HOW TO INNOVATE MATHEMATICS TEACHING TAKING SOCIAL AND CULTURAL CHANGES INTO ACCOUNT

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Abstract
In this paper we propose some classroom experiences worked out to connect children's out-of-school experience with classroom activities, through the use of suitable cultural artifacts. The double nature of these artifacts, that of belonging to the world of everyday life and to the world of symbols, makes it possible. So we can promote the movement from the real world situations to concepts, and back, from the mathematical concepts to the real world situations. But a different use of the artifacts presented the opportunity to also do mathematization from concepts to concepts. This manifested itself when symbols, embedded mathematical facts, have been turned into objects which are to be compared and manipulated, after which it is possible to reflect by pointing out the respective properties and by making conjectures. The artifacts can become then 'mathematizing tools', and bridge the out-of-school experience with school activities leading towards new mathematical goals.

1. Introduction

In this paper we propose some classroom experiences worked out to connect children's out-of-school experience with classroom activities, through the use of suitable cultural artifacts.[1] We deem that those conditions that often make extra-school learning more effective can and must be re-created, at least partially, in classroom activities. We find a significant convergence with the ethnomathematical perspective about this point. D'Ambrosio, 1995, maintains that "*cognitive power, learning capabilities and attitudes towards learning are enhanced by keeping the learning ambiance related to cultural background ... It is well documented the fact of children and adults performing "mathematically" well in their out-of-school environment, counting, measuring, solving problems and drawing conclusions using the arts or techniques [tics] of explaining, understanding, coping with their environment [mathema] that they have learned in their cultural setting [ethno]*". Related researches are for example Nunes, Schliemann, & Carraher 1993; Saxe, 1991, on street vendors' mathematics; Saxe et al., 1996.

Classroom experimental studies we have carried out during these years, of which you will find some examples in this paper, use opportune cultural artifacts. Cultural artifacts embody theories the users accept, even when they are unaware of them. Their use mediates the intellectual activities, and - at the same time - enables and constrains human thinking. Through these subtle processes social history is brought into any individual act of cognition (Cole, 1985).[2]

In our perspective, learning can be intended as a process of mathematical knowledge construction, by engaging the children in classroom activities that require an extensive use of culturally significant artifacts in order to encourage them to develop a positive attitude towards school mathematics, and some modifications are introduced in order to enable the children to extend, reflect, generalize, apply their knowledge.

By engaging children in meaningful classroom activities, mathematics turns out to be a fascinating human activity, less harsh and torn off from reality than usually believed, given the traditional practice in school mathematics.

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[1] In previous researches, we analyzed bugs and difficulties in the mastery of the meaning of decimal number, of the relationship between fractions and the decimal representations, in ordering decimal numbers. Overall, the results highlighted a clear gap between the mathematically rich situations that children experience out-of-school and the classroom practice.

[2] Also the multimedia computers are relevant cultural artifacts, which role in education perhaps has not been fully explored.

2. Connections between mathematics and reality

We stress that bringing real world situations into school mathematics is a necessary, although not sufficient, condition to foster "*a positive attitude towards mathematics, intended both as an effective device to know and critically interpret reality, and as a fascinating thinking activity*", as is specified in the Italian Programs for the primary school. We contend that this educational objective can be completely fulfilled only if we can get the students to bring mathematics into reality.

In other words besides

mathematizing everyday experience

it is necessary to have

everyday mathematics.

This is possible because there is a great deal of mathematics embedded in every day life. Effective learning situations in the classroom can be carried out by encouraging the children to analyze some '*mathematical facts*' that are embedded in some cultural artifacts. In this way mathematical knowledge can be inserted into a common knowledge, pupils can be stimulated and motivated and led towards a more conscious learning. The cultural artifacts that have been the objects of some of our experiences have made it possible to connect classroom mathematical activity with real life according to different modalities.[3] If they are used in a certain way they can favour the process which enables pupils to go from reference situations to concepts but also the reverse process, from concepts to reference situations, according to the '*horizontal mathematization*' approach proposed by the Freudenthal Institute Research Group. The double nature of these artifacts, that of belonging to the world of everyday life and to the world of symbols, makes it possible. An essential property of artifacts, which supports their bilateral influence and offers common bases to culture and discourse, is their being ideal (conceptual) and material. "*They are ideal in that they contain in coded form the interactions of which they were previously a part and which they mediate in the present. They are material in that they exist only insofar as they are embodied in material artifacts*", Cole [1995]. But a different use of the artifacts supported the opportunity to do also '*vertical mathematization*', from concepts to concepts. This manifested itself when symbols, embedded mathematical facts, have been turned into objects which are to be compared, modified or manipulated, upon which it is possible to reflect by pointing out the respective properties, by making conjectures.

