ARGUMENTATION AND CONCEPTUALISATION IN CONTEXT:
Developing mathematical meaning through everyday life experiences

Nadia Douek, I.U.F.M. de Creteil, France; ndouek@worldnet.fr

Abstract

Different trends in experimental research and theoretical frameworks (e.g. "Realistic Mathematics Education", "Situated Cognition") have been promoted in the last twenty years, all concerning everyday life experience as an opportunity for developing mathematical knowledge. Within this stream of research I would like to show, through examples, how individual and collective argumentation concerning everyday life experience, and natural and social situations, can play an important role in the process of mathematical conceptualisation in primary school. In this perspective, I would like to underline the importance of the teacher's guidance in two specific aspects: the transformation of experiences into reference experiences, and the development of argumentative skills. Examples will be related to the use of money in grade I; sun shadows in grade IV; and folded paper planes in grades III to V.

1. Theoretical framework

1.1. Definition of "concept"

We shall refer to Vergnaud's definition (Vergnaud, 1990) as the dynamic interwoven construct of "reference situations", "operational invariants" and "representations". Situations can be physical or social experiences, and operational invariants can be schemes.

From my point of view, in the complex and dynamic encounter of these three components of conceptualisation, representations and operational invariants or their production can themselves be reference situations or more specifically reference experiences forming various aspects of a concept. Examples will be considered later.

1.2. Description of argumentation

We cannot accept any discourse as an argumentation. Inspired by the Webster Dictionary, we intend the word "argumentation" in two ways: the process that produces a logically connected (but not necessarily deductive) discourse about a given subject; and the text produced through that process. "Argument" will be "*a reason or reasons offered for or against a proposition, opinion or measure*", and may include verbal arguments, numerical data, drawings, etc. So, an "argumentation" consists of one or more logically connected "arguments".

1.3. Differences and relations between experiences and reference experiences:

How do we distinguish experience from reference experience when we (as observers) analyse children's behaviour and schemes in solving problems? It is a reference when it is referred to explain, justify or contrast in an argumentation. We can imagine that if the experience is referred to in the child’s processes of thinking, of mentally problem solving, then it should also be a reference experience. But, firstly, this is not quite certain from a theoretical point of view: what is the stage of differentiation of knowledge when it is not put into language? Secondly, as researchers, we have (fortunately) no means to deal with this sort of unspoken knowledge. This criterion remains valid from basic experiences to high-level, formal and abstract ones. Experiences and reference experiences can be intellectual, symbolic, physical, or more likely a blend of these, and, evidently, are always mental. To become a reference, an experience must be connected to representations in a conscious way.

1.4. Argumentation, references and experiences

We make the conjecture that the subject develops argumentation skills and constitutes references through a dialectical process. Argumentation can be seen as a means to develop an experience into a reference experience through two facts: it involves the subject's view and consciousness in the experience; and it involves some representations in the experience.

These two facts create semantic roots to the representations relating the experience to the network of the subject's conscious knowledge. On the other hand, one clearly needs references as arguments and backings in an argumentative process.

In Douek (1999b) I introduced the expression "reference knowledge" to consider *"the knowledge which can be put into different forms (graphic, verbal, etc.), and agreed upon by the class, or used in argumentation in the subjective conviction that it is so".* According to this definition, a reference experience belongs to reference knowledge. On the contrary, some references (e.g. some formulae or statements) belonging to the reference knowledge of a student (or a class) can be used without being reference experiences when they are assumed as "arguments" in a procedure or in an argumentative chain. In this case no questioning about their meanings or conceptual implications is usually needed.

1.5. The teacher's mediation

According to Vygotsky (1985), we must stress the crucial mediating role of the teacher in developing argumentative skills and promoting the transformation of experiences into reference experiences.

1.6. Experience field

I shall use the definition of "field of experience" (Boero *et al*, 1995), especially as concerns the ideas of external context and the student's internal context. For a given subject (such as money - see 2.2.), the theoretical construct offers guidelines for following the long-term development of the student's "internal context" (i.e. her/his conceptions, schematas, etc.) in relation to the "external context" (signs, concrete objects, physical constraints, etc.).

