The development of Mathematical Problem Posing Skills for Prospective Middle School Teachers

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Introduction:

Mathematics is studied because it is a tool for understanding our world, for understanding how society operates, and for understanding and discussing science. Mathematics is also studied because of the beauty of its patterns, elegance of its ideas, and the fun one can have exploring its structures and behaviors. Mathematics teachers present the importance of mathematics in daily-life of their students and how they use mathematics to solve their own problems. Through the mathematics curriculum, the topic of problem solving in mathematics has attracted great attention because of its importance in mathematics skills and the use of its skills in solving the daily life situations. One of the most important emphases in NCTM Standards (1989) is to make problem solving a central focus of school mathematics. As such, it is a primary goal of all mathematics instruction (NCTM, 1989).

Many of the middle school mathematics teachers miss a wonderful opportunity not only to help their students learn problem-solving skills, but also to help them build confidence in handling unfamiliar situations.

Learning is affected by the opportunities students have to relate incoming information to what they already know and then restructure their existing knowledge or construct new ideas when appropriate. As the Professional Teaching Standards indicates, classroom discourse, or "the ways of representing, thinking, talking, agreeing, and disagreeing" (NCTM, 1991), is central to helping students develop their mathematical understanding and skills. This development, however, cannot be achieved without teachers asking a variety of questions and posing new problems that challenge students' thinking.

The destiny of any problem solving effort lies in the hands of the classroom teacher, in fact the success of any curricular reforms in mathematics "ultimately depends on classroom teachers"(Pejouhy, 1990).

While teacher educators generally recognize that prospective teachers require guidance in mastering the ability to confront and solve problems, what is often overlooked is the critical fact that, as teachers, they must be able to go beyond the role as problem solvers. That is, in order to promote a classroom situation where creative problem solving is the central focus; the practitioner must become skillful in discovering and correctly posing problems that need solutions.

This idea is not new, problem posing in mathematics has been researched in USA, UK, Australia, Japan and Singapore. Researchers have slowly begun to realize that developing the ability to pose mathematics problems is at least as important, educationally, as developing the ability to solve them. Kilpatrick (1987) and Silver (1993) are among many mathematics educators who have suggested that the incorporation of problem solving and problem posing situations into mathematics classrooms could have a positive impact on students' mathematical thinking. In their writings over the past twenty years, Brown and Walter have identified important aspects of problem posing in mathematics, and they have described essential components for a problem posing course (Brown and Walter, 1988). Many prospective middle school teachers lack the skills and confidence necessary to go beyond finding the solution to a given mathematical problem. According to Romberg and Carpenter (1986), a review of research on teaching and learning mathematics reveals the fact that the textbook is seen as the "authority on knowledge and the guide to learning. Ownership of mathematics rests with the textbook authors and not with the classroom teacher"(Romberg, 1986, 867).

In teacher training programs, we must de-emphasize the authority of the textbook and enhance the prospective teachers' content knowledge and problem posing skills so that they will have confidence in determining the direction for creative problem solving. Brown and Walter (1988) recommended that problem posing types of strategies be incorporated within the context of standard mathematics course her and there: "we look forward to the time that a special course on problem posing would be redundant" (B &W, 1988).

As it is known, there are two resources in mathematics problems for students: mathematics textbooks and mathematics teachers. I asked my students in mathematics teaching methods course in the college of Education to generate some good mathematics questions from a given life situation and to re-formulate another maths problem from a given textbook problem. There were bad questions and ill-formulated problems. After a discussion with students, I found that those students didn't have mathematical problem posing skills. Also because of the importance of problem posing skills for either mathematics teachers or students in middle schools, I tried to develop mathematical problem posing skills for prospective middle school teachers, so they will develop their students' skills in problem posing.

Objectives of the study:
There are two objectives for this study:
(1) To develop mathematical problem-posing skills for prospective middle school teachers.
(2) To design a discourse in mathematics problem posing that might be taught for mathematics teaching students in college of education.

Major Questions of the study:
(1) What are the mathematical problem posing skills needed for prospective middle school teachers?
(2) What is the effective strategy to develop mathematical problem posing skills for prospective middle school teachers?

