Creating realistic environments for the development of mathematical thinking among students with different abilities and aptitudes.

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Abstract
In the popular mind, at least, there is a growing opinion that computational skills are more properly mathematics than reasoning and aesthetics can be, and that people can either "do" mathematics or they cannot. While nobody would doubt the value of computational skills and processes, too strong an emphasis on this may diminish the levels of interaction that students can have with the subject. Mathematical thinking also includes a sense of the geometrical and dynamical, the aesthetics of space and relationships, and the logical skills of reasoning and proving, and research shows that different people with different aptitudes may succeed in one area but not another. A renewed focus on all the aspects of mathematics, and on the ways in which people express their understanding, will assist in identifying

- awareness of differences in aptitudes between people
- different possibilities in terms of the packaging of mathematical ideas
- ways of integrating mathematical thinking into other parts of the curriculum, so that the idea of mathematics becomes less frightening to those who do not believe they have strong mathematical skills.

Introduction

During the 1990s most of my research has been focussed on the backgrounds and educational experiences of New Zealand students who are gifted or highly talented mathematically. Mathematically gifted and talented students are different from other students; but the difference lies in their levels of competence, creativity, lateral thinking, speed and task-focused concentration rather than in the methods they use to learn and experiment in thinking mathematically. Because they are able to be much more articulate in describing what they are doing and how the educational environment affects their interest and performance, the things they say give us real clues for understanding what it is that other students, less talented in mathematics, find to be stumbling blocks in the process of learning to think mathematically. Thus, although some of the observations I report in this paper have been facilitated by the talent of the students interviewed, the conclusions can be shown to have application across the wider spectrum of students.

Different Abilities and Aptitudes

Research findings on aptitudes give a starting point for identifying ideas which will encourage lateral thinking about ways in which mathematics can be presented in classrooms. There are ample indications in research and in anecdotal evidence that there are real differences in people's thinking styles. It has been fashionable in some teacher circles to claim that each student's thinking style is unique. Although there will be an element of uniqueness in each student, just as there is with DNA, there do appear to be groupings which are identifiable. While some writers attempt to return discussions on intellectual ability to the plane of the old nurture versus nature debate, the greater body of research accepts that the genetic influence is important and that the options in the debate have moved from simply 'nurture' or 'nature' to the option of finding ways to recognise and identify the interconnectedness of nature and nurture. It is increasingly recognised that instinct and learning should not be seen as alternatives when describing how the human mind, and the person, develops (Marler, 1991; Edelman, 1994; Gelman, 1993). Glotov (1989) described this in terms of the genotype-environment interaction being a "third power", which cannot be reduced to the formula "genotype + environment". Genetic and biological make-up will influence how people relate to their environment, and their environment will in turn strongly influence which aspects of their innate abilities develop and function appropriately. The gaining of confidence in the willingness to accept the findings of such research has been prompted by the strength of medical research on the genetic factors involved in illness, and this has led to the realisation that the same factors will govern wellness.

Various researchers working in the area of mathematical ability have used different descriptions and analogies to identify differences in thinking style and the ways in which these both affect and are affected by each individual's environment and practice. Krutetskii (1976) identified three basic types of thinking and named them as Harmonic, Analytical and Geometric. Osborn (1983) identified fundamental differences in the nature of individuals' mathematical ability, and noted that "the methodology of teaching adopted by a teacher, influenced by his or her own profile, is liable to favour an understanding of and communication to, pupils with similar profiles, [and] work to the disadvantage of pupils with strengths in other components." Hermelin and O'Connor (1985) noted that "while non-spatial verbal reasoning is related to verbal IQ, the ability to deal with verbally presented spatial problems is not solely so determined." Sternberg (1986 and 1995) likened the characteristics of mental organisation which he found, to the principal elements in a governing process, namely those of legislating, executing, and evaluating. Bishop (1989) found that some geometrical competencies, such as visual ability, have "a highly individual and personal nature". Gross (1993), undertaking research in a similar area to my own in Australia also found specific differences in aptitude and preference in gifted students.

