, student-teachers were also called to analyze selected short video-clips of their prior sessions with the children. In short, it was their involvement in pedagogies of investigation and enactment (Grossman et al., 2009) that constituted the design of supporting the development of their practices.

The collected data to analyze and to document student-teachers' progress consisted of video-recordings of their sessions with the children and of our weekly meetings, as well as the student-teachers' planning and evaluation forms.

In analyzing student-teachers' learning, we tried to find evidence regarding how and to what extent their practices were changing in terms of the set goals. In order to identify the challenges they faced in their attempts to support students' learning, we also focused on analyzing the role of our supports in terms of setting clear goals for student-teachers' learning, continuously assessing their progress, and asking them to justify their pedagogical decisions.

## **4. Expected conclusions**

Initial results of the ongoing analysis suggest that the three student-teachers on whom we focused our analysis showed considerable progress across the goals we had identified for their practices. Notably, it appears that the student-teachers would need further support, with respect to setting clear goals for their student's learning, and the sequencing of tasks based on assessments of her progress.

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<sup>2</sup> The identification of these goals drew upon the research literature on the development of teaching practices, as well as upon difficulties detected as they were rehearsing activities among themselves in their prior semester.

## Acknowledgements

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## Pre-service elementary school teachers' ideas about fractions

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**Abstract.** This paper focuses on a study carried out to analyse pre-service Primary school teachers' knowledge about fractions. It addresses three questions: 1) What ideas do Pre-service Primary School teachers have about fractions? 2) How do these teachers understand ordering and equivalence of fractions? And 3) How these teachers understand the representation of fractions? A survey was carried out with 86 pre-service Primary school teachers from several parts of Portugal. The survey comprised 52 questions related to the concept of fraction, the invariants of rational numbers (ordering and equivalence), fraction representations, and the interpretations of fractions. Results revealed that Pre-service Primary school teachers have several difficulties concerning fractions. Several difficulties were identified in these pre-service teachers: some do not recognise the need to divide the whole into equal parts in fractions representations; they find difficult to relate parts of unit to 2 or more units; the property of the density of rational number set is not recognised by students, many believe they can count the number of fractions between 0 and 1; many cannot find the double of a given fraction; and many of them revealed difficulties with the interpretations of fractions, in spite of having to teach them in a close future.

**Résumé.** Cet article se concentre sur une étude réalisée pour analyser les connaissances préalables des enseignants du primaire sur les fractions. L'étude aborde trois questions: 1) Quelles idées les enseignants des écoles primaires ont-ils des fractions? 2) Comment ces enseignants comprennent l'ordre et l'équivalence des fractions? Et 3) Comment ces enseignants comprennent la représentation des fractions? Un sondage a été réalisé avec 86 enseignants du primaire de plusieurs régions du Portugal. L'enquête comprenait 52 questions liées à la notion de fraction, les invariants des nombres rationnels (ordre et équivalence), les représentations de fraction et les interprétations des fractions. Les résultats ont révélé que les enseignants de l'école maternelle ont plusieurs difficultés en ce qui concerne les fractions. Plusieurs difficultés ont été identifiées dans ces enseignants de pré-service: certains ne reconnaissent pas la nécessité de diviser l'ensemble en parties égales dans les représentations de fractions; Ils trouvent difficile de relier des parties de l'unité à 2 unités ou plus; La propriété de la densité de l'ensemble des nombres rationnels n'est pas reconnue par les étudiants, d'autres croient pouvoir compter le nombre de fractions entre 0 et 1; Beaucoup ne peuvent pas trouver le double d'une fraction donnée; Et beaucoup d'entre eux ont révélé des difficultés d'interprétation de fractions, en dépit d'avoir à leur enseigner dans un proche avenir.

### 4. Framework

This study aims to understand pre-service teachers' knowledge of fractions. For that, it addresses three questions: 1) What ideas do future Primary School teachers have about fractions? 2) How do these teachers understand ordering and equivalence of fractions in different interpretations of fractions? And 3) How these teachers understand the representation of fractions in those interpretations?

The concept of fraction is considered fundamental for a successful and proper development of children's mathematical thought. Nevertheless, is also one of the most complex concepts that children learn during the elementary grades. Knowing the concept of fraction demands the understanding of the logical aspects of fractions (ordering and equivalence) and the ability to use distinct modes of representation, in different

interpretations of this concept (Behr et al., 1983; Nunes, Bryant, Pretzlik, Wade, Evans & Bell, 2004; Mamede & Nunes, 2008).

Pre-service teachers must be competent in the domain of rational numbers in order to be able to develop fruitful practices with their primary school students. But are future teachers adequately prepared to teach rational numbers to their students? Little research has been developed in order to explore this issue regarding the Portuguese reality.

In the domain of rational numbers, the Portuguese mathematics curriculum for primary school demands the use of mathematical and pedagogical knowledge quite challenging for teachers. They are supposed to be fully acquainted with the representation, ordering and equivalence of fractions, as well as with different interpretations of fractions. Teachers are also supposed to help students to establish the link between different representations of rational numbers (fractions, decimals, percentage), and compute with fractions and decimals.

## 2. Teachers' difficulties with fractions

Literature suggests that very often teachers have the same difficulties of their students and have the same misconceptions (see Lamon, 2003). Previous research conducted with Portuguese elementary school teachers has revealed that teachers have several difficulties concerning the teaching of rational numbers. These difficulties comprise conceptual and didactical features. Pinto and Ribeiro (2013) analysed 27 pre-service teachers' mathematical knowledge about fractions – interpretations of fractions, representation of fractions, the concept of unit, ordering and equivalence of fractions, and the density of the rational number set. Results indicate that most of the teachers felt comfortable only with part-whole interpretation of fractions; only 50% of them could identify the unit when a part of the whole was given; 43% and 97% of teachers revealed problems with equivalence and ordering of fractions, respectively; and 73% of them possess erroneous ideas concerning the density of the rational number set.

Concerning the didactical issues, Cardoso and Mamede (2013) interviewed two primary school teachers to understand how they explored the concept of fractions with their 3<sup>rd</sup> graders, when quotient interpretation is involved, as recommended by the curriculum guidance. Their results showed that teachers did not possess any lesson plan to teach fractions, because usually they do not teach them. Teachers believe that their students were not capable of learning all the content included in the curricula, nevertheless they had never tried to teach them fractions before. Perhaps the devaluation of students' abilities could be hiding some of the teacher's difficulties with fractions. Indeed, one of the teachers could not identify a pictorial representation of a fraction in a quotient interpretation context. These authors pointed out that the fragility of teachers' knowledge regarding these issues compromises their teaching of rational numbers (see Cardoso & Mamede, 2013; Pinto & Ribeiro, 2013), and consequently, the development of students' rational number sense.

In this scenario, it becomes of utmost importance to have an insight into pre-service primary school teachers' preparation to teach fractions. The identification of pre-service teachers' difficulties is relevant to improve the future teaching and learning of rational numbers, as it interferes with the mathematisation process of the students.

