The Role of the surrounding space in the teaching of geometry

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Abstract : The results I'll talk about comes from the research lead by MH Salin and me, presented in the ICMY book "Perspectives in teaching geometry for the 21 century", Kluwers (ed). We shall make some criticism of the French curriculum, from the point of view of the pupil's ability to use their knowledge to explore real space, and from the following needs for geometry learning. Then, we shall ask what geometrical knowledge is useful to teach in primary school from the perspective of compulsory education. Lastly, we shall present some parts of a new curriculum based on these goals that we are now experimenting with.

I. Criticism of the present French geometrical curriculum of the primary school.

The French primary school is divided into three cycles, and we'll talk about the third one, which concerns pupils from 9 to 11 years old. After the primary school, compulsory education ends in four years spent in our "collèges de l'enseignement secondaire", divided into two cycles of two years.
So, at the end of it, the ministerial texts assigned that teachers make pupils able to:
- reproduce, describe and construct some common solids and some plane figures (cube, cuboid, parallelepiped, rectangle, square, triangle) ;
- identify them in a complex figure ;
- recognise the symmetry axes of a plane figure, complete a figure by axial symmetry ;
- use usual tools such as : tracing paper, squared paper, rule, set square, compass, protractor, solid patterns to construct some plane figures or some solids ;
- use some usual techniques of drawing (for example, parallels and perpendiculars with square and rule…)
- use advisedly the precise vocabulary given by the programs (with, in the supplement, the following words : face, edge, vertex ; side segment, middle, straight line, angle ; perpendicular, parallel ; the names of solids).
The actual curriculum that you can find in school books leans almost exclusively upon drawings on paper sheets, and upon the intuitive space knowledge about small objects (micro-space knowledge).
Geometric figures are taught by abstraction (by the imprint for example) of objects (shapes) or by drawings of polygons.

1 Our conclusions

This way of teaching has some advantages, but also leads to some drawbacks, for example :
a) The only lines which have meaning are closed lines, and segments as sides of polygons. A straight line which could be extended has no meaning.
b) this knowledge can't answer any real pupils' questions about their surrounding space and so it is not available for out of school drawings on paper ; it lacks meaning for pupils. If pupils learn to consider segments and to measure them, some important concepts such as extending straight lines and angles will not have any meaning for them.
c) whatever the teacher can say, most of the pupil's drawings of figures on paper are more effectively done with a ruler and a good usual view control than by a convenient use of tools (especially the set square) which geometrically guarantee the parallelism or the perpendicularly. The angles can't have any usefulness.
d) This way allows teachers to teach very quickly the common geometrical vocabulary about usual figures, but pupils and teachers don't use the concepts with the same meaning : pupils think of objects they see or they draw, while teachers have geometric figures in their minds.
e) The fundamental geometrical practice which needs to add lines to the given figures (inside or outside of them) to introduce some geometrical relations is not prepared and even will

be blocked by the conception of "objects-figures". The figures are strongly linked to eyesight, not to thought…

I-2 I'll give some illustrations of these conclusions :

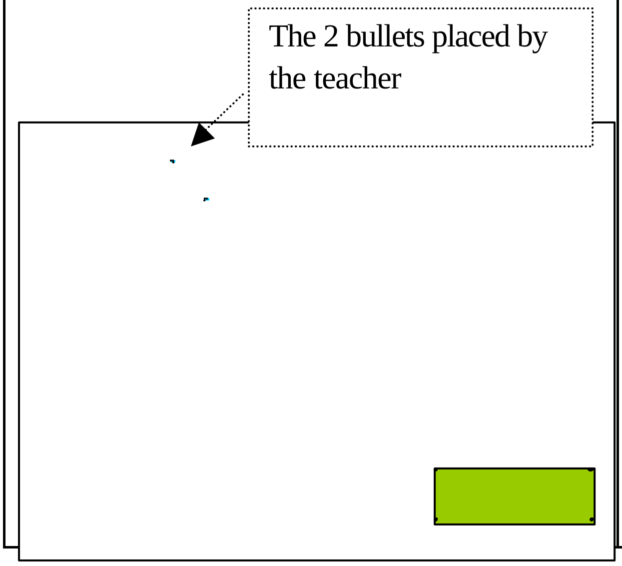
a) A very easy way to notice what is available from a paper sheet is to change the scale of the space.

Representation of the gym mat.

No representation is given to pupils. They just can touch the mat which lies upon the room's floor.