My own way of identifying the intuitive preferences (Daniel, 1995a; Daniel and Holton, 1995; Holton and Daniel, 1996) arose from realising that what was 'proof' to one was not necessarily self-evident or easily accessible to another, even among gifted students. Differences were also reflected in terms of such things as other interests, skills, memory and motivation. There was evidence of consistency in basic differences of approach and little evidence that these differences could be explained by variations in schooling or environment. Although I would not claim that each student studied fitted precisely into one type or another, there was evidence that students could be placed on a sliding scale between any two types, especially if the types where visualised as having a circular relationship rather than a linear one. The points I would mark on the circumference of such a circular diagram are Spatial, Rationalising, and Pictorial abilities representing, respectively, a dominant approach which principally uses reasoning and geometric skills, pictorial and reasoning skills, or geometric and pictorial. The correlation between skills and preferences were too strong not to believe that there were biological differences that had to do with hereditary factors, genetic make-up, and the evolution of skills in a way that was consistent with our understanding of long-term environmental adaptations. Thus I developed lists of characteristics which could be identified and tested, not only among mathematically gifted students, but also among other cohorts.

Spatial abilities group. Students grouped as Spatial had a high level of ability to notice detail, but to give succinct and logical answers as well, and to visualise in their heads. They talked about interactions and relationships between shapes and patterns and ideas, and had an enthusiasm for accuracy and for verbatim information. Some had a bent towards model-making of one sort or another, for example the making of polyhedra, and were more likely to make models simply to please the aesthetic and mathematical senses rather than the practical. They were among the most direct in expressing their opinions, and the most adept at drawing a final conclusion from the information supplied and then working backwards to discuss steps that had led to those conclusions and implications that followed from them. There was a high oral component in the best expression of their work and they were not always the quickest, or the most motivated, to record their work in written form. They could explain a greater variety of proofs, and could switch more easily from one approach to another to suit the particular problem, but they did not find individual competitiveness, for its own sake, a foolproof motivation for work. They had impressive recall of the detail of the physical components of classrooms they had been in their first years at school. They were particularly good at languages and computer programming, and were prolific readers in a wide range of topics. They seldom stood out in terms of performing skills such as in athletics, drama, music.

Rationalising abilities group. The students who could be placed within the Rationalising group philosophised less than those in the Spatial ability group and had more difficulty visualising, or claimed not to do it at all. In mathematics, they liked to have opportunities to try new types of problems and to be able to obtain some originality in their solution, but they also liked to have practical reasons for the work and preferred being able to use the application of information acquired for a purpose other than simply the pleasure of thinking, or of seeing the task completed. They made models of things that would work and be useful, and had skills in performance in such things as music. They did not have particularly good recall of the physical features of an early classroom, but were more able to tell something of what had happened in school. Such students were described by their peers and some teachers as extremely able and fast in their calculations, less likely than others to worry about the elegance of proofs, and more likely to look for the answer than to be preoccupied with the method. They were described by teachers as not having been the most unusual of the mathematicians that that teacher had taught, in that they were often content with an algorithmic approach to mathematics, but they were recognised as having been the most able in their class in terms of mathematics achievement.

Pictorial group. The students who were grouped as having Spatial or Rationalising abilities were frequently described as having been noticeably good at mathematics as early as their first two or three years at school, and some were described by their families as having exhibited quite complex or advanced mathematical skills as pre-schoolers. This was not so with students grouped as Pictorial. This group could often name an event that had triggered their interest in mathematics. They were the only ones among those who described themselves as visualisers, who described an overview of their school rather than a description of detail. Like those in the Rationaliser group, they were not self-conscious about performing in the arts and were more interested in things that had an application than simply in an aesthetic appreciation of what they did and saw. A noticeable characteristic was that they did not think in their heads without external stimuli. When solving problems they tended to build the next step on the visual image of the last step taken, and to think from a pictorial image rather than a mentally constructed one. One said he used to learn by “making a clear picture of the shape of the object” but at university could not keep up with that so now learned by rote. They were able to rise to the occasion in competitive situations, and were able to let individual success motivate them when necessary.

