The Effectiveness of an Instructional Program for Promoting Prospective Mathematics Teachers` Use of Metacognitive Strategies in Problem Solving

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Introduction

It is commonly recognized by many math teacher- educators that the development of prospective teachers`problem-solving ability and skills should be central to the preparation for teaching mathematics. NCTM`s Professional Standards for Teaching Mathematics(1991) have called for problem solving experiences to be" ingredients in the ways teachers of mathematics build and extend their knowledge of mathematics" (p.135). However, as Lester, Garofalo, and Kroll (1989) indicate, "many mathematics teachers have received little or no systematic training in problem solving when they were students, or when they were trained to become teachers"(p.6). Yet, ten years later, the above pointed out remark still applies. Typical mathematics instruction and all sorts of training that student-teachers receive at colleges focus much more on mathematical knowledge than on mathematical behaviour. Accordingly, it is highly expected that the recent wide- spread recommendations for problem solving to be the central focus of mathematics instruction, will find no way to our mathematics classes. Considering this situation, researches have emphasized the need for improving teacher-education programs in the way they offer prospective teachers the kind of knowledge and training necessary for improving their own problem -solving performance, and for enhancing their capacity to help their students in the same way. The present study aimed to explore the effectiveness of an instructional program designed to enhance prospective teachers` use of metacognitive strategies in their problem -solving behaviours.

Background

Researchers commonly agree that problem solving is a complex cognitive activity that requires much more than just the direct application of some mathematical content knowledge. Many suggest that cognitive performance in mathematical problem solving depends on having adequate knowledge (procedural or strategic knowledge) as much as on the presence of non-cognitive and metacognive factors that would inhibit or facilitate the appropriate utilization of that knowledge (Garofalo and Lester, 1985). Schoenfeld (1983) points out that many "driving forces" that determine success (or failure) in problem -solving performance are of metacognitive nature.

Metacognition refers to one's knowledge concerning his own cognitive processes and products, and to the active monitoring, consequent regulation, and evaluation of cognitive activities (Flavel 1979,cited in Cai 1994,p.167) . Lester and associates (1989) indicate that most psychologists consider metacognition to consist of two separate but related aspects: (1) knowledge about cognition (metacognitive knowledge); and (2) regulation of cognition (control and regulation aspects of metacognition). The second aspect refers to the type of decision behaviours one makes in order to: (a) become aware of decisions necessary for planning the solution processes, and of the effective use of such decisions; (b) monitor progress (monitoring actions and effectiveness of applying strategies); and (c) assess the situation and evaluate solution (Callahan and Garofalo, 1987).

During the last two decades, a growing number of researchers have been discussing and investigating the role metacognition plays in mathematical performance. The work of Silver (1982), Schoenfeld (1983&1985), and Lester and colleagues (1985&1989) have highlighted the critical influence of metacognitive beliefs and behaviours on individuals` problem-solving performance. Their work have established a strong evidence that such beliefs and behaviours are important determinants of success or failure in a wide variety of mathematical activities. Nevertheless, research by Schoenfeld (1983) has shown that the development of metacognitive skills and behaviours do not depend on age per se. He found that most students do not develop, by themselves, these skills to any degree. Therefore, the need for incorporating metacognitive-based activities into instructional settings has become a necessity.

Some of the prominent work involving metacognition-based instruction is presented by Mason and companions (1982). In their book "THINKING MATHEMATICALLY" they express explicit conceptions of some metacognitive strategies as part of their approach to develop mathematical thinking. They emphasized the role of reflection, self-questioning, awareness of process of thinking, developing internal monitoring, judging, and convincing self and others, in developing mathematical thinking at all ages. (Mason's work was a prime source of activities involved in the instructional program of the present study.)

Researchers have explored the extent to which school students can be taught to become more strategic and aware of their own problem-solving behaviours. Among those are Mevarech and Kramarski (IMPROVE Project 1977), Bondy (1984), Narode (1985), Linn (1987), Lester et. al. (1989), and McInerney et. al. (1997). Overall findings of these studies indicate that the use of metacognitive strategies and skills is greatly facilitated through instruction. Some other researchers, (e.g., Persichitte 1993), designed in-service programs to teach strategies that promote classroom teachers' metacognitive development. However, it seems that only a limited number of studies have investigated prospective-teachers' metacognitive beliefs and behaviours , and how these can be taught.