This map is rectangular, but nothing is told to the pupil about its shape.

Representation of the room where the experiment is done.



The mat lays in the position marked in green. The pupil is invited to put a little ball of « patafix » under each corner of the mat. After that, the teacher proposes the problem : The pupil will have to place the bullets at the opposite corner of the room, in order that, when the mat will be transported, the bullets would be exactly at its corners. The teacher has already placed two bullets. He has just to verify the two places chosen by the teacher will be convenient , and to place the two others. The usual tools of geometry (rule, square, compass, chalk, and measuring tape) are shown to him: to succeed, he can use them if he needs, and do anything but move the mat before the four bullets are placed.

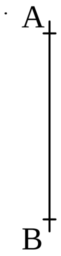
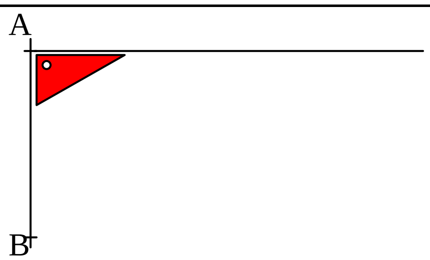
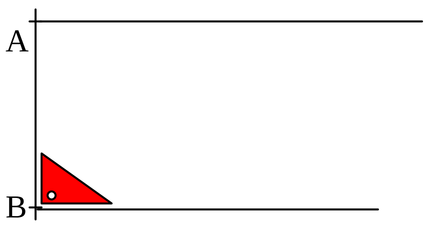
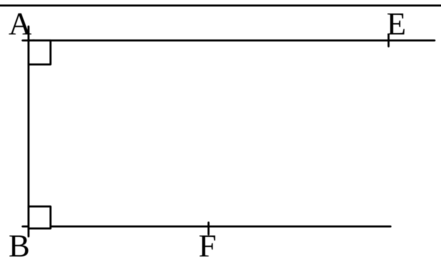
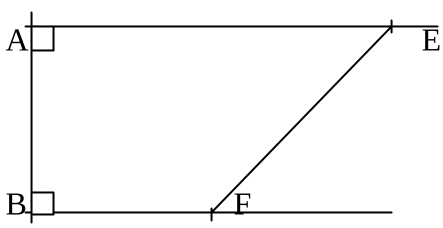
Results: we did the experience with a small sample (35) of 9-10 year old pupils, individually. Even after a trial and a verification by putting the mat on the foreseen places, most of the pupils didn't imagine they have to control the position of the two coins by using square and right angles. They only used lengths. Some of them used the square upon the mat, superposed it to the coin, but they could not use it to place the bullets.

We did a similar experience with a hundred pupils, and a school bench: the pupils had to place the position of the legs on the floor, before it will be transported. The pupils worked in teams of two.

We found that after one trial, only 50% of the pupils were able to explain how to succeed in this task. They said that they used the rectangular angles to control the orientation of the measuring lines they used, so as to foresee the position of the four legs of the bench. The other pupils only thought about lengths (and parallelism).

Have you ever asked pupils to draw a long straight line with a piece of chalk, from a point to another, with a one metre stick?

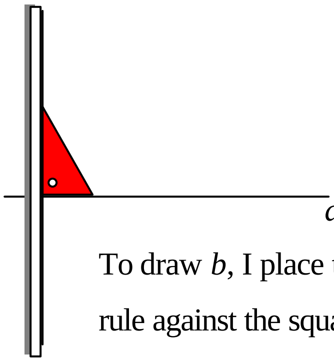
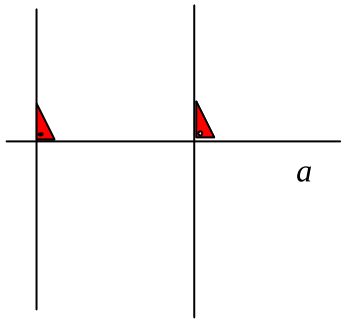
b) About the teacher's trials to drive the learning into geometry
Let us examine a sample extract from a very used and ambitious book on geometry for 10-11 years old pupils.
Extract from : Cycle des approfondissements CM1, page 62 ; Collection Diagonales ; Math en flèche ; Ed NATHAN 1993

Following the film scenario below, draw the construction on a sheet.	
	On a (straight) line, put two points A and B so that AB = 3 cm.
	On A, place the square in order to draw a perpendicular (straight) line to AB
	On B, place the square in order to draw a perpendicular (straight) line to AB.
	Mark the point E in order that AE = 6 cm. Mark the point F in order that BF= 3 cm.
	Draw the segment [AF].

