# Innovative Challenges for Mathematics Education into the New Millennium (Some User-friendly Ideas and Quotations)

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## Introduction

"I can believe anything as long as it's incredible" Oscar Wilde

We are now on the threshold of a new millennium - what does the year 2000 hold in store for us? Nostadamus is not the only one who foresees social disorder, war and unnatural events at the turn of the millennium. Some scientists are also predicting natural disasters associated with the aligning of the planets, increased sunspot activity and the catastrophic possibility of a reversal of the poles which would bring civilised life as we know it to an end. Others are predicting a worldwide computer failure due to the millennium bug, leading to air crashes, health and governmental failures and social and economic collapse. And if all that does not frighten us, there is the increasing pollution, the *el nino* effect, the opening of the hole in the ozone layer and the increasing probability of a meteorite striking the earth!

A similar state of fear and gloom prevailed before the year 1000 AD as the whole of Christian Europe waited in expectation and dread for the Second Coming of Jesus and the end of the world. As the year 1000 AD passed, dread was replaced with a feeling of universal joy and, according to chroniclers of the time, the whole of Europe became sprinkled with white - the white churches built to thank God for allowing the world to continue! Let us hope and pray that we too will survive the year 2000. Assuming we do, what will be some of the major challenges facing the future of mathematics teaching in schools into the new millennium? I will offer four crucial challenges for mathematics education - challenges that have already been the concern of projects stretching back 20 years or more, but which will inevitably gather momentum and become more important as the new millennium arrives.

## 1. The Challenge of New Technology

> "Don't be too proud of this technological terror you've created. The ability to destroy a planet is insignificant next to the power of the Force!" Darth Vader , Star Wars.
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> "I speak of none but of the computer that is to come after me, a computer whose merest operational parameters I am not worthy to calculate … A computer which can calculate the answer to the ultimate question." The Computer Deep Thought, in The Hitchhikers Guide to the Galaxy.

### 1.1 Inwards to the School Curriculum

Technology is not necessarily good, nor is it the answer to all our problems in mathematics education. Used wisely, however, technology in all its forms can enhance and assist our teaching. It is inevitable that technology will continue to develop at a faster and faster pace as quicker, smaller and (most significantly) cheaper computers enter our homes, our schools and our lives **[1]**. What can we expect from this challenge and how will it affect mathematics education?

In 1964 I learnt Fortran in a Numerical Analysis Course as part of my Mathematics Degree. Although we wrote programs in this new language, we never actually saw a computer. Programs had to

be transferred to punched cards or punched tape and sent in for processing by a new transistor-driven computer that filled a large air-conditioned room - but still had considerably less power than a laptop computer of today! Since then I have been involved in introducing and encouraging the use of computers and computing in schools, first with the School Mathematics Project, which pioneered this work in the UK, and later with Olivetti in Italy and many schools and universities throughout the world **[2]**. During that time, we have seen a growth in the use of computers in schools that has been quite remarkable.

Three years ago our secondary school in Australia introduced a radical programme to equip all teachers and every student from Year 1 through to Year 12 with state of the art Toshiba laptop computers. Each computer had CD ROM and floppy drives as well as a network modem which allowed direct access to email and the Internet. During the last two years, every student and teacher has also had access to a wealth of educational software. This included word processing and spreadsheets, user-friendly graphing and geometry programs, databases for work in history and the humanities, Power Point and Visual Basic editing and display programs and Problem Solving and Computer Assisted Learning Packages of the highest sophistication and flexibility.

As many of us have found in the past, however, new software, no matter how well-designed and educationally motivated, does not of itself bring about good teaching and nor does it always enhance student motivation. The fact that every student had a laptop in the class sometimes proved to be an embarrassment to some teachers, and a disaster zone for others. All of the teachers had to face the challenge of how the computer could be **successfully** integrated into our teaching of the normal curriculum, and this applied to all subjects in the school, not just mathematics.

The Languages, Art and Music departments used some highly innovative and creative programs to teach their students. The History and Geography Departments made use of Microsoft Access for database work while the English department made extensive use of Microsoft Word for written work, as have all the other subject areas. Mathematics students, for example, invariably prepared their 10-15 page written CAT reports for VCE Further Mathematics using Microsoft Word and Excel. While this has had many advantages, it has also given new and dangerous opportunities for plagiarism and cheating which we have had to cope with and overcome.

The Science and Mathematics departments have also made extensive use of spreadsheet programs such as Excel, graphing programs such as Graphmatica, and Geometry programs such as Geometer's Sketchpad. Although all of these programs are well designed and reasonably user-friendly, our experience was that none of them could be used alone to teach mathematics. In fact careful preparation of other written material was usually essential if we were to exploit the full potential of these programs and use them effectively in the classroom on a regular basis. We also soon discovered that it was not always appropriate to use the programs to teach traditional mathematics in the traditional way. Many of the mathematics problems we used to set in the past have now become almost trivial using powerful tools such as Excel or Graphmatica.