Experience field settings offer varied experimentation situations and occasions for argumentation (class discussions and individual writings). Under the teacher's guidance they promote the connection of new knowledge, in multiple ways, to the subject's network of knowledge, thus enriching the semantic roots of its representations. They also facilitate the evolution of experience into reference experience.

2. Examples in primary school

2.1. A general description of the teaching situations

The <u>first and second examples</u> were performed in Italian schools involved in the Genoa Project for primary school. The aim of this project is to teach mathematics, as well as other important subjects (native language, natural sciences, history, etc.), through systematic activities concerning "fields of experience" from everyday life (Boero, 1994). In Grade I, the "money" and "class history" fields of experience ground the development of numerical knowledge and initiate argumentative skills as well as the use of schematic representations. In Grades III and IV, the sun shadows field of experience grounds the development of argumentative skills and geometry concepts.

A fairly common classroom routine consists of: individual production of written hypotheses on a given task; classroom comparison and discussion of student products, guided by the teacher; individual written reports about the discussion; and classroom summary, usually constructed under the guidance of the teacher and finally written down in the copybooks. In most cases, classroom summaries represent the knowledge the students reached (with all possible ambiguities and hidden mistakes), and are not final institutionalisation phases (Brousseau, 1986), which are attained only in few circumstances. This style of slowly evolving knowledge without "sure" and final "truth" offers the opportunity to observe the transformation of the students' knowledge in a favourable climate: such transformations are normal, expected events. Moreover, argumentation reflects fairly well each student's level of mastery of knowledge, his/her level of reference use, and the class's common stable references.

The data used as a grounding for the analysis of the sequence derive from direct observation, the students' texts, and videos of classroom discussions. As the classes were two of the Genoa Group's "observation classes", a lot of information was accessible about previous activities: students' individual texts and copybooks, recordings of classroom discussions, and one video. As in all Italian primary schools, the student group stays with the same two teachers for the whole five years of primary education. Therefore didactical contracts are very stable and once established need no specific effort.Italian curricula are rather loosely prescriptive around a core of necessary knowledge.

The <u>third example</u> is a project intended for French primary schools. It is not yet rooted in a developed research project group. In French schools, students generally have one teacher who changes every year. The didactical contract can thus be revised, and this calls for strong effort on the part of both the children and the teacher. Although the French curriculum is split into two three-year periods (the first comprising the last year of kindergarten), it is in fact regarded as a sequence of annual curricula, and is very precisely drawn up. Therefore, projects clearly must fit in with this constraint, at least to reassure teachers and parents.

In the presentation of the examples, my comments will be written in helvetica.

2.2. Use of money and approach to the decimal-positional representation of numbers (Grade I)

A: How the teacher introduced the use of the money abacus for writing numbers:
"Yesterday we had to buy flour, eggs and sugar, which altogether cost 1900 lire. If each of you brought 100 lire would that suffice?" (there are 19 children in this class)... *"Yes, it makes 1900"* (children easily compute such a sum by using the oral sequence of numbers and fingers; hundreds and thousands are mere words designating coins). *"But some of you do not know how to write this number: some of you wrote in words, 'mille 900', others 'mille nove cento', etc. Today we shall use this tool:*