Why do we believe that the cultural artifacts are so helpful for students? We believe that the cultural artifacts we introduced are meaningful because they are part of the children's and adults' real life experience, offering significant references to concrete situations, or at least more concrete ones. This enables children to keep their reasoning processes meaningful, to monitor their inferences. As a consequence, they can off-load their cognitive space and free cognitive resources to develop more knowledge (Arcavi, 1994). Furthermore these tools stir up curiosity, attention, a positive attitude, even an emotional involvement towards mathematics. They bring forth the construction of new procedures, strategies, computational algorithms, whether they are standard or not, that sometimes are close to the learning processes emerging in the out-of-school mathematics practice.[4]

Furthermore children are asked to select other cultural artifacts in their everyday reality, to point out the embedded mathematical facts, to look for analogies and differences (for example, different number representations), and to generate problems (for example, discovering relationships between quantities). In contrast with the traditional classroom curriculum, children are offered endless opportunities to become acquainted with mathematics. This is a motivating starting point for the

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[3] We worked out another experience to introduce the decimal numbers through the use of a common tool such as the ruler, see Basso, Bonotto, Sorzio [1998].

[4] The cultural artifacts can be utilized also to build up interdisciplinary classroom activities.

development of the ability of reality mathematization, and for a change of attitude towards mathematics.

3 Some didactical proposals

Let's now analyze some of our experiences accompanied by the artifacts used and part of the given tasks. They have been carried out in the school year 1998/1999 in a fourth class of a primary institute (Trebaseleghe, PD), by the teacher Milena Basso, as on-site person in charge of updating, in the presence of the official teacher of the logic-mathematical area. This was repeated in a fourth class of a primary school teaching institute (Asiago, VI), used as a control class by the student teacher, Chiara Frigo, who in the course of her mathematics degree had completed her thesis on this topic. These experiences have led to situations that are typical examples of both horizontal and vertical mathematization; they have also favoured the arising of those special learning processes that Freudenthal, 1991, defines '*prospective learning*' or '*anticipatory learning*'.[5] But these experiences have also favoured the type of learning "*retrospective*", that occurs when old notions are recalled in order to be considered at a higher level and within a broader context, a process typical of adult mathematicians. Freudenthal, *op cit*, states that "*prospective learning should not only be allowed but also stimulated, just as retrospective learning should not only be organized by teaching but also activated as a learning habit*".
The objective in relation to the content of the proposals that will be illustrated in this paper was the bringing together of the formalisation of the algorithm for multiplication of decimal numbers. To produce this procedures of estimation and approximation were also introduced, because we maintain that these would allow children to think freely with an "*open mind*", without the need to concentrate too much really and truly on the computation, and moreover allowing a checking process of the order of magnitude of the same results, even before calculating them. Particular types of receipts were used moreover because they permitted us to work with the weight unit of measurement that, given its "concreteness", leads us to think it is easier to comprehend.
Even if these objectives are established only in a small or medium way they always however contribute to attain the final scope. Amongst these, we stress:

- the interpretation and correct use of mathematical language and its related symbolism,
- prediction of the required results,
- comparison between the different results obtained,
- comparison of the structure of decimal numbers with that of natural numbers.

With regard to the research method, the fact is emphasised that before beginning the experimentation certain behavioral norms are given for distinguishing the participation of the students and of the teachers involved. We are in agreement with Greer [1997] when he asserts that it is appropriate to invoke an '*experimental contract*' by analogy with the concept of '*didactical contract*'.
With regard to the role of the students these norms may be summarised by the fact that they are expected to:

- explain and justify their arguments,
- listen and do their best to comprehend the explanations of their companions,
- indicate points and concepts not fully understood, asking for clarification and deeper explanations,
- indicate and explain the corrections made when they realise their mistakes.