On the positive side, however, Excel, Graphmatica and especially Geometer's Sketchpad have allowed us to dynamically manipulate (respectively) all kinds of formulas, graphs and geometrical shapes, thereby helping to reveal their deeper mathematical properties. Using this software has also enabled us to solve many problems in a more powerful and flexible way, which the black/white board and OHP were unable to help us with in the past. In order to exploit this potentiality, however, a lot of

hard work has been expended over the past three years in developing, writing and testing worksheets. This has been not only for topics within the normal mathematics curriculum, but also more challenging problems that have extended and broadened that curriculum.

We have especially been looking at the ways in which the new software and associated written materials are contributing to changing the mathematics curriculum itself, and the overall balance of topics within that curriculum. It appears that we have already begun a revolution which will soon be coming to many, if not all, schools in the developed world, as the power of computers increases and their price falls. Paradoxically, while this revolution will be a challenge to every teacher in every discipline, for our students the so-called computer revolution is now becoming just a normal part of their everyday life!

### 1.2 Outwards to the Internet

In 1997 a new type of school was founded in Australia by a friend and colleague, Lynne Kelly, which promises to be a prototype for a **new** type of education for the new millennium. As far as we know, it is still the only school of its type in the world. Yet it must be a model for a way of learning that will grow and possibly may become a component in all schools (at least in the developed world). It is not Distance Education, valuable as that is, which uses correspondence materials, TV and radio, nor is it one of an increasing number of institutions which are putting their course materials on the Web and calling themselves "schools".

The new school is the **VSG** - the Virtual School for the Gifted which, despite its title, can in principle teach students of almost any ability, as long as they have the time, the motivation and a computer terminal connected to the Internet and to email. Just like a real school, the VSG has classes, courses, staff room and student lounge, chess club, art gallery, Student, Teacher and Friends Discussion Lists and lots more - but unlike a real school, all of these are **virtual**. A "class" in the VSG often contains students from all over the world, students whom the teacher never actually sees, but who use highly interactive email communication as their contact learning medium. The curriculum is also different, concentrating on the stimulating and the unusual: an analysis of humour, existentialism, fractal geometry, mathematics in puzzle format for primary students, visual basic, mathematics in society, ancient Egyptian culture, oddities in physics, Web page authoring, artificial intelligence, renaissance art, poetry, and so on. Why not have a look at the VSG at: **http://www.vsg.edu.au** and see a school that is beginning to meet the challenge of things to come in the next millennium!

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## 2. The Challenge of Mathematics

I respect conscious guessing because it comes from the best human qualities - courage and modesty…..Certainty is not a sign of success, but of lack of imagination…..The value of a logical proof is not that it compels belief, but that it suggests doubts. Imre Lakatos **[3]**.

Mathematics Education is about teaching **mathematics**, so how will mathematics itself fare in the next millennium? Kuhn's concepts of scientific revolutions, paradigm change & normal science have illuminated the History of Science as a humanistic creative adventure **[4]**. Lakatos and later writers have revealed the History of Mathematics to be equally fallible, exciting and adventurous, contrary to a dehumanised traditional view, and harking back to the heuristic ideas and examples advanced a generation before by Polya and others. Waclaw Zawadowski believes that “Mathematics is changing in

its style, and its role in our culture. The stress on axiomatics is less pronounced. The use of pictures, often neglected in the modernistic approach in favour of symbol manipulation, is again gaining ground. We have the tools and the means to show students the graphs of elementary functions right at the beginning of elementary analysis, not at the end. In the postmodern style we are much less formal, and we are trying to give school mathematics a rich and broad context" **[5]**.

It is clear that the didactic role in this new paradigm for the next millennium is vastly different from its limited role in formalist mathematics and the associated traditional mathematics teaching of the past. For example, Stefan Turnau criticises conventional textbooks that concentrate on the: "idle learning of useless statements", believing instead that we should "abandon theory as the core of mathematics since what we want is learning to theorise" **[6]**. Turnau has replaced the traditional authoritarian teaching of formal content with a new methodology for 15-16 year olds, incorporated in a new kind of textbook, which is already in use in schools. In this the object is to learn to *do* mathematics, to *solve* problems and thus to *theorise* when necessary, not to learn and memorise abstract formal theory and then practise it on examples.

I would like to pose some mathematical problems to illustrate this imaginative, heuristic philosophy of the new millennium. The object will be not just to solve the problem, but to stimulate new questions.

1. What is the remainder when $7^{504806}$ is divided by 5? Not a problem the computer can help with - so how do we do it?
2. Find as many mathematical examples as you can of the equation $ab = a$. Note, however, that a and b need not be numbers - so what else could they be?
3. Two duellists A and B have a 50% chance of hitting their target with every shot. If A shoots first at B, then B at A and so on, what is the probability that A will win (ie shoot B)? Can you find a "proof" with a minimum of words or numbers - what else could it possibly contain?
4. Start with any two digit number, multiply the unit digit by 5 and add this to the tens digit. Keep on doing this. What happens? This is open-ended **algorithmic** mathematics, which computer spreadsheets have helped develop and solve. What are the **attractors** in this problem and why?