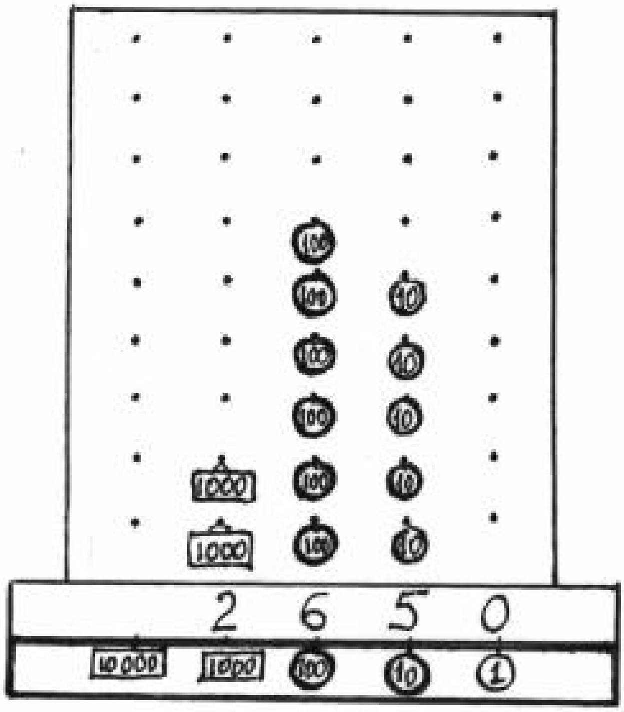


FIGURE 1

It will help us in calculations, and will be useful to help us understand the numbers. It is a 'money abacus'. We can see columns with coins or banknotes we know (10,000 lire, 1000, 100) but also some with coins we do not use but saw when we spoke about our grandparents' life: 10 lire and 1 lira coins. [...] Yes, Marco noted that the columns are ordered decreasingly [...] each column has exactly nine nails to hang nine coins (or banknotes) on. The money abacus will be useful for making clear our computations and what we think, what we say.... (invitation to argumentation).*We also have a rectangular frame to write the number in."*

During this sequence each child put a 100 lire coin on a nail of the hundreds column until it was full, which posed the problem of the relationship between the columns. Etc.

B: Experiences and reference experiences were needed to build the rule of the decimal positional representation of numbers, and their different levels of mastery at the time of the described work. These belonged to two fields of experience which made extensive use of numbers: money (with effective use of money in a grocery) and class history (dates, children's attendance, temperatures).

B1-Representations of numbers: oral and written numbers used in a wide range, from -10 (temperatures) to ten thousands (Italian money) and in various contexts. Written representations were either decimal-positional (given by the teacher, observed on the coins or banknotes) either written completely in words or in a mixed form, such as *"10 thousand lire"*. This use of numbers before entering the stage of making their decimal-positional representation rules explicit, even if repeated, must not be considered as a reference experience, but only as experience at that time. It will evolve towards a reference experience (for the decimal-positional rule of writing numbers) in two ways: through the need the children will encounter to separate and interpret the words and the signs so as to succeed in reproducing the newly discovered rule; and, as will be shown later on, through other problem-solving situations. For further developments, see **C.**

B2- Differentiating the number of coins from value and exchanging coins of the same value: these are already reference experiences for decimal-positional writing of numbers. The children had solved many problems in which coin exchange procedures played a crucial role as steps towards explaining or solving those problems.

B3- Other aspects of number (order, etc.) needed for decimal-positional writing of numbers: comparison of positions, comparison of lengths (used on the thermometer). Related to this, the schematic representation of numbers on the ordered line. This is a representation but it also supports experiences of displacement for counting and metaphors of additive and subtractive calculations accompanied by physical mimicking. These experiences are (at the time) going to become reference experiences.

B4- Additive and subtractive calculations. These are already references (as mental experiences as well as practical ones when buying things in a real shop , comparing temperatures, etc). Children solve such problems by counting on their fingers, using numbers as words and schemes such as that above (B3); they also know that 10+10+...(ten times) makes 100 and that 100+100+... (ten times) makes 1000.

In general, we may note how the teacher uses experiences and reference experiences: he refers to a practical experience he shared with the children a few times out of the school (buying things in shops, etc) and many other times in simulated or real purchases in the classroom; he also refers to additive decomposition (as in the case of 100's into 1900); he distinguishes amongst the various number representations usually adopted in the classroom.