## 3. Methods

A survey was conducted with 86 pre-service elementary school teachers (mean age: 22 years, 5 months), from several parts of Portugal. The survey comprised 52 questions related to fractions (18 related to the concept and properties of fractions; 13 concerning the invariants of rational numbers - ordering and equivalence; 13 about the representation of fractions and concept of unit; and 8 about the interpretations of fractions).

## 4. Results

The mean of correct responses was 31,5 (standard deviation of 6,56). Table 1 resumes the means and standard deviation of the proportions of pre-service teachers' correct responses.

**Table 1.** Mean (standard deviation) of proportions of correct responses by each Type of questions.

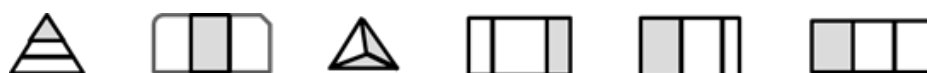
| Type of questions   | Mean (standard deviation) |
|---------------------|---------------------------|
| Concept of fraction | .68 (.17)                 |



|   |           |
|---|-----------|
| Invariants of fractions                             | .60 (.22) |
| Representation of fractions and the concept of unit | .58 (.21) |
| Interpretations of fractions                        | .62 (.15) |

There were 18 questions concerning the concept of fractions, but only 2 teachers (2,3%) could succeed in all of them; only 31,4% of the teachers could succeed in at least 75% of the questions.

In one of the questions regarding the concept of fractions, most of the participants presented an incomplete answer when asked to identify pictures that could represent  $1/3$ , as those of the Figure 1.



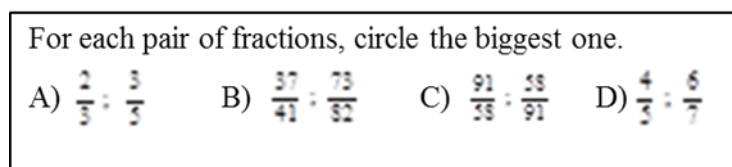
**Figure 1.** Pre-service teachers were asked to circle the pictures that represent  $1/3$ .

Only 12 teachers (14%) presented a total correct answer; 70 (81,4%) presented an incomplete answer considering only the last picture; and 4 presented responses indicating that the size of the part into which the whole is divided is irrelevant to represent fractions. In another question related to the concept of fraction in which it was asked "How many ninth are there in 3 units?", 31 participants (36%) answered correctly, 18 teachers (20,9%) presented no answer, 12 participants (14%) referred '27/9', and almost 29% gave other incorrect answers. When asked "How many fractions are there between 0 and 1? Justify your answer.", only 36 teachers (41,9%) answered correctly, 37 presented no answer (43%), and 13 presented an incorrect response (15,1%). Pre-service teachers' justifications reveal that some of those who gave a correct answer could not present a written justification, only 13 gave an explanation based on the property of the density of rational number set (15,1%); 47 presented no justifications at all (54,7%), and the remaining justifications were incorrect.

When asked whether " $4/8$  is 2 times bigger than  $2/4$ ", 34 participants (39,5%) believed that that was true, but 52 considered the sentence false; when they were asked whether " $4/8$  results from multiplying  $2/4$  by 2", 40 pre-service teachers (46,5%) could not recognise it as a false sentence. This reveals some of the difficulties of prospective teachers with fractions. It is not expected that future elementary school teachers could accept  $4/8$  as the double of  $2/4$ .

Regarding the invariants of fractions, 13 questions were presented to the pre-service teachers concerning ordering and equivalence of fractions, but only 1 teacher (1,2%) could succeed in all of them, and 10 teachers (11,6%) succeeded in 12 questions. Only 9,75% of the teachers could succeed in at least 75% of the questions.

Concerning the invariants of fractions, in the problem of the Figure 2, teachers had to compare fractions in each situation. In the case A) 56 teachers (65,1%) identified the correct fraction, 12 gave a wrong answer (14%) and 18 students (20,9%) could not answer; in the case B) 47 students (54,7%) identified the correct fraction, 14 gave a wrong answer (16,3%) and 25 students (29,1%) could not answer; in the case C) 52 students (60,5%) identified the correct fraction, 9 gave a wrong answer (10,5%), and 25 students (29,1%) could not answer; in the case D) 49 students (57%) identified the correct fraction, 14 gave a wrong answer (16,3%) and 23 students (26,7%) could not answer.



**Figure 2.** A problem of ordering of fractions presented to students.

In the case A) 30 pre-service teachers were not able to compare correctly  $2/3$  and  $3/5$ , which are fractions that children use in the 3<sup>rd</sup> grade; probably, for the same reason 37 teachers failed in the case D), comparing  $4/5$  and  $6/7$ . In some of the correct answers of these questions, students had to reduce fractions to the same denominator to compare them in order to produce a correct answer. Nevertheless, 4<sup>th</sup> graders cannot use this procedure to compare fractions. In case B), 47 pre-service teachers (54,7%) presented a correct answer, but 39 failed to compare these fractions (45,4%), being unable to recognise, for instance, that one denominator is

the double of the other, but the correspondent numerators are not. In the case C), 52 teachers (60,5%) succeeded, but 34 (39,6%) could not realise that one of the fractions was smaller than 1 and the other was bigger. This lack of knowledge when dealing with fractions compromises the implementation of meaningful classes about rational numbers conducted by these pre-service teachers. More results will be presented in the conference regarding pre-service knowledge about fractions.

## 5. Final remarks

Pre-service elementary school teachers need to improve their ideas about fractions. Several difficulties were identified in these pre-service teachers: some do not recognise the need to divide the whole into equal parts in fractions representations; they find difficult to relate parts of unit to 2 or more units; the property of the density of rational number set is not recognised by students, many believe they can count the number of fractions between 0 and 1; many cannot find the double of a given fraction; and many of them revealed difficulties with the interpretations of fractions, in spite of having to teach them in a close future. Discussion and the educational implications of our findings will be presented in the conference and in the version of the paper for the proceedings.

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# Comment interpréter le cycle de modélisation avec l'Espace de Travail Mathématique ? Etude de la trajectoire d'un problème.

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**Résumé.** À travers la trajectoire d'un problème, le lièvre et la tortue, nous étudions comment le cycle de modélisation peut être interprété à l'aide du modèle des Espaces de Travail Mathématiques. Il s'agit de préciser le rôle de l'enseignant dans la circulation du travail mathématique lors de l'activation d'un cycle de modélisation.

**Abstract.** Through the problem's trajectory, the hare and the turtle, we study how the cycle of modelling can be interpreted in the Mathematical Working Space model. It's a question of specifying teacher's role in the circulation of the mathematical work during the activation of a modelling cycle.