## 3. The Challenge of Society.

"The text and photographs about the Islamic world transmit a certain religiousness together with the message of art and mathematics (for example, on being asked "what did we talk about today" a child answered "about Mohamed")" 1990 Report on the use of MISP in Australia **[2]**.

The final transition in mathematics education from **content-centred** to **child-centred** to **society centred** mathematics is no less a paradigm shift than the earlier change from traditional teaching to problem posing and solving **[7]**. Mathematics educators are only now beginning to appreciate that mathematics teaching is a unique interaction of teacher and learner in a particular socio-cultural context - even though the Mathematics In Society Project (MISP) has been writing and testing thematic materials along these lines since 1980. What MISP showed was that mathematics can be introduced and taught **implicitly** using integrated real-life themes based on the social context of the students **[8]**. As the new millennium arrives, there are a growing number of individuals and groups working along these **humanistic** lines throughout the world, and a number of curriculum development and teaching projects that are taking into account the cultural and social-political-economic contexts of students. Particularly

inspiring work has been done by the MUED group in Germany, Mathematics in Action and CAM projects in Australia and the Spode Group and Exeter and Sheffield University initiatives in the UK **[2]**.

In 1986 the Mathematics Education Into the 21st Century Project was founded. Its overall purpose was to strongly support this humanistic vision of the future of mathematics education, and to find and disseminate practical ways in which this vision can be implemented into the new millennium. During the past twelve years, many individuals have contributed detailed analyses of the present state of the art of mathematics education worldwide, as well as predictions of futuristic trends in mathematics education in schools going into the next millennium. The first collection of these contributions was published in 1992 by UNESCO as one of their Studies in Mathematics Education, entitled "Moving Into the 21st Century" **[1]**. In order to publicise as widely as possible the many other innovative contributions a series of International Conferences will be held throughout the world leading into the next millennium beginning with our conference here in Egypt in 1999. Planning is going ahead for conferences in Australia in August 2001 and Sicily in September, 2002. The objective of these conferences will be to disseminate new and useful ideas in mathematics education already contributed to the Project as well as to serve as a forum for innovative work from all those attending.

## 4. The Challenge of Global Injustice

> "We have the hardest lesson of all to learn yet - that truth cannot be forced but must be allowed to plead for itself" Lawrence Durrell.

One of the most important pioneers of humanistic education, Paulo Freire, was involved for many years in Brazil teaching peasants to read and write, not merely as an instrumental activity but so that: "in the process of learning to read, they discover that they are creators of culture, and that all their work can be creative ... they are no longer willing to be mere objects". Freire "operates on one basic assumption: that man's ontological vocation is to be a Subject who acts upon and transforms the world" and in so doing "overcomes that which is dehumanising" believing that "every human being, no matter how "ignorant" or submerged in the "culture of silence" ... is capable of looking critically at his world in a dialogical encounter with others. In this process, the old, paternalistic teacher-student relationship is overcome" **[9]**.

This approach highlights the fact that even mathematics teaching serves a **functional** or **instrumental** role in society, a role that should therefore always be controlled and mediated by moral, ethical and hence **human** values. This point has been vividly illustrated during the past ten years in a growing public concern about the means and ends of technology, energy depletion, pollution, the environment and the social responsibility of both science and mathematics. The results of rapid industrialisation geared to *laissez-faire* capitalism and economic rationalism are now being seriously questioned throughout the world, especially in the Third World that has borne the brunt of mindless and greedy exploitation. Developing countries make up three quarters of the inhabited globe, contain two thirds of the world's population that are starving or close to starving and yet export eighty percent of their food to the rich developed nations, which consume eighty per cent of the total world's energy output **[2].** Consider the case of Brazil where three quarters of the population are underfed and where 10 million Brazilians own no land, while the richest 1% of the population own more than half the land.

Are these numbers of any concern to mathematics teachers? We should remind ourselves that all of us can suffer from the human pain of oppression, whether it is felt by women, ethnic minorities, handicapped or disadvantaged people - or whether it is caused by sexism, racism or social and economic oppression. In our schools in the so-called developed world, such pain often takes the form of alienation in working class students, immigrants and racial minorities, when we fail to relate our content and method of teaching mathematics to their background, experience and lives **[10] [11] [12] [13]**. The only viable solutions to these continuing and growing problems in the new millennium must be mediated not through replacement of one kind of oppression by another, but by a genuine love and regard for humanity, as exemplified in the work and words of Paulo Freire. These sentiments were echoed in the inspiring words of Eveleyn Tobin, a Maori colleague in New Zealand, when she said that "we must **celebrate** our differences", and they are powerfully summed up in the Maori saying:

> "What is the most important thing in the world?
> **He Tangata, He Tangata, He Tangata**, - people, people, people".

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