We may also remark that operational references were needed for the basic mechanisms of the money abacus: counting skills and the correct estimation of different <u>coin</u> <u>values</u> were needed to put coins on the nails of different columns, <u>exchange</u> practices were needed to relate successive columns of the abacus, the regularity of the <u>positions</u> of the nails in the same column as on the graduate line of natural numbers, the increasing filling of columns similar to <u>displacement of positions</u> through addition on the line of natural numbers.

The children's problem-solving schemes become reference experience (experience of their own action on symbolic or material objects) (see further developments in **C**).

C: Problem solving with argumentation situation

Let us consider a task proposed after the second sequence concerning the money abacus: working on the abacus, the children had to write the result of an addition (they still did not know the rules of adding by columns), namely the money spent for buying three pencils costing altogether one thousand and fifty lire. The task is identical to the problem solved collectively when the abacus was introduced, with the added difficulty that the result had a zero in the third column (here was the true -but hidden- problem!).

A first activity was devoted to individual written problem solving (argumentation is established in the didactical contract, in written as well as in oral productions, but is not yet well mastered). After reading the productions, the teacher chose three that took different positions in preparation for the following discussion (from which I chose some excerpts):

The teacher: *"Today we want to understand how to write this number (not in words!) [...] There were three sorts of reactions in your individual work: some of you wrote "I don't understand"; it is important to say so when we don't.*
Some (seven) said 'We should write it like this:1°50.' What do you think of that?
D.: We can hear that it should be written like this.

(reference to the use of words, a repeated experience)

C.: *One and fifty are not enough, one zero is not enough*

(Maybe he has in mind that one thousand needs plenty of zeros?)

[...] On the abacus we need a zero under column one...Where do we put the one?

A.: *To get a hundred we need two fifty lire coins.*

(reference to exchanges to situate their "fifty", which is puzzling: the coin exists, but not the column! In this way this child puts the rule of the columns in question)

C.: *the one hundred coin doesn't make fifty, we should exchange it for a fifty lire coin.*

(confusion between columns and coins, and error on exchange, but we are in the same questioning)

J.: *It is true that the hundreds column won't do, but five ten lire coins makes fifty, so we can put 5 on the tens column and zero on the ones column because there aren't any.*

(reference to the new rule of the abacus and to exchanges and calculations, etc.)

This activity helps in moving from the experience of using words and writing numbers within the use of money towards a reference experience of translating words into a structured system of signs and finally initiating the mastery of the decimal-positional representation of numbers. This is reinforced by the following excerpt concerning the third kind of answer:

The teacher: *"Marco thinks it should be written like this: 1 100, and explained it. Go on and say what you wrote."*

M. :"*I have put a fifties column in the abacus instead of the hundreds column :*

I	*I*	*I*	*I*	*I*
10 000	*1000*	*50*	*10*	*1* "

(For Marco, exchange and designation of values are sufficiently established so that, facing the difficulty of the empty column in the middle, he can reinvent a "convenient" money abacus system, with the same rules.)

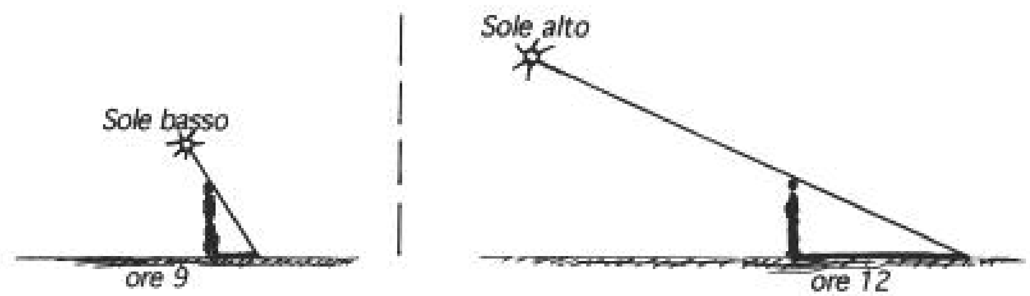
A discussion follows about the importance of sharing the same rules so as to be able to communicate with one another.