## 1. Introduction

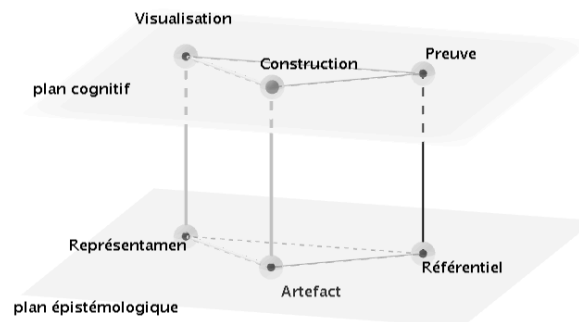
Notre recherche porte sur l'intégration de la simulation informatique par des enseignants au niveau grade 9 après une formation sur les probabilités et statistiques. Elle prend appui sur la situation « du lièvre et de la tortue ». On lance un dé : s'il tombe sur 6 alors le lièvre gagne sinon la tortue avance d'une case. Kiet (2015) et Gaydier (2013) ont analysé le travail de l'élève dans cette situation et notre étude s'intéresse à l'enseignant, s'appuyant sur la trajectoire de problème (Kuzniak & al., 2013). Elle recouvre l'ensemble des processus de transformation d'un problème à travers diverses institutions allant d'une formation pour des enseignants à sa mise en place dans les classes. Ce problème peut être résolu avec différents modèles mathématiques. Nous examinerons la circulation du travail mathématique provoquée par l'enseignant dans cette activité de modélisation ; notre analyse s'appuiera sur le cadre des Espaces de Travail Mathématique (ETM) (Kuzniak, 2011) et du cycle de modélisation (Blum & Leiss, 2007).

## 2. Cadre théorique

L'analyse a priori des modèles utilisés par l'enseignant et de l'implémentation de la situation en classe s'appuient sur les Espaces de Travail Mathématique (ETM). Ce modèle permet de repérer la spécificité du travail mathématique. Un ETM est une structure d'accueil de ce dernier et repose sur l'articulation de deux plans : celui épistémologique et celui cognitif. Le premier s'articule autour de trois pôles : le référentiel théorique (propriétés, théorèmes et définitions), le représentamen (signes) et les artefacts matériels ou symboliques (logiciel, techniques de calculs). Le plan cognitif contient trois pôles : les processus de visualisation, de construction et de preuve.

Cet ensemble est représenté par un diagramme formé d'un prisme à bases triangulaires dont les arêtes verticales font le lien entre ces deux plans et définissent :

- la dimension sémiotique, reliant représentamen et visualisation
- la dimension instrumentale, reliant artefact et construction
- la dimension discursive, lien entre référentiel et preuve.



**Figure 1.** Modèle des Espaces de Travail Mathématiques (Kuzniak, 2011)

Le modèle des ETM permet d'identifier les genèses activées par l'enseignant au regard de celles favorisées par les élèves. Nous analyserons les plans activés dans le déroulement de la résolution de la tâche pour décrire la circulation à travers les plans [Sem-Dis], [Sem-Ins] et [Ins-Disc]. Notre but est de repérer en quoi le travail organisé par l'enseignant pourrait ne pas être complet (Kuzniak et al., 2016). L'étude des ETM idoines prévus pour une classe (potentiels) et de ceux mis en place par des enseignants (après formation) nous permettra également de préciser certaines connaissances spécifiques nécessaires à l'enseignant pour mener à bien cette situation.

L'implémentation de cette situation en classe sera aussi décrite avec le cycle de modélisation (Blum & Leiss, 2007). Quels ajustements sont possibles au sein du cycle et en quoi la responsabilité de l'enseignant est-elle engagée ?

### 3. Méthodologie de recherche

Nous avons donc choisi d'analyser différentes mises en œuvre de cette situation à travers la notion de trajectoire d'un problème (Kuzniak et al., 2013). Un avatar est une réalisation particulière envisagée par un professeur. Nous avons choisi un premier avatar, qui nous permettra de décrire les choix d'un professeur pour sa classe. Nous précisons le modèle mathématique retenu par celui-ci, et décrivons la circulation prévue a priori dans l'ETM. Grâce à l'observation de sa séance, nous mettrons en évidence certains blocages et rechercherons les facteurs qui empêchent éventuellement un travail complet. Pour ce faire, nous nous appuierons sur les dimensions et plans du modèle des ETM.

Nos données émanent d'un questionnaire, d'entretiens pré et post séance de classe. Cette dernière a été filmée et couplée à des enregistrements sonores.

### 4. Description du travail prévu dans l'ETM idoine potentiel

L'énoncé présentait un parcours avec cinq cases et une d'arrivée. La séance était imaginée sur 2h.

*Phase 1 exploration de la situation* : (10 min) recherche individuelle où elle attendait une amorce de résolution au brouillon. Des dés étaient disponibles sur son bureau, permettant d'éventuels lancers.

Cette phase dans l'ETM idoine de classe, devait prendre appui sur la dimension sémiotique, voire se situer sur la dimension instrumentale pour certains, le dé étant un artefact pouvant favoriser l'expérimentation et l'émergence d'échanges autour de la situation ensuite.

*Phase 2 recherche en travail de groupes* : (3 élèves) (amplitude variable selon l'avancée, entre 50min et 1h50) avec restitution de production écrite et fichier numérique éventuel attendus.

L'enseignante avait imaginé que certains élèves (phases 1 ou 2) feraient des expériences aléatoires avec un dé jeté manuellement préalablement à une simulation. L'accès à un ordinateur a été prévu, en ce qu'il permettrait de créer une simulation numérique, si certains élèves le jugeaient utile. Elle pensait que les groupes s'achemineraient vers un travail dans le plan [sem-ins], avec l'usage du tableur, et espérait la production de simulations à partir de feuilles de calcul vierges, sans autre consigne que celle de la phase 1.

Ainsi, des ETM idoines de groupes devraient alors se façonner, grâce aux échanges d'idées individuelles sur la situation au sein des groupes. L'enseignante prévoyait de prendre des indices et d'interagir si nécessaire. Ces divers ETM idoines, organisés autour des travaux de 3 élèves, constitueraient des entités au service de celui de la classe, en particulier en phase 3. Elle avait préparé la veille trois simulations tableur présentant toutes 6 lancers indépendants pour chaque course.

*Phase 3 d'institutionnalisation:* Ce bilan, s'appuyant sur des productions de groupes sélectionnées par l'enseignante, était évoquée dans la préparation. Flou dans son contenu potentiel, il dépendrait du temps de consacré à la phase 2. Des représentants de groupes viendraient exposer à la classe leur travail, sur un temps limité et ce dans un ordre choisi par l'enseignante.

## 5. Description du travail effectif dans l'ETM idoine de classe

Voici une description partielle de l'ETM idoine de classe, repris phase après phase suite à nos observations de la mise en œuvre dans la classe.