Hypothesis:
1 - There are no statistically significant differences between students in Group A and students in Group B in the **post-test** of "Mathematical Problem Posing Skills" test.
2 - There are statistically significant differences between students in Group A and students in Group C in the **Post-test** of "Mathematical Problem Posing Skills "test.
3 - There are statistically significant differences between students in Group B and students in Group C in the Post-test of "mathematical problem posing skills" test for Group B.
4 - There are statistically significant differences between the mean scores of students in Group A in the Pre and post-tests for the post-test of "Mathematical Problem Posing Skills" test.
5 - There are statistically significant differences between the mean scores of students in Group B in the Pre and post-tests for the post-test of "Mathematical Problem Posing Skills" test.

Mathematical Problem Posing Skills:
Dunker described problem posing in mathematics as the generation of a new problem or the formulation of a given problem. (Dunker, 1945). In the same meaning Silver described problem posing as it is refers to both the generation of new problems and the re-formulation of given problems, posing can occur before, during or after the solution of a problem (Silver, 1993). In another thought, Stoyanova has defined mathematical problem posing as the process by which, on the basis of concrete situations, meaningful mathematical problems are formulated (Stoyanova, 1996).
For students, problem posing contains mental skills; where students may use the given conditions in the problem to reformulate the given problem.

Problem posing skills could be developed by giving students an ill-formulated or a partially formulated problem, and asking them to restart it (Silver, Kilpatrick and Shlesinger, 1990), English considers generating new questions from given mathematical tasks to be the main activity of posing problems (English, 1997).

From previous discussions, I would contend that mathematics teachers in order to develop quality-structured problem posing situations in mathematics classrooms could apply the following basics principles:

- Problem-posing situations should correspond to, and arise out of, pupils' classroom mathematics activities.
- Problem-posing situations should correspond to pupils' problem solving processes.
- Problem-posing situations can be generated from textbook problems, by modifying and reshaping the language and task characteristics.

Using problem-posing activities in teaching and learning mathematics requires the following skills, from various perspectives:

1) Use problem-solving strategies to investigate and solve the posed problems.
2) Formulate problems from every day and mathematical situations.
3) Use a proper approach for posing problems up to the mathematical situations.
4) Recognize relationships among different topics in mathematics.
5) Generalize solutions and strategies to new problem situations.
6) Pose complex problems as well as simple problems.
7) Use different subjects' applications in posing mathematical problems.
8) The ability of generating questions to improve problem posing strategies like:
- How can I finish the problem?
- Can I pose another questions?
- How many solutions can I find?

Strategies of Mathematical Problem Posing:

There are widespread demands that mathematics instruction should be reformed so that students will be enabled to learn mathematics with understanding by actively participating in tasks that incorporate important mathematics. The current reform movement in mathematics education, as espoused in NCTM's *Curriculum and evaluation standards for school mathematics* (NCTM, 1989) advocate the development of "problem-posing in mathematics" that approach to mathematics instruction is based on the view that students learn by resolving problematic situations that challenge their current conceptual understanding. "Students need practice in formulating mathematical problems for themselves. If they are always presented with well-formulated problems that contain just the information needed for a solution, how can they learn to deal with situations in which appropriate mathematical ideas and techniques are not obvious-that is, situations in real life?" (Silver, Kilpatrick and Schlesinger, 1990).

Generating new questions from given mathematical tasks is considered to be the main activity of posing problems as described by English (1997). Here are examples of generative questions:

- What are the important ideas in this problem?
- Where else have we seen ideas like these?
- Could we have used this information in a different way to solve the problem?
- Do we have enough important information to solve the problem?
- What if we were not given all this information to make a different problem?
- How might you change some of this information? What might the problem become then?

The set of empirical responses represents what prospective teachers who are inexperienced in problem posing can do in this particular problem posing activity.

In the review of literature some strategies in problem posing can be inferred. These strategies can be applied in performing this particular problem posing activity.

The strategies include how to "see" or find the problems that naturally follow Dillon, (1988) & Krutetskii, (1976) manipulating the given conditions and the goals of prior posed problems. Hashimoto, (1987): asking "what.if" and "what if not" Brown Walter, (1983): assuming new relationships of supply new story components during the looking back stage (Polya, 1945). Another strategy in posing a problem is to see relationships of information given and pose a problem that follows the relationship (Krutelskii, 1976). The way to see or find problems is similar to the " association" Strategy in the formulation of problems (Kilpatrick, 1987). This is at the discovery level of problem finding according to Dillon (1988). The problem is presented to the experimenter or other problem posers; all the posers need to do it to find it.

Another strategy is to manipulate the given conditions and the goal of previously posed problems. This is similar to the use of analogy (Kilpatrick, 1987) in generating new related problems.