Extract from : Cycle des approfondissements CM1, page 63 ; Collection Diagonales, Math en flèche ; Ed NATHAN 1993

I LEARN :

How to draw parallel (straight) lines with rule and square

1. With the rule,	2. With rule and square, I draw a (straight) line *b* *perpendicular* to *a*	3. With the square and the rule, I draw a second (straight) line *c* perpendicular to *a* :
I draw a (straight) line *a* 	 To draw *b*, I place the rule against the square, and I get the square out	*b* and *c* are *parallels* 

The teacher tries to help pupils to think of straight lines, and other elementary concepts…
You can see two ways to try to make geometry enter the curriculum :

- the naming (capitals letters for points, and small ones for infinite straight lines…), but you can imagine what it can mean for pupils. But there is no warning about that towards the teacher in the teacher's book…
- the guided action. But if every teacher of mathematics immediately sees the goal of the authors, no goal is given to these actions to pupils; they have to do it and then they will have to verify or (in more and more books) tell what they notice !

Teachers hope to make pupils learn by empirical experience the geometrical theorem :
"Two straight lines which are perpendicular to the same straight line are themselves parallel", and to make it important, the pupil would learn to control parallelism this way!
But the parallelism between lines is spontaneously known a long time before the perpendicularity, and so a lot of pupils cannot easily follow the teacher:
they can draw the lines, with no worse results and often better ones, much easier using only a rule, and their spontaneous knowledge of parallelism by view control, without a set square.
So, something obvious to them should become very complicated to do without any reason.
This remark is a general one upon common geometrical exercises.
Geometry becomes a time where pupils learn to do in a complicated manner obvious things, learn to tell in a complicated language what seems obvious (the properties of the rectangle for example….). The knowledge works against the doing. To know helps only to show that you are an obedient pupil, or to enter an esoteric society.

Conclusion of part one

Our question is now: Does the time spent on these goals allow pupils to get fundamental empirical experience about the main geometrical knowledge and its technical use?

II- What is the geometrical knowledge useful to develop by empirical experiments in the primary school ?

I'll quote some of them :
* the importance of similarity which allows us to find the measurements of the solution(s) of every geometrical problem by using scale drawings on paper
* the basic geometrical concepts : straight lines and their main relations (parallelism and perpendicularity), planes and their main relations, angles
* the triangles, and their constructions : this is the only polygon the construction of which is determined by the length of its sides (and the order of them).
* the usual polygon properties as summaries of some geometrical links, theorems.

From another point of view, Psychology informs us that babies develop very early a lot of space knowledge linked to trajectories of objects and from their moving in the around space. This space is a daily occasion to meet planes, lines and angles, linked with actions.
The curriculum of the primary school doesn't call on the spontaneous knowledge about interactions with space (meso-space) and doesn't develop them.
It should be quite difficult for teachers to work from this point of view at school, through the present curriculum, for several reasons:

- It is very difficult to make pupils work outside the class,
- It is difficult to find interesting and easy to solve geometrical problems upon polygons or solids of this scale,
- The French geometrical curriculum in secondary schools, in the university, and even in the teacher's training neglects all knowledge which can be of some practical use…we are from a very Platonic mathematical culture.