Tests of ways in which students solved specific mathematical problems reinforced their placement in these groups (Daniel, 1995b). This testing also showed patterns which grouped the students in other mathematical areas such as a connection between ability in arithmetic and in geometry, differences in the way in which they visualised, and differences in the way they developed thoughts and ideas. Again, students in the Spatial ability group invariably did more visualising and calculating in their heads before committing a solution to paper, while students in the Pictorial group needed to commit their thoughts to paper so that they could concretely look at the way in which their solution was progressing. Although all the students initially interviewed were able mathematicians, only one group, the Rationalising group, readily used algebra as a part of their intuitive solution path.

The probability that different types of mathematical aptitudes are an outcome of different “hard-wiring” of the brain, has quite radical implications for the teaching of mathematics, especially at more senior levels where the syllabus itself may be more suitable for one type than for another. Most of those with high mathematics ability tend to be multi-talented in the sense that they can see, mathematically, what other types of thinkers are doing to reach the solutions they reach. But students with less mathematical ability are less likely to be able to understand or learn through the methods of another group, so teachers’ understanding of difference in aptitude is even more important for them.

Differences in aptitude have implications for realising why work that is easy for one student is difficult for another, for deciding on curriculum content and examination methods, for allocating particular teachers to particular students, and for making assumptions that the same test mark in relation to one student will indicate a similar level of overall mathematical ability in relation to another. Add to this the fact that during the twentieth century, education has passed in many countries from being the privilege of the few to the expectation of the many, and one sees the enormity of the implications for making false assumptions about student progress. The sense of expectation of what can be gained through education is not limited to the individual receiving an education. Societies have an expectation that all citizen will become educated in the ways required, will delight in that, and will recognise that their right to employment might depend upon their acquiring a better and better education.

Understanding differences in aptitude may also help teachers understand themselves. One of the problems for the teaching of mathematics, in New Zealand at least, is the fact that too few teachers of mathematics are competent mathematicians themselves. My samples of gifted students reported a lot of antagonism from teachers; interviews revealed that most of the friction between gifted and talented students and their teachers stemmed from differences of opinion on mathematical facts. In each case that was able to be identified, it became clear that the student had known more about mathematics than the teacher. Many teachers, especially in primary schools, are aware that they do not understand mathematics themselves, and are fearful of showing this to students. Carr, Barker, Bell, Biddulph, Jones, Kirkwood, Pearson and Symington (1994) point out that teachers who are insecure in their own knowledge often use transmissive teaching methods to avoid discussion which might reveal their uncertainty and thus alter power relationships in their classrooms. One of the ways to strengthen the confidence of such teachers, is to encourage them to identify their own aptitude so that they feel more personal freedom to carry out investigations without being sure of the answer, to have the students work with them in developing mathematical ideas, and to own the respective aspects of mathematical thought or process with which they themselves feel both more and less comfortable.

It follows, of course, as Osborn (1983) observed, that teachers will present the same differences as students. Unless teachers are aware of the nature of the differences and of their own subjectivity in both understanding and valuing one approach or another, one solution path or another, or one area preference or another, the possibility of their assisting students to use mathematical tools and skills is lowered. Recognition of different aptitudes releases us from the pressure of thinking that all students, if they and their teachers try hard enough, will be able to grasp the concepts and learn the skills of all of the branches of mathematics. The challenge for the new century will be to sort out what we think students really need to know in terms of each identified aptitude, and recognise that differences in the approach to thinking will affect the way each group

- relates to branches of a subject area, both aesthetically and functionally
- understands, sorts and uses information
- reasons from information
- retains information for future use.

Creating realistic environments for the development of mathematical thinking.

In this second section of the paper, I discuss some of the implications that understanding different aptitudes has for enriching and endorsing classroom approaches.