In this study, two different strategies has been developed:

1. *'Posing mathematical problems from given textbook problems*: Kilpatrick (1987) shows that there are two phases in the solution process during which new problems can be created, the solver can intentionally change some or all of the problem conditions to see what new problem might result and after a problem has been solved, the solver can look back to see how the solution might be affected by various modifications in the problem.

 Based on that view, students will do the following:
1. Choose a problem from a mathematics textbook or mathematics workbook of grades one, two or three of the middle school.
2. Determine its conditions (The information given) and its unknown.
3. Change the problem conditions in two different ways:
   Uno.   Add more or new conditions to the original problem then formulate a new demand.
   Due.   Remove conditions (item or two) from the original problem then formulate a new demand.

For example.
1. Original textbook problem:
   Sum of Khalid's age and his father's age now is 35 years. If father's age is 4 times the son's age. What is Khalid's age and his father's age?
   * Conditions: Khalid's age + Father's age = 35
                 Father's age = 4 times son's age.
   * Demand : Khalid's age and father's age.
2. First posed problem, when we add extra Conditions :
   Sum of Khalid's age and his father's age is 35 years, translate this statement into a symbolic relation.
   New conditions: Khalid's age + father's age = 35.
                   Father's age = 4 times son's age
   New demand: Khalid's age and father's age after 5 years.
3) Second posed problem, when removing some conditions:
   If Khalid's age and his father's age now are 35 years, translate this statement into a symbolic relation.
   New conditions: Khalid's age + father's age = 35
   New demand: Translate word relation into symbolic relation

2. *Posing mathematical problems from semi – structured situations*:
Stoyanove (1996), described semi – structured situation as an open situation is given and it takes these formats:
- Open – ended problems (i.e. mathematical investigations);
- Problems similar to given problems;

-   Problems with similar solutions;
-   Problems related to specific theorems ;
-   Problems derived from given pictures;
-   Word problems.

This strategy was developed with students as the following:

1. A semi – structured of daily life situations was presented to all students.
2. Students were asked to complete the situations from their views to be able to pose problems from that formed situation.
3. Each student has to complete the structure of the situation then to pose some questions of that situation.

Students can create problems by omitting the questions from given situation, questions much as the following can be asked:

" How can I finish that situation " ?

"Can I pose another question?" Or " write down all problems you can pose related to this situation".

Example:

Her is a semi-structured situation:

" A man buys a bicycle for EP350 , after one year he sold it to his neighbor".

-   Complete this situation from your view to be a math situation.
-   Generate two or three questions from that situation.

Sample:

The sample consisted of sixty Mathematics College students enrolled in mathematics teaching methods courses (They were in the fourth and third grades in mathematics department at college of Education). They are divided in three groups; each group was twenty students. Group (A) and group (B) were the experimental groups, group (C) was the control group.

All students were teaching mathematics in the middle schools in " teaching practice" course, and has same experiences in teaching mathematics middle schools in there teaching program.

Treatment:

Experiment design in this study was as it is shown in table (1):

Group C	Group B	Group A	
20	20	20	N. Students
No pre-test	Pre-test	Pre-test	Pre-Test
No treatment	Given semi-structure situations	Given textbook problems	Problem-posing strategy
Researcher	Researcher	Researcher	Teacher
No time limit	4-weeks	4-weeks	Time teaching
Post-Test	Post-Test	Post-Test	Post-Test

Table (1)

Table (1) shows that there were two different experimental groups each had different problem posing strategy: Group A treated through given-textbook problems strategy, also there is a group C which was a control group and no specific treatment has been given to its students;

Experimental study was as follows:

Students of group A has been chosen from students of grade three.

Students of group B has been chosen from students of grade three.

Students of group C has been chosen from students of grade four, they are more experienced in problem solving.

All of students were either male or female and that wasn't a significant variable in the study.

A prepared pre-test in mathematics problem posing skills has been presented in the same time for each of group A and group B.

Group A has been treated using "Given textbook problems" strategy. The given problems were selected from middle school mathematics textbook. This group has four weeks training on the strategy; the researcher gives full instructions on how to pose problems from a given problem, which included presentation and discussions, for examples from routine and non-routine problems. Each student from this group has to develop ten posed problems weekly; the total posed problems by this group was 600-posed problems, among them:

10% of the posed problems were ill—formulated problems.

40% of the posed problems were usual posed problems, while

50% of the posed problems contained new ideas in their conditions or in demands, which were considered good, posed problems.