*Phase 1 d'exploration de la situation :* (10 mn) Dans cette phase de recherche individuelle, peu d'élèves ont utilisé un dé pour effectuer des expériences. Des incompréhensions des règles du jeu furent fréquentes et multiples avant même que les élèves mathématisent la situation (asymétrie de la règle d'avancement sur le parcours). Le travail a débuté ou dans le plan [sem-ins] ou dans le plan [sem-dis]. Pour passer de la situation modèle au modèle réel, certains ont lancé des dés, notant pour chaque course les gagnants, tandis que d'autres, dans le plan [sem-dis] donnaient spontanément des probabilités erronées (pour que le lièvre gagne 1/6 voir 6/36, pour que la tortue gagne 5/6), mobilisant le modèle d'équiprobabilité sur le dé.

*Phase 2 recherche en travail de groupes :* (1h50) Elle est ici partiellement relatée.

La structuration de la feuille de calcul, a dévoilé des difficultés d'interprétation des règles du jeu auxquelles l'enseignante a dû faire face.

Un premier ETM idoine de groupe présentait une simulation tableur avec 7 lancers de dés par course alors qu'au maximum 6 sont nécessaires. L'enseignante a alors imposé un recours aux dés physiques pour jouer des courses à la main, afin de modifier un confinement dans la dimension instrumentale. La condition d'arrêt a alors été réajustée à 6 lancers dans le tableur. Un travail a eu lieu dans le plan [sem-dis], en appui sur des artefacts (dés et objets matérialisant les animaux), non prévu dans la circulation initiale.

Un autre groupe, souhaitait, au tableur, faire des relances de dé uniquement si le 6 n'était pas sorti, et a demandé de l'aide face à des difficultés. L'enseignante a alors imposé 6 lancers systématiques par course, privilégiant un modèle (celui de la loi binomiale) jugé plus facile sur le tableur que celui initialement choisi (lié à la loi géométrique). La circulation initiée a été détournée par l'enseignante, privilégiant le plan [ins-dis] au plan [sem-ins]. Les élèves ont exécuté les 6 lancers, verbalisant une résistance au modèle imposé.

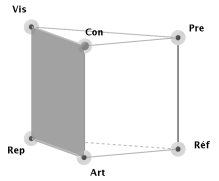
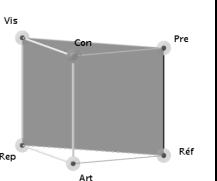
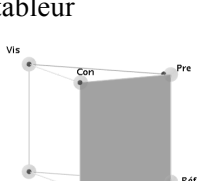
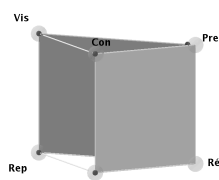
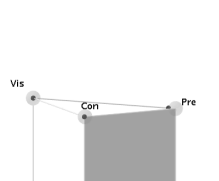
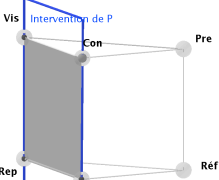
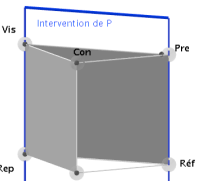
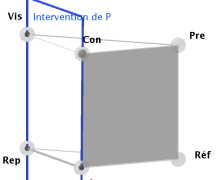
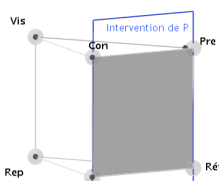
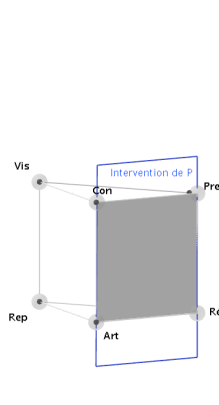
*Phase 3 d'institutionnalisation :* Dans chaque ETM idoine de groupe, l'enseignante a pris appui sur le fichier tableur présent et la loi faible des grands nombres, faisant relancer les simulations, observer et verbaliser la stabilisation des fréquences. Aucune réponse au problème n'a été partagée en classe entière. Le tableau 1 complète cette étude.

## 6. En conclusion

Le logiciel choisi n'est pas neutre et influe sur la circulation du travail mathématique à travers un cycle de modélisation non linéaire. Dans les ETM idoines de groupes, les circulations, a priori différentes ont été rendues homogènes par les interventions de l'enseignante. Si elle a semblé favoriser l'émergence de diverses procédures de résolution, elle n'a retenu que l'approche fréquentiste. Les simulations ont été ajustées, avec un modèle imposé (l'enseignante le considérant plus aisé dans le tableur). La phase 3 aurait pu permettre de confronter les valeurs observées par approche fréquentiste et des calculs exacts de probabilités d'un ETM idoines de groupe, mais l'enseignante n'a pas exploité ces derniers, privilégiant un travail dans les plans [sem-ins] et [ins-dis].

**Tableau 1.** Circulation dans les ETM idoines de groupe/classe, interventions du professeur (P)

|  | Situation modèle<br>Modèle réel | Modèle réel<br>Modèle<br>mathématique | Modèle<br>mathématiques<br>Résultats<br>mathématiques |
|--|---------------------------------|---------------------------------------|---|
|  |                                 |                                       |   |

|                           | Phase 1, début phase 2   | Phase 2 usage du tableur   | Phase 3  |
|---------------------------|--|--|--|
| <b>Circulation</b>        |    |    |   |
| <b>Interventions de P</b> |   |   |  |

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## Continuing education for teachers and the teaching of statistics at elementary levels

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**Abstract.** This article aims to analyze a teacher development experiment that explores statistical literacy oriented to develop the professional perspective in a group of twenty-three mathematics teachers working at the early levels of elementary education in public schools in the state of São Paulo, Brazil. As data collection instruments this qualitative-methodology approach used a questionnaire and recorded activities proposed by the teachers. Data analysis used the studies of Zeichner & Linhares on Reflective Practice, D'Ambrosio & Skovsmose's Critical Mathematics Education, and Statistical Literacy as proposed by Batanero, Godino & Gal. Results showed that there is a gap between the statistical literacy tenets presented at curricular orientation discussions and teacher practices, which is developed on the basis of activity templates available through external evaluation models and textbooks that emphasize the use of procedural statistical concepts in mathematics classes at Elementary Education levels. The need to review the curriculum of teacher development courses became evident: teachers should be able to not only repeat what is prescribed by official documents, but also and mainly, to be able to put into practice - with critical awareness and autonomy - activities that enable students to develop a critical reading of reality, a key factor to fully exercise their civil rights.