(1) Almost all the gifted and talented students I have interviewed valued talking about mathematics, and believed that they had not had enough opportunities for discussion. The oral component in the learning process is high, especially in the Spatial group. There were indications that some students, especially in the Pictorial group, needed to write things down in order to remember them. Others, especially in the Spatial group, reported that they wrote things down so that they could forget them. They made records in order to release the memory from the need to remember! Yet increasing amounts of our classroom practice, our judgements of diligence, and our assessments of students' progress are based on the assumption that what students write in their

exercise books and what they can reproduce in pen and paper assessment tasks, is the best they can do.

In New Zealand, the 1960s and 1970s are often recognised as being the period of best teaching: the teachers were of a post-Second World War age with the confidence to challenge the old and analyse what they had not liked as students, yet they had not moved so quickly into the technological age that they had lost the idea of "chalk and talk" methods of communicating. Overhead projectors, photocopying, computers certainly have their place but they will not have improved students' education if their replacement of the chalk will mean also the replacement of the teacher-facilitated and student-orientated talk. In this technological age we have begun, because it has become so possible, to see the accumulation of vast quantities of information as being of the essence of a good education. The possibilities that now exist for passing on vast amounts of information threatens to engulf the idea that education is about building up ideas, about developing the thinking process rather than filling the mind with information for its own sake.

(2) Not all of the old techniques of classrooms need to be abandoned. For some people, oral rote learning will endorse the patterns of number and give more lasting access to number and to basic computational skills. The extent to which patterning is endorsed, for some, by oral repetition fits with the intuitive aptitudes identified by the different types analysis. For example, some of the gifted and talented students, especially those in the Spatial group, were fascinated with counting aloud at an early age (Holton and Daniel, 1996). It is more likely that their maturation rate was different from that of average children than that their ability to learn by oral repetition and their delight in the sound of words they could connect one with another, were different.

It is still important to recognise memory as one of the chief components in the thinking, reasoning, appreciating, understanding, learning, applying process. Developing memory, learning to encode information in the memory, and practising using things remembered in various situations is one of the reasoning-focussed skills that mathematics can help build. It was clear from the answers given by gifted and talented students to the question "What do you remember about the first classroom you can remember?" that recognising the nature of the difference of what was remembered by each type of person is a key to understanding what that person is likely to remember. While all of us know that memory is one of the most important factors in enabling learning, lateral thinking, and application of what is learned and thought, there is very little substantial understanding of the way in which the brain encodes, stores, and utilitises memory or even of which parts of the brain are involved in this (Carter, 1998).

(3) Some of the technological developments that have been made in the last twenty years have been beneficial to advancing an interest in mathematical skills. For example, computers have given many gifted and talented students an opportunity to use their talent creatively, and other students of the same aptitude group the capacity to work and experiment in three dimensions, a characteristic which makes learning particularly attractive to people with Spatial abilities and aptitudes. The nature of these developments, and especially the options of graphic representation made possible by computers, may also lead us to a greater acceptance of varied methods of presenting mathematical proof. Traditionally we have required mathematical proof to be represented symbolically, but many students would gain a greater sense of achievement, which might in turn lead to greater efforts on the part of such students, if we accepted diagrams and verbal explanations as proof, especially at Primary School and lower Secondary School levels. As Shin (1994) said in putting a case for diagrams as proof, "In making inferences in ordinary life, human beings make use of information conveyed in many different forms, not just symbolic form" (p 188).

(4) A helpful educational environment should offer the freedom to name what one can and cannot do. In many subjects there are areas which appeal to one person but not another. In languages, some people have a strong interest in literature but struggle to be thoroughly competent in grammar. In art, some people find they are quite able to paint but have few skills in sculpture. In history, chronology is vital and important to some but of little interest to others, except as a tool to

use but not to be remembered in detail. It is my belief that in mathematics we have tended to make too many different approaches into essential parts of the one package.