NCTM`s Professional Standards (1991) highlights the fact that experiences teachers have while learning both mathematics and mathematics teaching, have a profound impact on the way they teach. Thus, a full consideration of the influence of metacognitive beliefs and behaviours on mathematical problem solving would require examining the ability of prospective teachers to adopt a metacognitive posture toward mathematical performance. Unfortunately, as many people would agree, prospective teachers do not have adequate metacognition training or experiences that enable them to design and implement instructional activities that foster their students` problem-solving behaviours.

The results of ***a pilot study***, conducted by the researcher, have supported the above perspective. The participants (ten prospective math teachers) were observed systematically as they solved a number of non-routine mathematical problems. The findings revealed by this pilot study were:

- Participants were very keen to consult the researcher regarding whether they were following the right course of action.[*lack of control and of self-evaluation*]
- They always sought to know from the researcher whether their answers were correct. [*seeking external judgment and lack of self-evaluation*]
- When stuck, they were not able to abandon the inefficient tactics or to change their course of action. [*inability to evaluate progress / failing to respond for being stuck*]
- They exhibited less attention to understanding and planning than they were expected to as mathematically capable. [*lack of comprehensive strategies*]
- Once they made a conjecture about a solution, they jump to a conclusion without testing the validity of such a conjecture.
- They experienced a great difficulty in solving many of the problems though the mathematical knowledge involved do not exceed that of the school mathematics level.

The above findings, together with the previously discussed views, set forth the ground for the premise that pre-service teachers need explicit and systematic instruction in problem solving, with an emphasis on metacognitive aspects. The main purpose of this study was to explore the effectiveness of an instructional program designed to promote prospective teachers` use of metacognitive strategies in mathematical problem solving. The study aimed also to assess the impact that the instructional program has on prompting prospective teachers to rethink some commonly held beliefs about mathematical problem solving.

More specifically, the present study attempts to answer the following questions :

1. **How effective is the proposed program to enhance prospective teachers` tendency to use metacognitive strategies in problem solving?**
2. **To what extent does the promotion of such tendency affect prospective math teachers` problem- solving performance?**
3. **To what extent does the proposed program affect prospective teachers` beliefs about mathematical problem solving?**

METHOD

Subjects

20 prospective math teachers participated in this study; each of them holds a B.Sc. in mathematics education. All of them, but one, were recently graduated. They were attending a one-year course called Practical Application in Mathematics Education. This is a four hours a week course taken as part of their qualifying requirements for a higher diploma in mathematics education at Tanta College of Education. The study was conducted over the period extending from October 1998 to May 1999.

The Instructional Program

A 3-phase instructional program was developed by the researcher. The aim of the program was to help prospective math teachers enhance their metacognitive behaviours in problem -solving performance. That is, the program provided instruction in cognitive as well as metacognitive processes. More specifically, the program incorporates: (a) Metacognition instruction [as suggested by Lester and associates (1985 &1989), and Gray (1991)] that designed to increase one's cognitive self awareness and ability to monitor and evaluate his own performance. (b) Teaching strategies for problem solving [as suggested by Polya (1957), Mason and associates (1982), and Schoenfeld (1980)].

The development of the program was guided by a number of considerations gained from related literature. These considerations include:

- Metacognition, as a component of mathematics instruction, should involve active learning on the part of students. Such learning helps students become aware of, reflect upon, and consciously direct their thinking as they engage in problem solving.
- Self-conscious reflection upon one's own thoughts and ideas is the essence of metacognition.
- Oral and written communication facilitates such reflection.
- Monitoring our own thoughts as well as those of others contributes to the development of our metacognitive skills.
- Metacognitive behaviours can be taught systematically through problem-solving activities.

It seems reasonable now to differentiate between what is cognitive and what is metacognitive. Garofalo and Lester (1985) indicate that, cognition is involved in doing, whereas metacognition is involved in choosing and planning what to do, and monitoring what is being done.

The program comprises ***three phases***: Readiness , Problem Solving, and Reflection.

***Readiness Phase*:** In this phase, direct instruction was provided to introduce participants to both problem-solving heuristic and metacognitive strategies. Then specific training on using such heuristics and strategies was arranged for participants. Instructional activities used in this phase were taken from a number of sources. Among these were the following:

- Polya`s work in his book `How to Solve it` (1957).
- The activities developed by the `Shell Center for Mathematics Education at Nottingham, England : `Problems with Patterns and Numbers` module (1985).
- The activities presented by Mason and Associates in their book `Thinking Mathematically` (1982). It is important here to point out that Mason's approach was of particular interest. His model is based on a vision that incorporates both cognition and metacognition as components in teaching and learning problem solving.