**Résumé.** Dans cet article on vise à analyser une expérience de développement de l'enseignant qui explore l'alphabétisation statistique axée sur le développement de la perspective professionnelle dans un groupe de vingt-trois enseignants en mathématiques travaillant au début de l'enseignement primaire dans les écoles publiques de l'État de São Paulo au Brésil. Guidé par une méthodologie quantitative, en tant qu'instruments de collecte de données on a utilisé un questionnaire et des activités enregistrées proposées par les enseignants. Pour l'analyse des données on a utilisé les études de Zeichner & Linhares sur la pratique réflexive, l'éducation mathématique critique d'D'Ambrosio & Skovsmose et l'alphabétisation statistique proposée par Batanero, Godino & Gal. Les résultats ont montré qu'il existe un écart entre les principes de l'alphabétisation statistique présentés lors des discussions sur l'orientation curriculaire et les pratiques des enseignants, qui sont élaborés sur la base de modèles d'activités disponibles à l'aide de modèles d'évaluation externe et de manuels qui mettent l'accent sur l'utilisation de concepts statistiques de procédure dans les classes de mathématiques aux niveaux d'enseignement primaire. La nécessité d'examiner le curriculum des cours de développement des enseignants est devenue évidente: les enseignants devraient pouvoir non seulement répéter ce qui est prescrit par les documents officiels, mais aussi et surtout pouvoir mettre en pratique - avec une conscience critique et une autonomie - des activités qui permettent aux étudiants développer une lecture critique de la réalité, un facteur clé pour exercer pleinement la citoyenneté.

## 1. Introduction

The evidence that statistics is increasingly present in the everyday life of persons has made the teaching of this subject at Elementary Levels more prominent in debates within the field of Mathematics Education. Presently, the easy access to digital technology and the use of specific software that generates calculations and graphs have contributed to the use of statistical methods for information and knowledge sharing. The use of concepts and statistical procedures focuses on the analysis of several situations, both for simple information and data involved in reality and for more complex kinds. Many times, information is transmitted in various media formats through graphs representing situations in different knowledge fields. These graphs must be read and interpreted so people can understand information in a critical manner when making decisions and to exercise their citizenship. Gal (2005) emphasizes the importance of statistical literacy for the development of citizenship as " [...] needed if adults are to be fully aware of trends and phenomena of social and personal importance" (Gal, 2005, p.49). According to that author, statistical literacy is to know how to critically interpret and evaluate information by comparing it against the arguments related to the data presented in various contexts so we can adequately discuss the meaning of statistical information. Hence, to be statistically literate is not a simple task: it requires knowledge construction. In Brazil, even though Statistical Education is included in the official curricular documents of elementary school, this content has not been effectively treated during mathematics classes in the perspective proposed by Batanero, Díaz, Contreras & Roa (2013) or Batanero & Godino (2005) and Gal (2005), who emphasize practices that allow students to experiment with investigative processes to learn statistical concepts. Statistical Literacy is a social demand based, for instance, on Freire's ideas (1994) about the importance of having education to develop individuals capable of reading the world in a critical manner. These ideas are also sponsored by researchers of Mathematical Education such as D'Ambrosio (2014) and Skovsmose (2008).

In the present Brazilian scenario in of research has shown that statistics concepts developed at school by mathematics teachers have the kind of approach that, in general, only prioritizes procedural use based on the application of formulas and calculations. This practice has its root in the initial development of teachers' education and in an entrenched practice based in educational concepts, which focus on teaching reproduction of models and procedures. Such a fact reinforces the existence of a gap between the need of the school to offer statistical literacy to students and the actual teaching practices. The gap, in its turn, constitutes a set of issues that has instigated researchers to re-think the initial and continued education of mathematics teachers.

Having these issues in mind, this article presents a study that has been developed within the scope of a teachers' continuing education course with participants of projects promoted by the Brazilian government known as Education Observatory. One of its projects takes place in the Postgraduate Studies in Mathematical Education at Anhanguera University of Sao Paulo, in partnership with the Department of Education of the State of Sao Paulo, and encompasses the development of actions both in research-related fields and in teacher education in the reality of elementary school. The goal of this research is to analyze an educational experiment on Statistical Literacy oriented to the professional development of a group of 23 teachers, presently teaching mathematics at the early levels of elementary school (6 to 10 years of age). Before the analysis is presented, however, we will show the basis of the curricular guidelines related to the teaching of Statistics, since the development experiment was planned and performed based on their tenets.

## 2. Curricular guidelines

Since 1990 we have used the National Curricular Guidelines (PCN, in Portuguese), as a guidance for teachers in their educational tasks. According to the official document, mathematics has an important role in the basic development of Brazilian citizens when it states that, *"in order to exercise citizenship it is necessary to know how to calculate, measure, argue with, and treat information statistically"* (BRASIL, 1997, p.25). It also underscores that *"students should not only learn how to read and interpret graphic representation, but also be able to describe and interpret their reality using mathematical knowledge"* (BRASIL, 1997, p.49). For that to happen, the teaching of mathematics should enable the development of an investigative spirit, which means, in the case of statistical collection and organization of information, to create records to transmit information, and the elaboration of simple and two-entry tables and graphs, text production based on the interpretation of graphs and tables (BRASIL, 1997). The new national curricular guidelines are currently under revision and a document entitled Common Core National-Based Curriculum



(BCNN, in Portuguese) is in its final phase of development. As far as statistics is concerned, the BNCC (Brasil, 2016) suggests the development of statistical work through the collection and organization of data from a survey based on students' interest with emphasis on those contents from the first school year (age 6). The fact is that curricular guidelines produced in a clear and well-argued basis by experts do not guarantee that teachers will incorporate them to the point they will be put into practice in the classroom. Also, it is worth pointing out that textbooks insist on favoring procedural characteristics on the use of statistics. Besides this, when an activity is presented as fulfilling the investigate features to be developed with students, teachers end up restricting the activity and focusing only on its procedural features, strongly influenced by their background education. Hence the need to invest both in their initial development years to review this issue and in their continuing education to enable in-service teachers to construct new references related to content, and their teaching and autonomy in the re-creation of their own practice.

### 3. Research and development

This qualitative research was carried out in the context of a continuing education course, based on Zeichner (1993) and Linhares (2011) regarding the reflective practice related to Statistical Literacy and the sharing of experience among the participants. A total of 23 teachers who teach mathematics in the early years of public schools in the city of Sao Paulo took part in the experiment. Data collection was made through the following: profile-questionnaire, protocols of teachers' activities and recordings (videotape and texts) of on-site meetings. The diagnostic activities clearly showed that the teachers in the group had many doubts regarding the reading and construction of graphs. This finding made us redirect the development proposal in two goals: to provide teachers with the construction of statistical literacy and to analyze and reflect upon their practices in the teaching of statistics.

### 4.

#### and results

#### Analysis

Information collected throughout the course allowed us to analyze the teachers' knowledge regarding the themes and literacy process of descriptive statistics, such as different forms of graphical representation, the measures of central tendency and some initial aspects of measures of dispersion. The chart below brings an example of one activity proposed in the course.