With regard to the role of the teacher they must

- encourage the initiatives of the students,
- comment on and summarise the students' contributions, emphasising the arguments offered

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5 We think that this type of learning is better enhanced by a "*rich*" rather than a "*poor*" context, and the cultural artifacts and the activities we introduced in the classroom can be considered '*rich contexts*' or '*rich materials*' in Freudenthal's sense.

- making clear eventual contradictions.
Moreover they must emphasise other possible strategies for solving the same problem that crops up, and get students into the habit of making comparisons between these strategies.
I want to now show some snapshots of how things work out in some class lessons, showing only some paradigmatic protocols of various types of argument and of various procedures used by the children.

Lesson 1
The following receipt is presented:

SPECIAL BREAD

kg	L./kg	L.
0,478	4000	1910

T=0,004kg

In this lesson the students were asked to read the receipt, to interpret the symbols and numerical expressions that they are to compare, to explain them and finally to try to establish what operation the machine has made to produce the amount to be paid.

Lesson 2
The following receipt is presented:

BRESAOLA BEEF

kg	L./kg	L.
0,196	41800	......

The instructions with regard to this receipt are as follows:
1) *Will mother spend more or less than 41,888 lire? Explain your answer.*
2) *Without doing the calculation in columns, are you able to find a quick way, without doing exact calculations, to find the missing price?*

Lesson 3
The following receipt is presented:

BONED PARMA HAM

kg	L./kg	L.
0,210	38900	......

The instructions in this case are as follows:
1) *In your opinion, is the missing cost bigger or smaller than 38,900?*
2) *Without doing exact calculations do you know how to find the approximate value of the missing cost?*
3) Now try to calculate the exact cost.

The following protocol of Caterina is significant:

"38900	*0,210kg=210g*		
↓:10			
3890	*38900×*		*38,9×*
↓:10	*0,210=*		*210=*
389	*00000*	→	*000*
↓:10	*38900*		*389*
38,9 → cost of 1 g	*77800*		*778*

00000 *8169,0 → 8170*
8169,000

"I removed three zeros because I divided the number 38900 a good three times to obtain grams." [6]

Caterina first found the prices corresponding to the various units of measurement, then she carried out the operation in two different ways and indicated their connection with an arrow. In the event the two operations support each other in the sense that each confirms the result of the other. The child gave a further explanation of how she managed to place the decimal point using the relationship between kilograms and grams. She managed to apply the rules she had learnt also in situations that she had never seen before; taking up the new media without doubt and using them easily to find the results. Her protocol is also a beautiful example of learning of the *retrospective* type. Caterina passes from horizontal mathematization, limited to the context of the receipt, to vertical mathematization, which is something other than pure and simple manipulation. It is also to be noted that she knows that the result obtained is a valid mathematical result, but that in the real world, in the receipt, it is more likely that 8170 will be written as the exact cost; one grows used to the fact that in the two contexts [reality and mathematics] different numbers are used. This fact was noted and confirmed also by other children.
Almost half of the children in the class of Asiago replied to the second question in the following way: "*Mother spends L.8169: if 100g costs 3890 lire, 200 will cost 7780; I must add the cost of 10 grams that cost 389. Mother spends 8169 lire*". This children used a strategy that is not part of an approximation procedure, but is based on the sum of various prices associated with parts of the weight purchased, applying that is the distributive property of multiplication over addition. This method of proceeding allowed the children to tackle multiplication between decimal numbers and to guess the algorithm; in this case we have learning of a *prospective* type.

Lesson 4
Finally there is presented the following receipt:

kg	L./kg	L.
TREVISO TARDIVO CHICORY		
1,200	7980	......
PISTACCHIO		
0,730	14500	......
SICILIAN ORANGES		
2,240	1980	......
TOTAL		......

The instructions with regard to this receipt are as follows:
"Will 20,000 lire be enough to buy these vegetables and fruit? Write how you would work out the answer without calculating in columns".

The strategies used by the children to fulfil these instructions are different in type and in procedural complexity; some are characterised by a more or less sophisticated use of estimation and approximation. This indicated the great potential inherent in the children, often not able to be anticipated and schematized beforehand.
Let's have a look at the protocol of Daniele, the only one that represents the best synthesis of work up to now dealt with.