The main features of the instructional setting at this phase were: (a) explicit instruction in how to select and to carry out a strategy, (b) whole -class discussions (assessing the value of a strategy), (c) modeling successful problem-solving behaviour(undertaken by the researcher / instructor), and (d) training on self-questioning.

***Problem Solving Phase* .** Participants were engaged in solving a wide variety of non-routine mathematical problems. They were to practice, on their own, the use of various cognitive and metacognitive strategies. They were prompted to maintain conscious and periodic self-checking of their strategies, and reflect upon and evaluate them. They were prompted also to come up with as many different ways to solve a problem as possible.

Problem tasks used were selected mainly from " The Monthly Calendar of NCTM`s Mathematics Teacher". The topics covered were geometry, algebra, calculus, and numbers and patterns. A variety of instructional techniques were used at this phase; among these were the following :

- Pair problem solving, in which partners alternatively acted as a solver and (active) listener, verbalizing what they were thinking and explaining their strategies.
- Cooperative group work, in which subjects were engaged in mutual questioning and explanations of tasks and solutions.
- Writing out [their] solutions along with a descriptive explanation of what they were doing and their reasons for doing it.

***Reflection Phase*.** Participants were encouraged to:

1. Reflect on what they have learned and how it could affect their future teaching activities.

2. Explain in writing how to employ specific strategies to solve particular problems.

3. Model a teacher's role in metacognition-based problem solving instruction. Such modeling required participants to prepare lesson plans for problem-solving sessions. They were given opportunities to present their lessons before their colleagues. Discussion of various aspects of teaching activities, and how they relate to metacognition- based instruction, was followed.

Data Collection Procedures

Multiple sources of data were used in this study in order to assess the effectiveness of the instructional program.

[I] 2-parallel forms of a written problem-solving test were developed to be used as pre- and post- instruction measures of cognitive-metacognitive behaviours. Each form includes six non-routine problems (3 geometric, one algebraic, and two pattern-problems). An ***open-ended survey*** consisting of 15 questions was also developed to be administered following participants solving of each problem.

As they solve a problem, participants were requested to write down their thoughts and how they go about the solution. They were also requested, then, to give a retrospective report in response to the survey questions.

Data collected by the problem solving test and survey were first analyzed using a four-section model (Fig.1) to assess metacognitive behaviours exhibited by participants. (This model, suggested by Garofalo and Lester (1985), proposes a cognitive-metacognitive framework for analyzing performance in various mathematical tasks.) The sections of the model are: 1.orientation, 2.organization, 3.execution, and 4.verification.

A six-point scale, 0-5, was used to rate the ***metacognitive level*** in each category (section) for each problem. So, the maximum score was 30 for each category and 120 for the whole test.

The data were re-analyzed using a seven-point analytic scheme to assess the level of ***problem-solving performance***.The scheme comprises three performance` levels: Understanding (0-2 points), Planning (0-3 points), and Answering the question of the problem (0-2 points).

[II] An open-ended survey was used to assess participants` ***beliefs about mathematical problem solving***. The development of this survey was based on the work of Kloosterman and Stage (1992), Schoenfeld (1989), and Ford (1994). (The survey was tried out with a group of twenty student-teachers, and modified to improve reliability.) In its final form, the survey contains 12 statements organized into three categories representing three components (scales) of beliefs about mathematical problem solving. These components are conceptions of :

(a) ***the nature of problem solving*** (3 items), e.g., “ There is more than one way to solve a problem, and some problems have more than one correct answer”; and “Getting the correct answer is the most important part of problem solving”.

(b) ***characteristics of effective problem solving*** (5 items); e.g., “Successful problem solving depends largely on the memorization of concepts and algorithms involved”; and “Solving a problem in mathematics is like exploring an unknown country, everybody has his own way of achieving a solution”.

(c) ***ways for promoting problem solving performance*** (4 items); e.g., “Learning problem solving in mathematics is like learning a new cooking recipe, a teacher or a book gives step-by-step instruction, and a students just follows the instructions”.

A respondent is requested to give free written accounts of his/her view on the idea presented by each statement. A 5-point scale was used to rate a participant’s responses to each item. Accordingly, the maximum scores for the three scales were 15, 25, and 20 respectively.