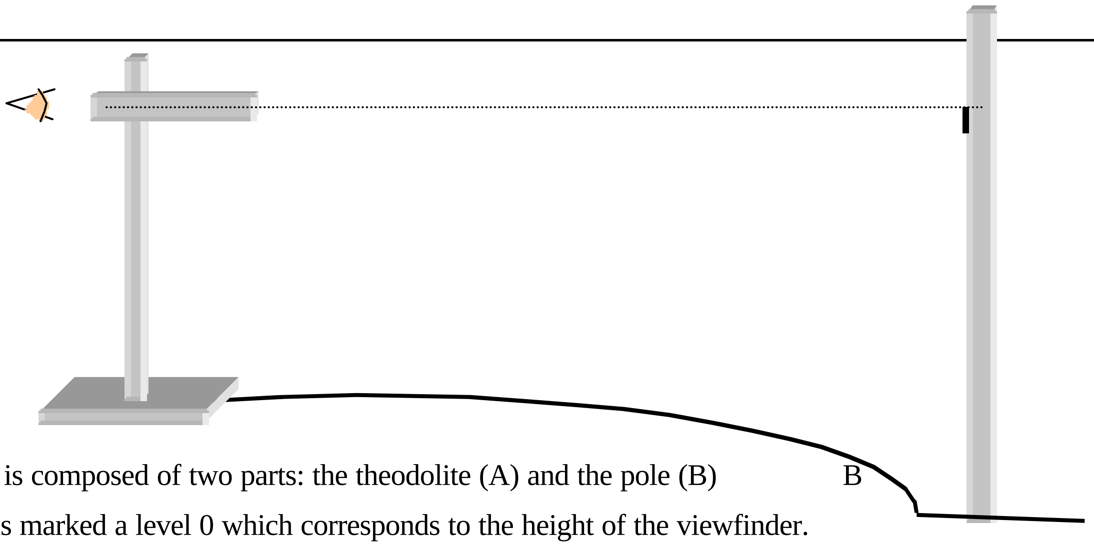
III- Towards a new curriculum

We are nowadays experimenting a new curriculum which allows the teacher to make 9-11 years old pupils familiar with the fundamental notions of geometry as straight lines, angles, planes, points as intersections of lines, parallelism and perpendicularity.
The way chosen relies on the surrounding space (meso-espace) and the micro-space of the paper sheet ; this last space constitutes a mean of representation as a strong meaning of this term, allowing simulation and exploration.
We lean upon a very old idea of geometry, which is said to be linked with very ancient Egypt: the measurement of spaces. We especially explore measurements of lengths and angles, by introducing already made instruments: the theodolite for difference of vertical levels, a sort of shade-sextant to measure the angular height of sun and its azimuth, and two kinds of indirect measure of distance, one of them based on Thales and the other on the congruence of triangles.
The choice of these instruments has been done in order that

- they can be used by pupils older than 9, with their spatial spontaneous knowledge, which can be easily improved,
- the geometrical knowledge that they suppose can be taught and constitutes a good base for geometry later taught,
- pupils can solve problems which are interesting for them,
- the use of paper is strongly linked with spatial anticipation and exploring geometrical rules.

<u>Présentation of the instruments and of the new goals :</u>

Thedolite



The instrument is composed of two parts: the theodolite (A) and the pole (B)

On the pole B is marked a level 0 which corresponds to the height of the viewfinder.

It needs two persons to be used. The pole is vertically laid upon the ground where you want to evaluate the difference of level with the one of the theodolite.

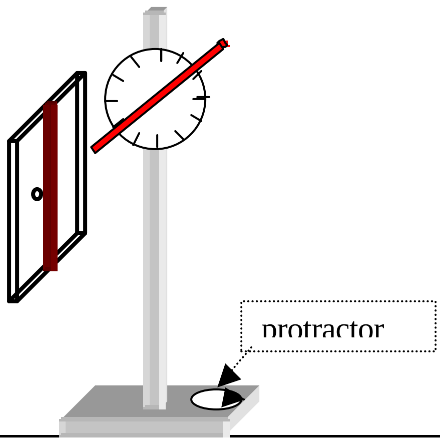
1) regulation of the zero of the pole : the theodolite and the pole are placed on an horizontal ground.

In the point A, the operator places the view finder horizontally (and fix it for the whole manipulation), and directs it towards the pole. The point of the pole which is on the view line is the zero. It will be the reference point from which you will locate the difference of level.

2) measurement of the difference of level : the pole is placed where you want to estimate the level difference. The theodolite operator aims the pole with the horizontal viewfinder and directs the pole operator who slides up or down a marker on the pole until the marker is on the view line.