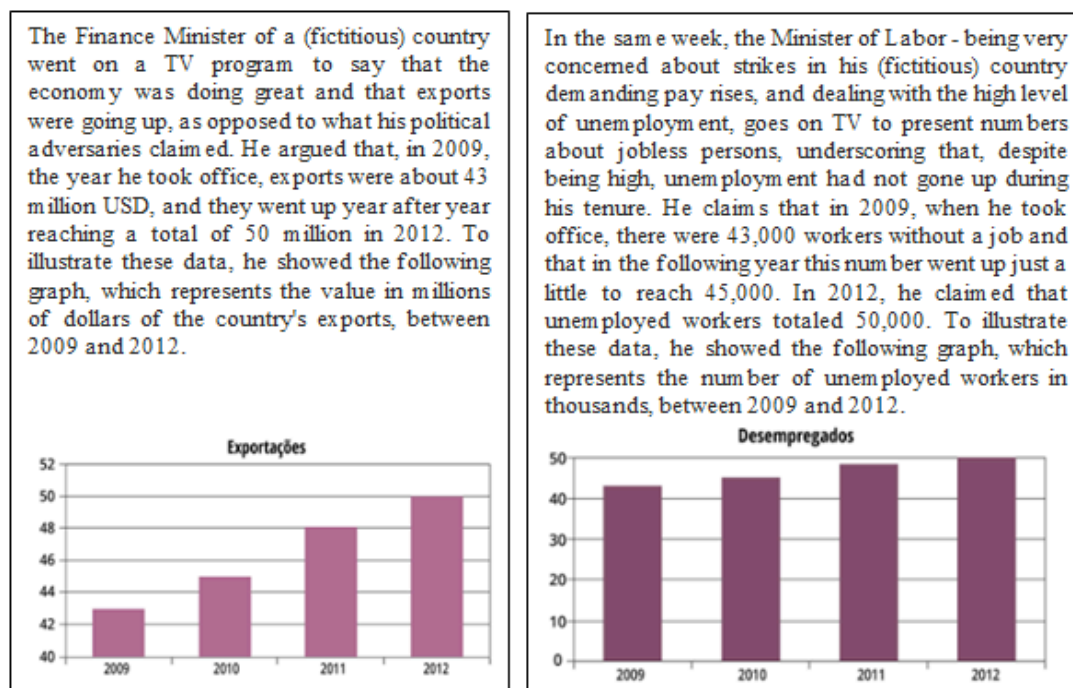


Figure 1: Activity Scenario

The reading and interpretation of the two graphs in Figure 1 generated lots of discussion in the group, mainly regarding the labelling of data in the graph and the scale used in its construction. This reflection enabled participants to become aware of the need to be statistically literate to be able to read information in a critical

manner. In further proposed activities, we noticed that teachers emphasize the building of bar graphs, including data used for continuous variables or for frequency distribution grouped in class intervals, instead of using line graphs or histograms.

We also noticed that the argument used by the majority of participants to analyze and make decisions to solve situations that involved central tendency values was based on the analysis of the values of mean, mode and median used in isolation. It was necessary to retrieve problem-situations that favored the need to relate the values of these measures to accurately understand the data presented.

As for the focus of the development to provide the sharing of practices by statistics teachers with their students, we found out that there is an effort to work with the children's daily life issues such as birthday dates, favorite games, and others. However, when the teachers propose a data survey to the students, the activity is reduced to organizing a list and building a graph, usually in columns, and reading information, such as locating the most and least voted. This fact was discussed during the development course. This reflection allowed teachers to acknowledge that the proposed activities in their classes are directly affected by activities used in external evaluations and in textbooks, whose focus is in the reading of information in tables or column-graphs and problem-solving, simply from reading. This finding shows that there is a gap between the tenets of statistics teaching understood as a kind of literacy presented in the curricular guidelines and the teacher's practice that is developed based on models of activities available in external evaluations and in textbooks that tend to reinforce the procedural use of statistical concepts in mathematics classes at elementary levels.

## 5. Final remarks

The statistical literacy process provided the participants with the contact with everyday situations that demanded decision-making for which they needed to read critically the information presented. This approach in the formative context stimulated the reflection on the teaching practice and the re-reading of the curricular guidelines. The participating teachers had the opportunity to experience a process of statistical literacy development. This study showed that curricular guidelines should be analyzed and understood by in-service teachers and in initial development courses. For that, the need to review the curriculum of teachers' initial development courses is key to ensure that future teachers will be able not only to repeat what is prescribed by official documents, but also and mainly, to put it into practice with critical awareness and autonomy and to propose activities that enable students, from the very early school years, to build statistical literacy so that children can develop a critical reading of reality and exercise their citizenship.

## Acknowledgements

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## **Collaborative inquiry: A professional learning approach for middle school mathematics teachers**

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**Abstract.** This research studied the use of a collaborative inquiry approach as a tool in the professional learning of middle school mathematics teachers to challenge their pedagogical practice in the middle schooling context. It involved the building of a professional learning community among middle school mathematics teachers to challenge and share their pedagogical practices. This grounded theory study adopted the collaborative inquiry approach via the use of site-based programmed planning time to enable the teachers to discuss, deliberate and share pedagogical practices so as to grow their professional practice. The research was undertaken over a three-year period, and included the use of team planning recordings, student focus groups, teacher survey responses and individual teacher interviews as data to triangulate the teachers' collective professional journey and trajectory of growth and development in their pedagogical understanding and practices. The findings of this research confirm the importance of such an on-site built-in professional learning model which harnesses the benefits of the site-based context in establishing a professional learning community that enables a group of teachers to consolidate professional learning needs for their collective growth. This approach takes into account the teacher differentiated learning needs and has the added benefits of creating a positive team and school culture that connects a critical mass among the teachers to reflect their practice. Such professional learning approach was found to be effective in developing a shared positive disposition and potentially initiated pedagogical practice change. It adheres to adult learning principles such as focussing on relevant topics to their work and encouraging dialogues to share personal experience. The impact of the teachers' pedagogical change on the students' engagement was found to be largely dependent on the wider context of the culture and attitude of the school, the teachers' ability to link mathematics to other disciplines, and the mental models and beliefs of the teachers.

**Résumé.** Cette recherche a étudié l'utilisation d'une approche d'enquête collaborative comme outil d'apprentissage professionnel des enseignants des mathématiques du collège pour remettre en question leur pratique pédagogique dans le contexte de l'école intermédiaire. Il s'agissait d'établir une communauté d'apprentissage professionnel parmi les enseignants des mathématiques du collège pour contester et partager leurs pratiques pédagogiques. Cette étude basée sur la théorie ancrée a adopté l'approche de l'enquête collaborative par l'utilisation du temps de planification programmé sur le site pour permettre aux enseignants de discuter, de délibérer et de partager des pratiques pédagogiques afin de développer leur pratique professionnelle. La recherche a été entreprise sur une période de trois ans et a inclus comme données l'utilisation d'enregistrements de planification d'équipe, des groupes de discussion d'étudiants, des réponses aux enquêtes auprès des enseignants et des entrevues individuelles avec des enseignants, pour trianguler le parcours professionnel collectif des enseignants et la trajectoire de croissance et de développement dans leur compréhension pédagogique et pratique. Les résultats de cette recherche confirment l'importance d'un tel modèle

d'apprentissage professionnel sur site intégré qui exploite les avantages du contexte basé sur le site en établissant une communauté d'apprentissage professionnel qui permet à un groupe d'enseignants de consolider les besoins d'apprentissage professionnel pour leur croissance collective. Cette approche prend en compte les besoins d'apprentissage différenciés des enseignants et offre des avantages supplémentaires en créant une équipe positive et une culture scolaire qui relie une masse critique parmi les enseignants pour refléter leur pratique. Une telle approche d'apprentissage professionnel s'est révélée efficace pour développer une disposition positive partagée et un changement de pratique pédagogique potentiellement lancé. Il adhère aux principes d'apprentissage des adultes, notamment en mettant l'accent sur des sujets pertinents à leur travail et en encourageant les dialogues pour partager leurs expériences personnelles. L'impact du changement pédagogique des enseignants sur l'engagement des étudiants a largement dépendu du contexte plus large de la culture et de l'attitude de l'école, de la capacité des enseignants à associer les mathématiques aux autres disciplines et des modèles et croyances mentaux des professeurs.