As a final comment, we may remark that, when children have to explain their solution to an exercise, argumentation is a mode of relating past experiences to the new questions and representations, and brings to light questions ("why" and "how" sort of questions) concerning the words and writings they naturally used as stable unquestionable knowledge. This may "shake" reference knowledge and give an opportunity to develop it.

2.3. Sun shadows and angle in grade IV

The following task was set in a fourth-grade class (19 students) engaged in a long-term activity of geometrical modelling of the sun shadows phenomenon:

"At the beginning of class work on sun shadows, Stefano (a VI grade student) thinks that shadows are longer when the sun is higher and stronger. Other students think the contrary. In order to explain his hypothesis, Stefano produces the following drawing:



(sun low at 9 a.m.) (*sun high at 12)*

FIG. 2

and writes: "As we can see in the drawing, the sun makes a longer shadow when it is higher, that is at noon, when it is also stronger". *We know very well that shadows are longer when the sun is lower (early in the morning and late in the afternoon). So, in Stefano's reasoning there is something* *that does not work. What is wrong with Stefano's reasoning, and particularly with his drawing? Try to explain yourself clearly, so that Stefano can understand."*

The task of refuting Stefano's elaboration was not easy, especially because the meanings attributed by Stefano (an actual student from another class who produced the protocol) to the words "high" and "low" were compatible with what students could see: it is true that at 9 a.m. the apparent height of the sun from the ground (or the horizon) is always lower than at noon! Moreover, it is true that the statement *"when the sun is higher, the shadow is shorter"* was a reference for this class, already used in many argumentative activities; but no detailed work had been performed on its meaning.

The aims of this task, agreed with the class teachers, were: to get the students to question the statement *"when the sun is higher, the shadow is shorter"*, especially as concerns the meaning of *"high"*; to approach and develop the concept of inclination of sun rays as a tool to explain and overcome Stefano's mistake; and to question some aspects of the conventionality of the geometrical schema used by Stefano and by the class (we will call it the "shadow schema") - particularly, the sun at the end of the segment representing the sun ray.

As concerns links between argumentation and conceptualisation, I was interested in exploring how argumentation modified the meaning of *"high"* from length of a vertical segment to angular height, and how it gave *"inclination"* a precise geometrical meaning. These transformations also affected the understanding of the sun shadows model represented by the shadow schema, and led many students to reconceptualise the model.

Preceding classroom activities concerning sun shadows, angle and shadow schema

Classroom activities on sun shadows started in grade III: games, observations of shadows at different times of the same day and on different days of the year at the same time; drawings, measurements, etc.

The construction of the shadow schema had undergone the following steps:

a- following (with the hand) the border of the "shadow space" (*"spazio d'ombra"* in Italian) of their body and other objects, and representing it in a drawing;

b- indicating (with the arm) the direction of the sun, and representing the scenario in a drawing;

c- then integrating a) and b) (three months before this teaching experiment): the teacher introduced the shadow schema as a tool for interpreting some aspects of the sun shadow phenomenon (see Scali, 1997).

The sign originally proposed by the teacher did not contain the sun (indeed, in a frontal position the sun cannot be seen!). But immediately afterwards the students started to add the sun as a reference for the origin of sun rays. In order to begin challenging the conventionality of the representation of the sun within the shadow schema, the teacher asked the class to imagine the object being shifted from one schoolyard to another and to hypothesise the effect that this would have on the length of the object's shadow at a given moment. Experiments followed (for further details, see Douek, 1998).

Method

The aim of this experiment was to detect productive links between context-related argumentation, mathematical modelling (here concerning sun shadows) and conceptualisation (inclination of a straight line to a plane). In line with this aim, the individual task described and discussed above was proposed (with written answers in 100'). The next day in class, the usual discussion was held about the individual productions; the teacher led this discussion (75') with the aim of comparing different lines of argumentation produced by students and getting them to understand the crucial role of the "inclination" of sunrays in overcoming the ambiguity of the expression "sun high". Then the students produced individual reports (in 45') about what they had learned and understood during the discussion. Finally, the teacher led a classroom discussion (75') based on individual reports in order to examine the questions raised by the initial task and propose a connection between "inclination" and "angle".