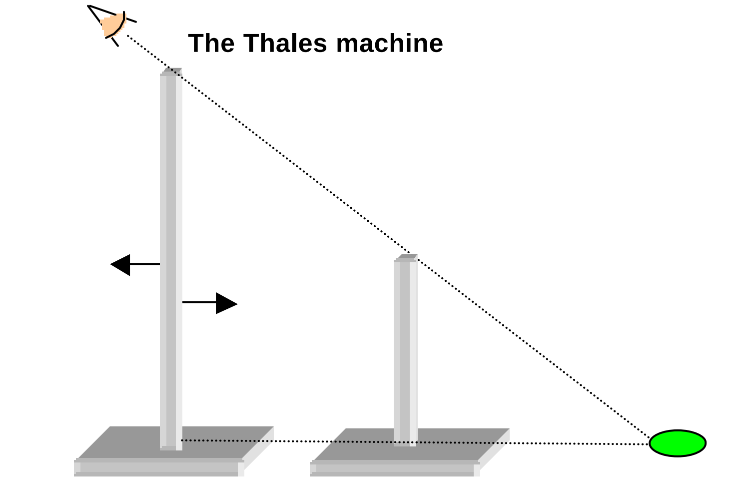
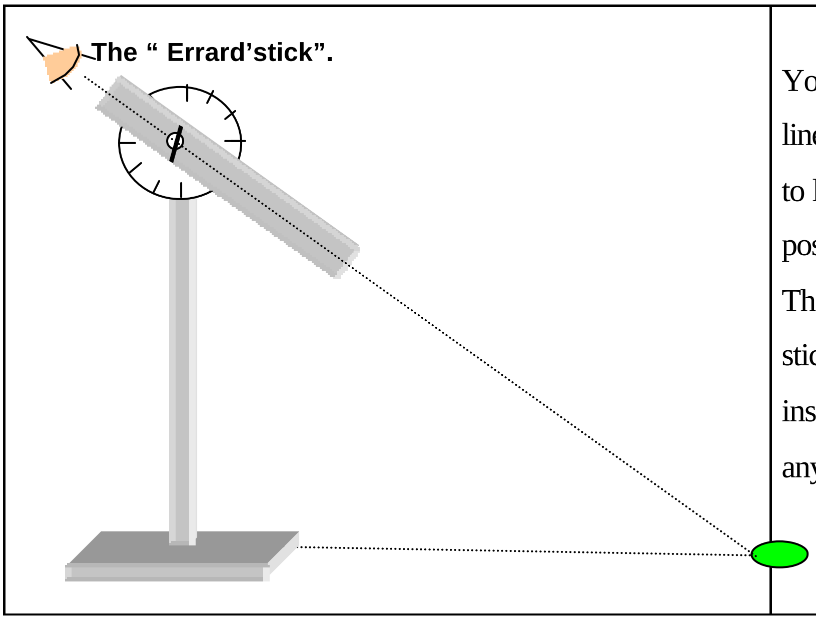
The level difference between the two points is the distance "h" between the zero 0 and the marker: between the two points, the pole has been got up or down from this length.

The shadow sextant



Measurement of the height angle of the sun : 1) You rotate the instrument until the shadow of the tube is parallel to stick one. You get the azimuth on the compass.

2) you minimise the shadow of a tube by rotating it around an horizontal axis, and you get the height on the protractor.

	The short stick height is half the long one. You move the long one until the two sticks are aligned with the point you want to locate. The distance between this point and the short stick is the same as the distance between the two sticks.
	You make the stick turn until the view line (alidade) is aligned with the point to locate, then you fix the stick in that position. The distance between the foot of the stick and this point is linked to the instrument and can be measured anywhere you move it.

The principle of the teaching can be described as follows:

- introduction of a situation-problem,
- introduction by the teacher of a technical way of solving: the instrument
- learning of the instrument use,
- introduction of the geometrical problem: how does it work, what characteristics of the instrument can be /can’t be changed if you don’t want the result changed.
- experimentation of the links between the main characteristics of the instrument by exploring them on drawings on paper sheet. Links between the drawings and the instrument: modelling.
- teaching of the new geometrical knowledge associated with this part of the curriculum.

Instrument	Situation-problem	Geometrical knowledge
Theodolite	Measurements of the differences of ground level	Straight lines, horizontal and vertical one's. Alignment The rectangular which moves a length from a side to another.
The "shadow sextant"	To be able to locate the sun in the sky. Firstly measurements during some days, using these measurements to anticipate the position of the sun.	Angle between the ray lines and the horizontal, Plane, Function
The Gerbert's machine	To be able to find the distant between the puck and the little stick, without penetrating the zone between them…	Thale's property : the ratio between the stick lengths on the one hand, and the horizontal lengths on the other hand. The triangles
The Errard's stick	To be able to find the distance between the puck and the little stick, without penetrating the zone between them…	The triangle The triangle congruence

REFERENCES :

- R. Berthelot and MH Salin, The role of pupil's spatial knowledge in the elementary teaching of geometry, in "Perspectives on the teaching geometry for the 21 century", an ICMI study, Carmelo Mammana and Vicino Villani, 1998, Kluwers ed.
- About the instruments :

  Gerbert : Opera mathematica (10-11^{e} s)

  Errard : Géométrie (1619)

  Fourey Emile, Curiosités géométriques, Vuibert (1907, 1994)