The survey was administered to the participants prior, and directly after the end of the instruction.

Fig. (1) A cognitive-metacognitive framework for analyzing mathematical performance
(Garofalo & Lester, 1985)

Orientation: Strategic behaviour to assess and understand a problem
A. Comprehension strategies
B. Analysis of information and conditions
C. Assessment of familiarity with task
D. Initial and subsequent representation
E. Assessment of level of difficulty and chance of success

Organization: Planning of behaviour and choice of actions
A. Identification of goals and subgoals
B. Global planning
C. Local planning (to implement global plans)

Execution: Regulation of behaviour to conform to plans
A. Performance of local actions
B. Monitoring of progress of local and global plans
C. Trade- off decisions

Verification: Evaluation of decisions made and outcomes of executed plans
A. *Evaluation of orientation and organization*
1. Adequacy of representation
2. Adequacy of organizational decisions
3. Consistency of local plans with global plans
4. Consistency of global plans with goals

B. *Evaluation of execution*
1. Adequacy of performance of actions
2. Consistency of actions with plans
3. Consistency of local results with plans and problem conditions
4. Consistency of final results with problem conditions

Results

Metacognitive behaviour

The means and standard deviations of scores on the metacognition scale and its subscales were calculated for the pretest and posttest. These are shown in table 1 together with the results of paired-sample t-test comparisons (King,1969).

Table 1

Means and standard deviations of scores (n=20) on the metacognition scale

categories (scales)	pretest		posttest			
	mean	st.dev.	mean	st. dev.	t-value	p<
Orientation	10.4	2.5	20.1	2.9	14.69	0.001
Organization	12.0	2.46	20.9	3.97	9.17	0.001
Execution	12.2	3.76	21.2	3.87	10.36	0.001
Verification	4.95	3.9	14.05	3.03	13.65	0.001
Metacognitive level (total)	39.5	8.2	76.1	12.4	14.67	0.001

*The higher score on each subscale is $6\times5=30$
The highest total score is $30\times4=120$

As can be seen in the table, the subjects started the instructional program with quite low level of metacognitive behaviours. They were so deficient particularly in the metacognitive behaviours at the verification component (M=4.95 out of 30). The posttest mean scores are considerably higher than those of pretest. The significant differences between all pairs of means (as evident by the values of t) indicate substantial improvement in the prospective teachers'use of metacognitive strategies and skills during problem solving. This trend is evident for the four aspects (categories) of metacognition: Orientation (t=14.7), Organization (t=9.17), execution (t=10.4), and Verification (t=13.7).

Overall, by the end of the instruction, the subjects were able to exhibit a significantly higher level of metacognitive usage as compared to that exhibited at the beginning of instruction.

Problem -solving performance

In order to determine the effects of the instructional program on the prospective teachers' overall problem solving performance, paired- sample t-test was used as a measure of comparison between the mean scores on the problem-solving scale of pretest and posttest.

Table 2

Means and standard deviations of scores on problem-solving scale

Pretest		Posttest		t-value	p<
mean	st.dev.	mean	st.dev.		
17.5	4.6	28.5	5.5	8.5	.001

*Maximum score is $6 \times 6=36$

Evident from the table, the level of problem-solving performance of the subjects has significantly improved from the start to the end of the instructional program (t-value is 8.5). This improvement in the level of problem-solving performance (cognitive behaviour) is consistent with the significant increase in the metacognitive behaviours exhibited by the subjects (prospective teachers) as they performed the test tasks. A general trend was found. The calculated value of correlation coefficient between the amount of changes in metacognition scores and that of problem-solving performance scores (r=0.46), indicates a positive relationship between the level of improvements in the two aspects of problem solving: cognitive and metacognitive.

Beliefs

Means and standard deviations of scores on each scale of the beliefs survey were calculated for both pre- and post-instruction measures. Paired sample t-tests were conducted to measure changes in subjects' beliefs about mathematical problem solving from the start to the end of the instruction. Table 3 contains these results.