Two episodes were selected in which new arguments, linked to conceptualisation and refinement of the model, were generated through argumentative activities.

DOMENICO'S EPISODE

During the first discussion, Domenico produced the following statement: *"the length of the shadow depends on the position of the sun".* In formal terms, we might write: L=L(P). Then, invited to explain at the blackboard (where Stefano's drawings were reproduced), he drew another position for the sun (below the sunray in the 9 a.m. drawing) and said: *"if I change the position of the sun, the length of the shadow changes too".* The content of this statement, brought to a formal level, would be: if P1 ? P2, then L(P1)? L(P2). Two students realised that this was not always true. This was not discovered at the formal level, but probably through the dynamic exploration suggested by Domenico's gesture accompanying his words: the students could imagine other positions compatible with the same length! The teacher encouraged discussion about this discovery. It was spontaneously recalled (as reference experience) by students twice during the first discussion and twice during the second.

THE 'TALL MAN AND SHORT CHILD' EPISODE

During the first discussion, a student said that the height of the sun (*"for us"*) can be seen through the *"inclination"* or the *"height"* of the arm pointing to the sun. Then Steven (who had discovered, in his first text, that the prolongation of a "lower" sunray overtakes the "higher" sunray) proposed a correction: not *"high arm"*, but *"arm towards a high position"*. Making a connection with a preceding point in the discussion (Ambra's comment evoking the gesture of pointing to the sun), the teacher suggested observing what would happen if he pointed to a low position of the sun with his "high" arm, while Steven was pointing to a "higher" position of the sun with his "low" arm. This observation (supported by many comments, especially concerning the idea of the prolongation of the two arms) created a new reference in the classroom.

Argumentation and conceptualisation: an outlook

Argumentative activities can be analysed from two points of view: how students exploited their reference knowledge in order to fulfil the task; and how reference knowledge changed during the sequence originated by the task.

In their first individual productions, the students organised their argumentation about Stefano's mistake in four different ways: producing and exploiting graphic evidence, based on graphic schemas as shared and almost institutionalised references; making connections with past experiments and interpreting them in order to produce appropriate arguments; exploiting reference statements as arguments to put in a logical chain; and expressing a global dynamical conception of the phenomenon in order to support their view against Stefano's.

During this experiment, evolution occurred in reference knowledge, following different argumentative patterns:

- by questioning the meaning of shared statements (the *"When the sun is higher the shadow is shorter"* statement gradually took on a more precise meaning through long reflection about the *"sun high"* clause, repeatedly stimulated by the teachers);
- by integrating in the discussions, under the guidance of the teacher, arguments brought up by some students that belong to their "private" knowledge, such as the idea that *"at noon, sunrays are more inclined"*;
- by generating and integrating new arguments.

Domenico's episode and the "tall man and short child" episode represent two meaningful examples. In both cases, a statement produced by a student was questioned *("If I change the position of the sun, the length of the shadow changes too"; "The arm is more inclined when it is higher").* The consequence was that a new argument was generated and incorporated in the reference knowledge under the guidance of the teacher, who emphasised the argument and encouraged students to take it into account when they came back to it .

We note that this analysis shows the importance of the social context, managed by the teacher, in ensuring the evolution of reference knowledge, which occurred in strict relation with processes of conceptualisation and refinement of the model.

The field of experience of sun shadows: inherent complexity and its relevance for argumentation and conceptualisation

In dealing with the sun shadows field of experience, the students involved in the teaching experiment were faced with great complexity. We will consider three different, interrelated aspects of this complexity and try to detect their effects on student performance.

Plurality of spaces (cf Berthelot & Salin, 1992): observed shadows were in meso- and micro-spaces. Studying the "cause" of shadows meant considering macro-space: the solar system. The external graphical representations of the phenomenon were on the micro-space of paper. Plurality of spaces entered individual productions and classroom discussions in different ways: for instance, the "tall man and short child" episode established a relationship between the inclination of the sunray in the shadow schema, the evoked gesture of pointing to the sun and the visible situation of a "lower" inclination represented by a "higher" arm.