[6] Also Alex and Daniele replied "*I have removed three zeros because between the kilograms and grams you must take three zeros, that is the grams are smaller by three times with respect to kilograms.*"

"I took 1kg of chicory because therefore we take into consideration 7980 lire which is the cost of 1kg, 1hg costs 798 lire and I have 2 therefore I must do 798× 2=1596+7980=9576 lire, which is the cost of the chicory.
I took 7,30hg of pistacchio and changed it to 7,50hg, the cost of 1kg is 14500 lire, but I didn't take 1kg but changed it to L.1450, the cost per 100g, then it is necessary to find the cost per hectogram of half a hectogram which is 1450:2=725 +1450× 7=10150 =10875 the cost of pistacchio.
I have 2kg of oranges so must do 1980× 2=3960 lire the cost of 2kg then I took 2hg, 1hg costs 198× 2=396 lire costs of 2hg then I have 40g which I change to 1/2 hg that costs 99 lire, then I must do 3960+396+99=4455.
9576+10875+4455=24906.
No, 20000 lire is not enough".

Note how the child easily handles approximation, even if only with the numbers indicating weight. In contrast with her companions who approximated with the 100g, Daniele makes a finer choice and decides that a good approximation is that of half a 100g. One also notes that the child prefers to write 7,30hg in preference to 0,730kg, and she will not be the only one to make this type of approximation. Children prefer to deal with natural numbers than with decimal numbers, and if they really have to work with the latter, they avoid the notation 0. etc.

4. Reading across the data

During all of the work we were presented with results that went behind the original objectives that were proposed. This was because the type of teaching chosen was not an end in itself, nor static, but very flexible, open to more possibilities, because very often it was determined by the students themselves, by means of their intuition, expectations and curiosity. Because of limitation of space we can only dwell on one aspect that has characterized our experience, that that concerns the use of estimation and approximation, even if other aspects (those of the role of language, written and oral, the reading and interpretation of data) which merit our attention.
The use of estimation seems to have been well covered by all the children. The request to compare two numbers, not knowing the exact value of one of them, brought some thoughts and arguments that were not obvious. In this case the presence of a connection with reality has been a great help, even more it was the extra gear that made the question more easily solvable. It was then the anticipated request to "estimate" that which had allowed the process to start that removed the misunderstanding that multiplication always produces a larger result. The use of the procedure of approximation allowed the children to argue more freely, to think and concentrate on the mathematical concepts rather than on the pure and simple computation that might distract from the final objectives. The surprising thing is that the actual possibility to be free to "choose" had baffled and alarmed the majority of the students, who were used to working with well defined schemes and to expect and find a unique solution. At first the fact that one can do the operations also when the numbers were not exact had moreover aroused much perplexity in the children, and also that it was possible to find approximations for whole numbers and not just decimals. The first resistance encountered was that of "*the numbers can't be changed*". The children were in fact strongly anchored in the structure of the natural numbers, for which they know the operations and the relevant algorithms, for which they asked themselves "why" they should approximate the calculations when they know how to do it in an exact way.
The great contribution made by the process of approximation, however, was that of helping the students to first guess and then later formalise the "*rule of the decimal point*", to realise that is the algorithm for multiplying out decimal numbers. The comparison between the approximate answers and those obtained by means of the chosen algorithm decisively helped the children to put the

decimal point in the correct place. The use of the method of estimation and approximation allowed the pupils to predict the results and above all lead to a discussion of the proper operation whenever the result obtained from the procedure used was not compatible with the predictions previously held. Thanks to this new way of proceeding the children have had the possibility to choose the strategies most appropriate to the particular possibilities. This was revealed to be particularly useful for the teacher who, analysing the ways of approximating and the choices used, was able to evaluate the level of learning and the capacity to mathematize attained by each pupil. Finally, the use of the method of estimation and approximation brought the pupils to make comparisons with their previous knowledge, that they still had, hence making the problems associated with their levels of understanding stand out [see the case of "equivalencies"]. In the course of the whole-class discussion it was revealed how some knowledge could be only formal, lacking in any significance, connected only to a mechanical process, not aware, of any algorithm; thus it is possible to recall some of their aspects in filling up the emptiness "not seen". The method used in these experiences allows us to go back and forth along the paths of knowledge, making use of different instruments, belonging to distinct sectors. Thus it is possible to systematise and consolidate concepts, making clear what is not understood and filling up the eventual gaps. Going in this direction old knowledge is revisited that becomes attached to the new, working at the same time in the past and in the future, thus *retrospective learning* is favoured, that makes it more difficult to forget what has been learnt.