Table 3

Means, standard deviations, and paired sample t-tests for differences in beliefs' scores between pre- and post- instruction measures

Scale	pre-measure		post-measure		t-value	p<
	mean	st.dev.	mean	st.dev.		
Nature of problem solving (M.S =15)	9.75	1.21	12.55	1.1	7.35	0.001
Successful problem solving (M.S= 25)	14.05	1.67	20.6	2.6	13.1	0.001
Promoting problem-solving performance (M.S=20)	11	1.08	15.8	1.5	13.9	0.001
Total (M.S= 60)	34	3.07	48.7	4.6	14.2	0.001

*M.S is the maximum score

As shown in the table, all differences between the corresponding pairs of means on each of the beliefs' scales achieve statistical significance. These results indicate that prospective teachers demonstrated significant improvement in their conceptions of the nature of problem solving. They became more aware of the attribution of causes of successful problem solving, and more knowledgeable of the ways to promote problem- solving performance.

Discussions and Conclusions

This study aimed at examining the effectiveness of an instructional program designed to enhance prospective teachers' metacognitive behaviour in mathematical problem solving. The instructional program proposed and explored in this study sought to introduce direct and systematic instruction of metacognitive behaviours in the course of problem solving. Each of the three phases involved in this program has its distinctive impact on enhancing metacognitive behaviours in mathematical problem solving. The first phase presents explicit knowledge of, and training on, the use of different problem-solving processes, both cognitive and metacognitive. In the second phase, prospective teachers engage in mathematical problem solving during which they are to develop metacognitive skills for themselves. They have the opportunity to recognize and employ a full range of metacognitive strategies and skills. Assuming the role of a teacher, in the third phase, gives prospective teachers an opportunity to reflect upon, and become more aware of what has been learned. They are to practice oral and written communication.

Based on the data gathered in this study, ***several conclusions can be drawn***.

First, metacognitive-base instruction does have a positive effects on the ability of prospective teachers to use metacognitive strategies and skills in their problem-solving activities. The prospective teachers were very poor at their problem-solving behaviour. It was particularly disturbing what was observed at the pilot study, that conducted prior to this study, and at the start of the instruction. Prospective teachers were so deficient not only in their regulatory skills of monitoring and assessing, but also in their overall problem-solving performance. As the program was progressing, they began to incorporate metacognitive decisions in their problem-solving behaviours. They, gradually, demonstrated greater increase in the use of control and self- regulating strategies and skills. By the end of the instruction, they exhibited greater use of behaviours such as the following :

- discovering the main idea in the problem question,
- representing information in different forms,
- thinking through the meaning of the question before beginning to solve a problem,
- expressing awareness of what was to be done, and how?,
- assessing appropriateness of a strategy,
- thinking of different ways to solve a problem,
- monitoring progress toward the goal, and
- showing stronger goal-awareness, and goal-modification behaviours.

Informal observation showed also that prospective teachers learned to become clinical interviewers themselves. They learned how to listen to each others, ask helpful and clarifying questions as well as how to facilitate oral and written communication.

Second. A significant improvement in prospective teachers' problem-solving performance was observed. indicating that they became more able to solve mathematical problems and employ successful cognitive strategies. The results indicate also a positive relationship between the tendency to use metacognitive strategies and skills, and the level of improvements in prospective teachers' overall problem-solving performance.

Third. The data suggest that the metacognitive-based instruction has a positive influence on beliefs about mathematical problem solving held by prospective teachers. The significant gains of the beliefs' scales suggest that the increased use of metacognitive behaviours in problem solving can be fruitful in terms of the improvement of prospective teachers' beliefs about problem solving.

To summarise, the results of this study indicate that the metacognitive-based instructional program helped prospective teachers increase their awareness and control of thinking while adopting more positive beliefs about mathematical problem solving.

Concluding remarks

In preservice teacher education, preparing prospective teachers to teach with a problem solving focus has become a challenge that should be faced. Innovation in problem solving does not just happen. Teachers must possess sufficient knowledge and skills that enable them to play their prospective role. Unfortunately, mathematics instruction at colleges of education focuses almost exclusively on mastery of mathematical content knowledge rather than mathematical thinking or problem solving. It is undeniable that problem solving requires specialized knowledge and skills; among which, these related to metacognition. Teacher educators should not expect that student-teachers would be able to develop many of these knowledge and skills for themselves.

The findings of this study suggest that explicit attention to the enhancement of metacognitive behaviour in problem solving should be given in the preservice and in-service education of mathematics teachers. There should be specific programs, similar to the one proposed by this study, that provide direct instruction and training on various cognitive and metacognitive aspects of mathematical behaviour. Such instruction should be also carried over into all areas of mathematics teaching at colleges of education. The current pre-service programs should be investigated with respect to their effectiveness in affecting prospective math teachers' beliefs and (mathematical) thinking.

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