Group B has been treated using " posing problems from semi-structured situations strategy". Situations were designed either by students or by the researcher. It should be from the daily life of students. It contains some variables to construct mathematical situations. This group has four weeks training on that strategy; The researcher gives full instructions on how to complete posing problems from the formed situations, each student has to complete four posed problems during the four weeks training, total posed problems were 80.

Group C had no problem posing strategy, they just studied other problem solving in mathematics.

The pre-test in mathematical problem posing skills has been presented to groups (A), (B), and (C) after training in the same time.

Mathematical Problem Posing Skills Test:

The objective of this test is to measure the ability of students to apply the suggested two-strategies in mathematical problem posing skills. It contains two parts: First part, measure mathematical problem posing skills based of "posing problems from given textbook problems" strategy. Second part, measures mathematical problem posing skills based on "posing problems from a given semi-structured situations" strategy.

	N. of Questions	Test time	Test marks
First part	10	100 minutes	100 marks
Second part	10	100 minutes	120 marks
Total	20	200	220

Table (2)

Table (2) show that each question in the first part takes 10 marks; each new posed problem takes 5 marks:

Uno)   Complete mathematics presented situation 2 marks
Due)   Posed first problem   5 marks
Tre)   Posed second problem   5 marks

In the First part of the test, instructions for students were:

From each problem, you have to do the following:

(1) Add another suitable information (item or more), and then change the problem demand to formulate a new mathematical problem. Write this posed problem in the first box below the original problem.

(2) Remove any (item or more) from the given information of the problem, then change the problem demand to formulate a new mathematical problem. Write this posed problem in the second box below the original problem. So, for each problem you will pose two different new problems.

In the second part of the test, instructions for students were:

For each semi-structured situation, you have to do the following:

1 - Complete the situation, so it will be a mathematical situation (i.e. add mathematical events

and make it an accepted complete situation), then write it in the first box, below the non-complete situation.

2 - Pose a problem (1) from that situation using the information you added to the situation, then write this problem (1) in the second box, below the non complete situation.

3 - Pose another new problem (2) that situation using information you added to the situation, make sure that problem is different from the first posed problem (at least in demand), then write it in the third box, below the non-complete situation.

Data Analysis and Results:

Data of Pre and Post tests for Groups A, B and C has been collected and t-test has been used in comparing differences between mean of Groups.

Post-test A and Post-test B	Paired Differences				Sig.(2-tailed)
	Mean	Std. Deviation	N	t	
	3.4	36.44	20	.417	.681

Table (3)

Table (3) show that t is not significant. That means there aren't significant differences between students in group A and students in-group B in the post test of "Mathematical Problem Posing Skills". Although there were two different strategies used with students in both groups. Student's performances in post tests have improved.

Test Value=0						
Mean Difference	Sig. (2-tailed)	t	Std Deviation	Mean	N	
175.80	.00	33.42	23.52	175.80	20	PostestA
83.75	.00	18.29	20.74	83.75	20	PretestA

Table (4)

Table (4) show that t is significant and there are a differences in mean scores of "Mathematical Problem Posing Skills test" for students in group A in pre and post tests for post test. Strategy of "Posing Problems from a given mathematical textbook problems" is effective for group A in their performance of problem posing skills.

Test Value=0						
Mean Difference	Sig. (2-tailed)	t	Std Deviation	Mean	N	
172.40	.00	28.79	26.77	172.40	20	PostestB
81.50	.00	17.86	20.39	81.50	20	PretestB

Table (5)

Same results for group B, where t-test was significant and performance of group B students has improved from pre to post test. Strategy of "posing problems from a semi-structured situations" is effective strategy.

"Posing problems from a given mathematics textbook problems" strategy is considered to be effective strategy in developing problem posing skills; where t is significant from table (6).

PostestA and Postest C	Paired Differences			
	Mean	Std. Deviation	T	Sig. (2-tailed)
	85.65	28.72	13.33	.00

Table (6)

Differences between mean scores in post-test of group A and post-test of group C which doesn't had any problem posing strategy are significant.

"Posing problems from a semi-structured situations" strategy is an effective strategy in developing problem posing skills; where t is significant from table (7).

Postest B and Postest C	Paired Differences			
	Mean	Std. Deviation	T	Sig. (2-tailed)
	82.25	43.82	8.39	.00

Table (7)

Differences between mean scores in post-test of group B and post-test of group C are significant.

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