5. Once more the relationship between reality and mathematics

A significant example of the relationships between reality and mathematics is provided by the children of the class of Asiago. Often in their reflections, either in the individual protocols or in the collective discussions, the children have made reference to tenths, hundredths and thousandths, in reading and translating a decimal number, also when it referred to a measure of weight, that is kilograms, hectograms and so on. When this group of lessons was planned, a unit of weight was chosen to be used because we wished to retain a more concrete idea, which was easier to understand and to use than a "pure" decimal notation. The initial preference of this class for a more "abstract" decimal notation seemed to contradict our expectation of the fact that "reality" should be able to help us understand better mathematical concepts. We have then verified that the initial preference for decimal nomenclature was due to the fact that Asiago is a mountainous resort and the pupils of that class go skiing and are used to competing; they therefore are versed in a "concrete" image of decimal writing, determined from the temporal scansion. This can only confirm that the concept of ‘concrete’ is relative; different levels of concrete can be set for different people and, for the same person, according to their age, educational level, and intellectual, working experiences.

It is a task for the teacher to know, at the end of being able to profitably take account of the teaching, the life experienced by the pupil. On the other hand reality and mathematics are certainly not the same thing. Mainly in the higher degrees of education, the teacher has to try to overcome the limits of the simplification of mathematics that is embedded in reality to grasp the special features of this discipline [abstraction, generalization, formalization, ...]. During our varied experiences there are occasions that allow us to emphasise this fact, compatible with the degree of learning of the pupil. More often is it noted that the results obtained from the students and those present in the receipts differ and that the motive for this is to bring us back to that fact that reality and mathematics are different worlds. They enter deeply into each other but they are governed by different laws and principles. A mathematical result could be a number with a decimal point even if it indicates a price, whereas this does not happen in the world that we frequent, given the actual Italian monetary system [in reality the introduction of a unique European coin, the Euro, is rapidly changing the situation]. It is therefore underlined that mathematics is dominated by precision, whereas often reality is the fruit of compromises determined by various factors, often of a practical nature. Although mathematics gives its language, its symbolism to reality, in the two contexts it expresses itself in different ways. The children are aware that when they go to the bakery they never ask for *"0,478 kilos of bread" but rather* "*half a kilo of bread"* or "*ten rolls*".

If it is true that in our experimentation the first attempts occurred inside real life and that this has provided good hints for the various types of mathematization and has allowed the children good control of inferences and the results obtained, it is just as true that it is necessary to abandon reality, once the necessary supports are acquired, to reach abstraction and be able to work more freely with numbers. In effect once the child has become secure and has absorbed the appropriate knowledge, this enables them to move to a level of greater depth [which becomes "common sense" of a superior order] and they no longer have need of reality [or rather it is better to say a "common sense" of an inferior order] to support their own elaborations. We believe that the real world will go well as a launching pad also at higher scholastic levels, but once in action one should and one must, above all with certain children, use it less. One saw for example how Caterina thought about the data in front of her and had the capacity to elaborate and manipulate numbers, created from the motivation that derives from mathematics and that henceforth has nothing more to do with the real world. It is important to note that the separation between reality and mathematics varies from child to child; whether in the time necessary to bring it about, or the mode of that separation, whether the actual level of abstraction attained, they are individual, the route is narrowly personal. This makes the task of the teacher particularly complex, but also more interesting and stimulating.