Variety of sources and data representation: the students had observed the movement of the sun (especially at sunrise and sunset) and shadows (throughout the day). They had worked with numerical data (tables of shadow lengths), "graphic" data (the shadow fan is not just a drawing, it bears data), schematic representations of the relationship between the position of the sun and the length of the shadow for a given object (especially the shadow schema). They had also worked with different statements concerning the situation: "natural" descriptive statements (e.g. *" every day the sun crosses the sky");* "rigorous" descriptive statements (e.g. *"sunrays are parallel"*); relational statements (e.g. *"shadows are shorter at noon than at 9 a.m.").* In this experiment, the complexity inherent in the variety of sources and data representations suggested a great variety of arguments (and consequently, of lines of argumentation) in the individual productions. During classroom discussions, this complexity fostered the crossing of different lines of argumentation.

Variability of the epistemic validity of reference statements: this source of complexity was mainly a result of the teachers' decision to guide the class through a slow, gradual process of "institutionalisation" of knowledge. In this experiment, students continuously used statements of different epistemic validity: some of them had previously been institutionalised; some had not, and were thus subject to doubt; and others were consolidated by common use as evidence (although these might have been reconsidered later as sources of error or ambiguity). The effects of this kind of complexity are revealed in the classroom discussions. The one that arose most often was the need to verify and validate: 25 comments (11 in the first discussion and 14 in the second) from 13 students appropriately questioned statements produced by classmates (as in Domenico's episode and the "tall man and short child" episode).

2.4. Use of paper planes to construct reference experiences for various concepts

Here are the main features of this project. First, we must establish a didactical contract similar to that for the Genoa Project classes: argumentation (written and oral) must be encouraged and fear of errors must vanish; interest in and respect for each other's points of view must be valued. A personal copybook should be devoted to this activity; drawings are encouraged, partly to personalise the copybook, essentially to promote non-institutionalised modes of representation within a "scientific area". We conjecture that it will enlarge the range of mental experimentation the children allow themselves and thus the range and quality of arguments as well (see Ambra's case in Douek, 1999b).

I- Everyone tries to construct paper planes.

Here we want to address and value children's interests and skills.

II. Observation of the planes' flight performances and of their forms, discussion, individual writings and drawings regarding each discussion point and further plane construction. The question *"How can we* *compare your planes?"* opens the way to an important part of the work, namely finding criteria to classify the planes. But the pretext is flight competitions!

We now have a series of sequences with numerous aims: let the children use their own productions, introduce -if not yet existent - argumentation behaviour and its social rules; approaching a scientific problem: why some planes fly in different ways....; access to mathematical concepts they are usually acquainted with (like length, distance, height, time, forms of plane trajectories) but do not differentiate in a complex context.

We will return to these goals later, but first we need to sharpen some geometrical tools.

III. Production and reproduction of paper planes, either from another plane or from technical schematic instructions . Observation, comparison, comments and discussions (written and oral) of results.

Here we approach rather classical plane geometry taught in French schools : these tasks will all pose problems of recognition of geometrical forms, specific lines in squares and rectangles, symmetries, comparisons of length, of angles etc... these geometric objects should be dealt with here, it is not new. What is difficult though is to distinguish them in a complex setting that is not specifically mathematical.

IV. Refinement of criteria and measurement problems.

Here we return to the goals presented in Part B, but this time openly. Part C should help enter a more precise type of work, better use of words and arguments.

V. Conjectures and argumentations about the factors that influence flight.

This scientific domain can be approached easily through experiments to be set by the children. Some effects are easy to detect, like symmetry and the width of the plane. What is interesting for us is to help the children use various tools, and specifically mathematical tools, in a complex domain so as to bear out their argumentation.