The research with gifted and talented students showed that even though they all excelled at mathematics, they were unlikely to be equally strongly motivated in all branches of the subject and showed marked differences in levels of creativity in specific areas, even when they were unaware of this themselves. For example, in interviews with peers of the members of one sample, I found that the peers' judgements of the aesthetic value of the sample students' computer programming skills correlated with the same evidence which led me to the types groupings that I have adopted. Computer programming is based on the ability to work with algorithms, but even so not all these gifted and talented mathematicians were able to programme equally well: those with Spatial ability had an intuitive feel for developing solutions that were elegant, whereas those from the Rationalising group were much more concerned simply with whether or not the solution performed the function required at the time. It is worth noting here, that those who had the strongest basic numerical skills tended to be interested in shape and space relationships (that is, the areas which have substance in geometry) rather than in the sort of reasoning that has led to the development of algebra.

Sorting mathematics into different packages so that there are some choices for all students at an earlier age than is presently the case, would also allow for greater creativity in providing enrichment for gifted and talented students without their being obviously removed from the classrooms of their peers in age. In smaller countries like New Zealand, the opportunities for gifted and talented students is limited. In our research we found that these students were very conscious of wanting to access more enrichment opportunities but not wanting to be removed, or frequently shown to be significantly different, from their peers. The greater the number of reasons there can be for organising various groupings, then the more there will be opportunities for highly talented students to be extended without feeling socially disadvantaged.

(5) The present heightened awareness of non-European mathematics has given another incentive to re-think what it is that we wish to teach in mathematics. Our sharp focus on those methods which developed in Euromathematics has tended to diminish our perception of the importance of gaining aesthetic value from things mathematical and of acquiring simply those skills which one actually needs. One of the factors which led to boredom among gifted and talented students in classrooms, was a rigid conformity to the mathematics of the curriculum, without opportunities to move into adjacent areas even during less formal discussion. Multicultural research and studies have reminded us that mathematics is to do with the visual and the rational, with design and pattern-building as well as with computational methods; that people worldwide have developed and used methods and patterns which met their practical and their aesthetic needs (Gerdes, 1999; Eglash, 1999).

The mathematical aspects that we have been reminded of through the research focus on multicultural and ethnomathematics, will in fact play a large part in the mathematical thinking and needs that most of our school students will have during the period of their adult employment. For example, carpenters will need skills in number and measurement, but in mathematics classrooms do we remember that carpenters will do their tasks much better if they have also been encouraged and trained to look at the relationships between one shape and another, and adjust these in endless ways until the most appropriate, rather than simply the most obvious, emerges? When mathematics is defined more broadly than a concentration on Euromathematics has encouraged, then mathematical ideas and influences that have become less familiar to us but have been long embedded in custom, language and valuing systems, take on a status similar to those ideas with which we are presently more familiar in the mathematics classroom context.

(6) A factor which can influence the way in which the 'packages' of mathematics can be broken up in a curriculum is an understanding of the difference between the role of the practitioner and that of the commentator or appreciator. One does not have to be able to plan and build in order to appreciate the beauty and artistic value of what others build. One does not have to be able to design and stitch in order to judge the fit and appearance of a garment. Similarly, one does not have

to be able to use all the processes of mathematics to be able to appreciate mathematical concepts and outcomes. But the opportunity to have had one's ability to think mathematically heightened by classroom experiences is a part of the process of developing the ability not just to "do" mathematics but to think in a way that has been influenced by the reasoning and proving aspects of mathematical thinking. A friend of mine expressed it this way: "We have been taught to think that things can be tidy. If only we learn our lessons properly all our knowledge can be put in tidy parcels. There is a presumption that if you give people the same information they will be able to perform at the same level. But nothing is tidy. People can agree on definitions but that does not make the outcome definitive."