6. About the use of cultural artifacts in our proposals

As we have noticed thanks to the reported examples, the use of these artifacts can be articulated in various stages, each rich in educational potential and suitable for objectives with different contents. [7]
In a first stage, the cultural artifacts represent indeed the out-of-school reality; children can be asked to simply recognize the mathematical facts that are embedded and codified in the artifacts (often in this stage, mathematization is of the horizontal type). In a second stage they can be asked to interpret and reflect on the mathematical facts, both in themselves and in connection with real world situations (horizontal and vertical mathematizations are possible). In a third stage children can be asked to put mathematical facts in relation, make conjectures about procedures, notice properties. During the phase where cultural artifacts are deprived of some information [like in some tickets] they lose their fixed structure and no longer faithfully represent out-of-school reality [8], although they are strongly tied with real world situations. Now artifacts become more explicitly tools of mediation and integration between out-of-school and classroom knowledge, between school inner experiences and external experiences, and, if correctly used, can create new mathematical goals, thus becoming real *mathematizing tools* and enhance vertical mathematization processes.[9]
Given their properties, the cultural artifacts can be introduced and utilized in classroom as a starting point to facilitate children's construction of new mathematical knowledge, often through an '*anticipatory learning*' process. According to our experience, children exhibit also flexibility in their reasoning processes, by exploring, comparing and selecting among different strategies.
We think that activated procedures are mastered on the long term and can hence become part of the student's cultural heritage thus being re-constructed more easily in case they are

[7] It is worth saying that artifacts are as related to mathematics [as in other disciplines] as one is able to find these relationships.

[8] supermarket reality never quite lacks data; they are well clear on the ticket according to a precise positioning.

[9] As to the analysis of the triple role of the ticket, *as cultural artifact, as mental organiser and as mathematizing tool*, see Basso-Bonotto, 1996.

forgotten. In fact, when explication and reflection are done on one's own reasoning the used procedures are consciously acquired.
Through the use of appropriate cultural artifacts curricula starting from children's out-of-school experience and leading towards the understanding of the mathematical facts embedded in real world situations can be developed. Children develop more awareness about the mathematical facts embedded in the cultural artifacts, and personal understanding, which are useful in dealing with new problems in school as well as out of school.

7. Some concluding remarks

From an educational point of view, the classroom practices we implemented have increased our understanding about the mechanisms to make the connections between out-of-school and classroom activities more effective. The learning has thus come out to be a process, articulated in different phases, where mathematical knowledge is generated by the class collectively, by connecting in a stricter way the daily experience to the mathematical classroom practice. However, we do not want to suggest that our research is a prototype for all the classroom activities related to mathematics (but not just limited to it). In educational practice, innovative experiences must be joined by more traditional activities, of reinforcement, of computation, standardized exercises.
However we do believe that, given their paradigmatic value, by enacting some of these experiences in classroom, the children are offered an opportunity to change their attitudes towards school mathematics. These innovative practices involve children's intentions and attention, stir up dilemmas, and give meaning to mathematical symbols introduced. Furthermore, offer children the opportunity to tune their understandings towards the real world situations they face in their everyday experience, as Resnick, 1994, has noted.
The use of these artifacts is not easy or, in any case, is not of easy implementation for the teacher that has also to try to modify his/her attitude to mathematics and that is influenced by the way he/she has learned it. The teacher has to be ready to create and manage *open* situations, that are continuously *transforming* and of which he/she cannot foresee the final evolution or result. As a matter of fact, these situations are sensitive to the social interactions that are established, to the students' attitudes, reactions, and their ability to ask questions, to find links between school and extra-school knowledge. In this way the lesson sequence cannot be prepared in advance in all of its aspects, nor from above. It should rather plan for various "branches" to be then drawn together through a process whose management is quite hard; therefore the teacher has to be and to feel very strong and qualified both on the mathematical contents and on the educational objectives that are potentially contained in these artifacts.

Finally the teacher must not forget the moment of the *institutionalization* of knowledge being constructed together with the pupils: it has to be a shared moment so that these maybe interesting, stimulating and involving activities can be duly finalized.
Many teachers that have already experimented with our proposals have realized that through these experiences they can not only stimulate the curiosity of their pupils but also their own curiosity and turn pupils and themselves from passive to active elements. We think that both students and teachers should have the impression that the topic is being created during the teaching-learning process, that is being conceived and developed during the class. It has to be underlined that teachers that have tried our proposals, besides their ability to re-think their role, have proved they still had a remarkable intellectual and scientific curiosity. This has enabled them to focus on both new educational and new mathematical goals so as to become researchers; in other words, school has become for them too a learning community.

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