3. General remarks about the three examples

The three examples are different from the following perspectives:

- they are not at the same research level: the first and the second have a great amount of work behind them. The Genoa Research Group began to elaborate these settings twenty years ago, experiencing them in many classes, analysing class work, studying student behaviours etc. By contrast, the third is a new setting; partial experimental work began just one year ago;
- they need different conditions of school and class organisation: the second and third are well suited only if the school can afford to work within open curricula (instead of conforming to precise goals within a set time schedule, separating knowledge domains and isolating mathematics). The first can fit into a tighter time/curriculum constraint and rigidly separated domains. Anyway, children already have sufficient everyday life experience in this field!
- they correspond to different levels of mastery of argumentation, and have various relations to other topics: the first is well suited to children just beginning to use argumentation. The second and the third need a higher level of mastery of argumentation, and various domains of school knowledge are tightly intermingled (specially in the case of the third).

4. Conclusion

We need to choose suitable contexts to build the teaching of mathematical concepts, so that it is based on a rich set of reference situations. The concept to be taught must be related to a context that is meaningful for the student, so that the experiences related to the concept are very likely to belong to the student as a subject, or may be easily appropriated, for instance by being easily connected with the student's web of "common knowledge" (cfr. Vygotsky, 1985).

The teacher has a crucial role as a mediator, to:

- help relate the experiences and the operational invariants to representations;
- help transform suitable experiences into reference experiences through argumentation, provided that suitable problem-solving tasks have been chosen;
- (as the promoter of socially shared knowledge) introduce children to the art of argumentation by encouraging it, using it and rectifying their use of it when needed. Indeed, argumentation is a powerful, if not the only, means of conceptualisation.

One final, more general remark: not only do I think that the choice of suitable contexts and development of argumentation facilitate and deepen conceptualisation, but I also believe that a high level of skilled argumentation is a good condition for learning the topic of proving in secondary school. We may say that argumentation is a "reference experience" that is precious for learning proof (see Douek, 1999a).

REFERENCES

Berthelot, R. & Salin, M. H.: 1992, *L'enseignement de l'espace et de la géométrie dans la scolarité obligatoire,* Thèse, L.A.D.I.S.T., Université de Bordeaux-I

Boero, P.: 1994, 'Experience fields as a tool to plan mathematics teaching from 6 to 11', in L. Bazzini & H.G. Steiner (Eds.), *Proc. of the Second Italian German Bilateral Symposium on Didactics of Mathematics,* IDM Bielefeld, pp. 45-62

Boero, P.; Dapueto, C.; Ferrari, P.; Ferrero, E.; Garuti, R.; Lemut, E.; Parenti, L.; Scali, E.: 1995, 'Aspects of the Mathematics-Culture Relationship in Mathematics Teaching-Learning in Compulsory School', *Proc. of PME-XIX,* Recife, vol. 1, pp. 151-166

Brousseau, G.: 1986, 'Fondements et méthodes de la didactique des mathématiques', *Recherches en didactique des mathématiques*, 7, 33-115

Douek, N.: 1998, 'Analysis of a Long Term Construction of the Angle Concept in the Field of Experience of Sunshadows', *Proc. of PME-XXII,* Stellenbosch, vol. 2, pp. 264-271

Douek, N.: 1999a, 'Some Remarks about Argumentation and Mathematical Proof and their Educational Implications', *Proc. of the CERME-I Conference*, Osnabrueck (to appear)

Douek, N.: 1999b, 'Argumentation and conceptualisation in context: a case study on sun shadows in primary school', *Educational Studies in Mathematics* (in press)

Scali, E.: 1997, 'Choix des taches et organisation des intéractions dans la classe pour l'appropriation des signes de la géométrie...', *Proc. of CIEAEM-49*, Setubal, pp. 186-194

Vergnaud, G.: 1990, 'La théorie des champs conceptuels', *Recherches en Didactique des Mathématiques*, 10, 133-170

Vygotsky, L. S.: 1985, *Pensée et langage*, Editions Sociales, Paris