We cannot always judge in advance the point at which a particular student should abandon the thought of continuing in mathematical study, or the point at which s/he will reach sudden recognition of the ways in which mathematics is interesting. When I first began research on gifted and talented students, the university teachers of one sample were willing to offer predictions about who would "go straight through to a Ph.D." and who would be less likely to do so. Seven years later, none of those predictions are true even though, in one way or another, each of the students in that sample is using their mathematical skills creatively. In terms of the predictions, most of those students could be called academic failures; and yet when one knows what they are actually doing, one would be more inclined to wonder whether or not the issue is that the academic system has failed itself. The lesson for education is to judge its own validity by the ways in which people can use their thinking in adulthood and employment, rather than by pen and paper assessment of students. I believe we must be more ruthless in judging delivery and assessment setting practices, rather than in pre-empting the timing of decisions that a student has mathematical ability, or lacks it.

(7) A benefit that is possible when smaller 'packages' of mathematical thinking are constructed, is the encouragement this gives to develop more of the mathematical skills in an interdisciplinary way, and thus to see the benefits of being able to transfer skills and understanding from one subject area to another. During the last six years a major research project, NEMP (National Education Monitoring Project), on the standards of achievement of Year 4 and Year 8 students (9 to 12 year olds), has been being conducted in New Zealand. Some of the results show clearly that mathematics is basic to achievement in non-mathematical areas. A report (Flockton and Crooks, 1997a) describing the results of a Social Studies assessment exercise seemed to indicate that this age group understood little about the way in which the New Zealand political party system operated in terms of achieving a majority government after an election. In fact, the assessment exercise depended upon the students' ability to understand and explain fractions, and when one turned to the mathematics report of the same research project (Flockton and Crooks, 1997b), one realised that the thing both reports had in common was their examples of students' inability to understand fractions. There is no reason why fractions should be taught simply in mathematics. In my view, it may have been much more effective if it had been planned that fractions would be taught in the Social Studies periods of the school timetable, so that they were an aspect of understanding life around one, rather than an aspect of number.

A perusal of mathematics text and puzzle books shows a tendency for most problems to be associated with physical objects, but one of the things that was noticeable about many of the gifted and talented students interviewed, was that they took the ability to analyse and be logical, to explain and reason into other areas of their interests and used mathematical language to describe other ideas and situations. Their affinity with the idea of thinking mathematically meant that they made more effort to see what the patterns were in non-mathematical conversations. An understanding of the way in which the progressive deployment of reasons is used in proof increases our skills in thinking, writing and conversing on any topic and helps make any student less susceptible to simplistic or single-idea ideologies. Strong connections with teachers of other subjects and the planning of interdisciplinary curriculum units would help sustain the sort of flexibility in thinking and information-gathering that is required to achieve this sort of goal in teaching.

Relating mathematics and education.

One of the problems facing all of those involved in classroom activities in the twenty-first century is the assumption expressed frequently at present, that education is about teaching and learning. Something far more fundamental than teaching and learning needs to be going on in all classrooms, including mathematics classrooms, if the activity is to be described as real education. Education is something that is more like "noticing, experimenting, learning, thinking, applying, understanding, teaching, appreciating, knowing, using, ..." (Daniel, 1998). Mathematics is an important part of any curriculum not simply because of the specific mathematics skills and processes that can be acquired but also because of the wider and more fundamental skills of thinking and doing, independently, that are being developed. A year 7 teacher I know has a large sign high in her classroom which says "In this classroom the children do the thinking." She put it there because she realised that all of the students in her class came to school at age five, able to do things for themselves and to think what to do next, but in the six years it took to reach her class, they had learned to wait to be told what to do next. de Bono (1992) says, "my favourite model for a thinker is that of the carpenter. Carpenters do things. Carpenters make things." (pp 65). He goes on to describe the way in which the frequent thinking about and using of only a few basic operations, tools, structures, attitudes, principles and habits leads carpenters to be able to make both simple and very complicated objects. With de Bono's belief in mind – that doing and thinking are the important things in education – it is easier to see ways to accept that different students will make different connections with the subject not only according to their level of intelligence, but also according to their ability and aptitude; and easier to obtain a view of the ways in which doing mathematics assists students to develop wider, less specialized skills and appreciations even when it does not lead them to a place which enables them to achieve in higher mathematics.

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