## **1. Aim of study**

The objective of this research was to evaluate the effectiveness of collaborative inquiry approach in developing middle school mathematics teachers' understanding and implementation of effective pedagogy.

The following research sub-questions were addressed:

- a. What challenges and benefits would the implementation of the collaborative inquiry as a means of professional learning approach present for the school and the teachers?
- b. What would be the essential elements in implementing effective collaborative inquiry as a professional learning approach for teachers in the school?
- c. How would these essential elements be incorporated in a theoretical model to inform the effectiveness of collaborative inquiry as an professional learning approach for middle school mathematics teachers in schools?

This study used the grounded theory approach in conjunction with qualitative research methodology in the collection and analysis of data (Creswell, 2008). This qualitative design determined the impacts of collaborative inquiry approach of professional learning on site regularly for a group of mathematics teachers' development of understanding and implementation of effective pedagogies (Nelson, 2003; Nelson, 2008; Nelson, Slavit, Perkins, & Hathorn, 2008; Shadish & Luellen, 2006).

## **2. Methodology: Nature and Appropriateness of Grounded Theory Approach**

I adopted the constructivist approach to grounded theory research, first used by Glaser and Strauss in 1967, in that all the reality was constructed and both constructed perspectives of the respondents and I were equally valued (Oktay, 2012). Grounded theory was chosen for this research because it was designed to study the interactions between individuals and their practice setting employing the symbolic interaction theory (Oktay, 2012). I focussed on the interactions of the teachers within a collaborative inquiry setting as a team of year level teachers focusing on the interrogation of their own pedagogical practice and its effectiveness.

This multi-stage process with the gathering of data after the selection of topic using the researcher's theoretical sensitivity. My literature review focused on the current understanding of various concepts relating to the research problem and the emerging thinking as each phase of the data was being analysed and new understandings emerged. Consequently, it is essential that a heightened state of theoretical sensitivity by me eradicated the theoretical bias. Teppo (2015) suggested that as the research progressed, literature review would provide additional source of data for locating pattern for the emerging ideas and concepts from the data analysis. During the final stages of the research, further literature was reviewed to place the emerged theory in the existing theoretical framework of the area of study and extending the current framework as advocated by Teppo (2015).

Theoretical sampling in grounded theory was used as the study progressed when the core concepts of the

theory emerged through a range of sampling strategies such as surveys, observations of teaching and dialogues and focus groups (Oktay, 2012; Teppo, 2015). Theoretical saturation was reached when the theory and data fitted together (Oktay, 2012). At this stage, the theory emerged and was consolidated.

This research design was divided into three phases; and in each phase, inductive logic was used to hypothesise the theory based on literature review and/or data analysis; then applying deductive logic based on the data collected and analysed to generate and/or refine the theory. After that, the research design was reviewed and refined to further test and refine the theory.

### 3. Data Sources

This longitudinal study undertaken at a Kindergarten to Year 12 co-educational school on the Gold Coast, Australia. The school has been founded 35 years ago and it has four distinct phases of learning, namely early years, junior years, middle years and senior years. The research participants were teachers and students from the middle years. These teachers taught middle years' mathematics, i.e. year six to year nine.

The twelve middle years' mathematics teachers were invited to the information session about this research after the Head of College had granted approval for this research to take place at the site. The teachers were provided with the approach of the study, i.e. the researcher would utilise the timetabled year level weekly team planning sessions as collaborative inquiry sessions so that the teachers would not need to invest extra time to be involved in this research project. The types of data source and frequency of data collection at each stage of the research is summarised in table 1 below.

**Table 1.** Summary of data sources.

| Stage of research | Types of data source           | Frequency   |
|-------------------|--------------------------------|---|
| Year 1            | Collaborative inquiry sessions | Weekly over approximately 37 weeks in an academic year. |
|                   | Student focus group sessions   | In two groups at the end of the academic year.          |
| Year 2            | Collaborative inquiry sessions | Similar to year 1.                                      |
|                   | Teacher online survey          | Once in the middle of the academic year.                |
| Year 3            | Teacher individual interviews  | Twice, six months apart, in the year.                   |

There were eight teacher participants over the life of the study. However, as in most longitudinal study, teachers' relocation and changing schools, resulted in the reduction of original number down to four in the final year.

The collaborative inquiry sessions were recorded and analysed using the grounded theory approach to analyse and synthesise as outlined above. Furthermore, student focus groups, teacher online survey and teacher participants' individual interviews were the other data sources to ensure internal validity and reliability of the theory emerging from the data analysis. Triangulation of data from the various sources was applied at each phase of data analysis to ensure that the emerging theory was validated at each state to eliminate bias, error and anomalies. During each phase of the data analysis and interpretation, I always maintained a heightened state of critical self-reflection and reminded myself to stick to the appropriate modes of representation to honour the ethics of research (Taylor & Wallace, 2007).



#### **4. Findings**

My first key finding is that the benefits of collaborative inquiry as a professional learning tool for teachers at a school site outweigh its challenges which can be managed through careful planning (Cox, 2010; Small, 2011; Wilkins & Shin, 2011). Consequently, schools should endeavour to implement such a professional learning approach for all the teachers. The school leadership's role of supporting the teachers' professional learning and growth is to provide a conducive and relevant learning platform and structure to make learning relevant while meeting adult learning principles (Rickley, 2008; Terhoff, 2002). A built-in approach for teachers' learning is deemed more relevant when it is structured into the teachers' work routine than is the bolt-on professional learning approach of external presenters at conferences, workshops and seminars, as these presenters do not have the same understanding of the contextual constraints and preferences (Kruse, Louis & Bryk, 1994; T. H. Nelson, et al., 2008).

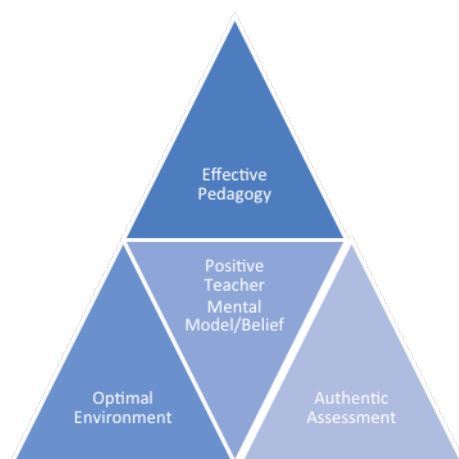
My second finding is that for collaborative inquiry to be effective at a school, the teacher professional learning must incorporate one or more of these four elements depending on the context and setting of the school within its internal and external environment. These elements are positive teacher mental model and belief, effective pedagogy, optimal environment and authentic assessment. All teachers are pre-disposed by their background to hold a certain mental model; they can either have growth or fixed mindsets (Brahier, 2005; Loughran, 2005; Strauss, 2001). Collaborative inquiry will be an effective professional learning tool for teachers if the teachers' mental models and beliefs are aligned with the school culture and values as well as its preferred pedagogical practice (Bessoondyal, 2005; Bishop, 2006; Darling-Hammond & McLaughlin, 2011; Louis, Anderson & Riedel, 2003; Provest, 2003). To create an optimal environment, the teachers need to have a tool kit of effective pedagogy (Bessoondyal, 2005; Mowlaid & Rahimi, 2010; Nickson, 2000; Stacey, 2008; Utlay, 2004). The pedagogical practice of teachers dictates how they run their classrooms and how the tone of the classrooms is set. As a team of teachers sharing the same understandings about the students at the same site with the same expectations from legislative requirements, the teachers in this study shared collaborative insights about teaching and learning challenges unique to the site. It can be concluded that it would be helpful for the teaching community to share ways of developing assessments, assessment practice and teaching approaches that would address the site-based needs of students. The teachers can work together to develop strategies to address students' weaknesses by using appropriate differentiation strategies incorporating higher order thinking elements to assess the students. Similarly, the teachers through their collaborative insights can link teaching practice to examination skills required to assist the students to achieve (Brahier, 2005; Moroco & Solomon, 1999; B. S. Nelson, 1999; Wong, 2009).

The last finding was the development of a theoretical model shown figures 1, 2 and 3. The model was developed based on the findings of the grounded theory study supported by a progressive literature review to verify my thinking and progress of theory sampling and saturation. However, the model has not been tested in context for refinement so as to apply this emerging theory in practice. It is grounded in the theory emerging from this research, and articulates that, for collaborative inquiry to be an effective professional learning tool for teachers at school, the students must be at the core of the inquiry represented by the sphere in the centre of the model. It has not fully explored the various ways collaborative inquiry could be implemented in the specific school context and the logistics involved in making this tool come to life. Hence, a model of collaborative inquiry akin to the one developed in this study can be developed and shared with school leaders to support Australian Institute of Teaching and School Leaders' advocacy of teacher professional standard.

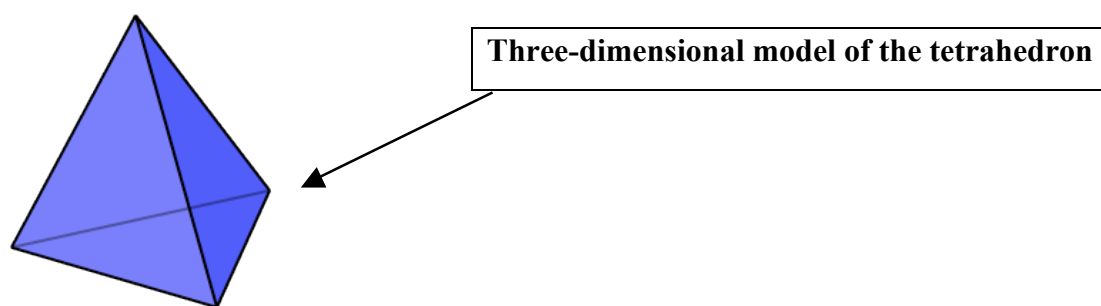
#### **5. Conclusions**

Collaborative inquiry provides the teachers with an effective professional learning approach which meets their learning needs in context and embedded in their job schedule. To teach, the teachers need to first engage the students in learning. To engage students, teachers need to create optimal environment conducive for learning which require effective pedagogy to make learning relevant, practical and useful. The effective pedagogy contains both generic and school relevant pedagogical strategies consistent with the school's mission, vision and values. Students' learning must be continually assessed authentically to ascertain their progress in the mastery of the learning. However, teachers bring with them mental models and beliefs which require alignment with the team or school's values and approaches so that consistency of practice can be attained. Collaborative inquiry is well placed to challenge these mental models and beliefs while offering the teachers regular forums to close the gaps between their perceptions and practice. All these elements about teaching and learning are captured in the model in figures 1, 2 and 3.

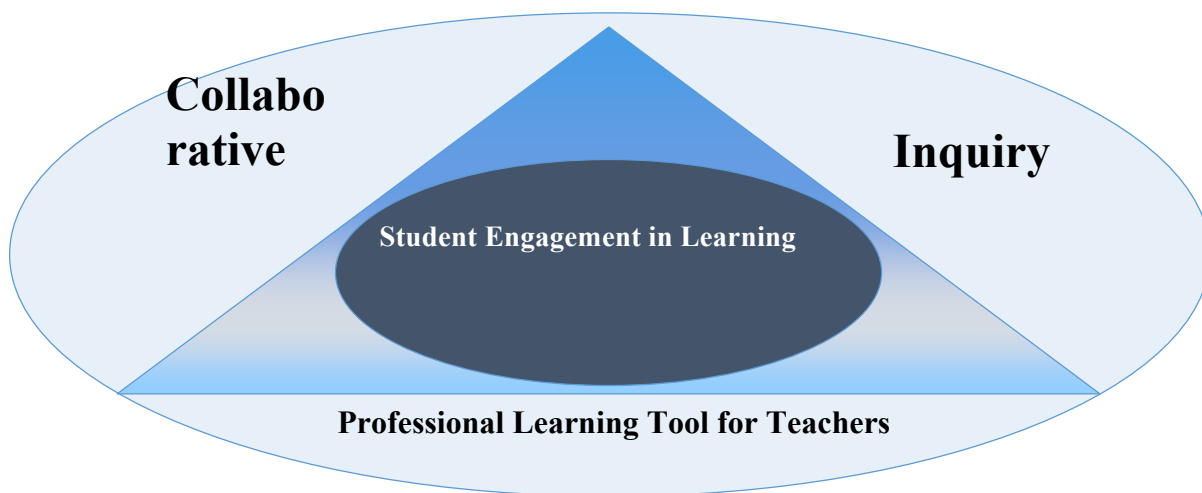




**Figure 1.** Net (two-dimensional representation of a three-dimensional model) of the tetrahedron containing inter-connecting evolutionary elements.



**Figure 2.** Three-dimensional model of the tetrahedron containing equally important inter-connecting evolutionary element



**Figure 3.** Collaborative inquiry: A professional learning tool for teachers.

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