MATHEMATICAL GRAPHS OBTAINED FROM EXPERIMENTAL SCIENCES

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Introduction

Habitually the students of scientific and technological areas study contents relative to mathematics that are after applied to other subjects. Nevertheless, from the historical perspective of the science development, most mathematical elements employed nowadays appeared due to the necessity of solving and justifying -through a mathematical language- problems that were in origin focussed on an inductive way.
In relation to experimental sciences, such as physics or chemistry, there are many principles that must be expressed through mathematical equations. This fact very often constitutes a great difficulty for the students, mainly during secondary school, when they have to learn those disciplines. They are obliged to study subjects not easy to understand and that, in addition, have many mathematical developments. Although the students have attended maths courses, normally the mathematical contents are taught in an abstract way, pure mathematics being very distant from applied mathematics. So, they are not very interested in mathematics, the result is boring and not easy, and they have even more difficulties in understanding those other scientific subjects.
Thus, it is important to find out ways to improve the effectiveness of mathematics education. To introduce mathematical content through some specific problems of physics or chemistry could be one of these ways. With this procedure two aims can be reached. On the one hand, mathematics will be seen by the students to be closer to reality. On the other hand, when students learn chemistry or physics, they will be able to understand better the behaviour of the matter. In sum, this approach can be an effective aid to facilitate both maths and experimental sciences learning.

Objective

The objective of this paper is to analyse the advantages, from a didactic viewpoint, of using examples of interdisciplinary character that could help the students to reach a better comprehension of many mathematical concepts.

Work proposal

In physical chemistry there are many chapters where mathematical aspects are involved. According to our purpose, one of the most interesting cases is the construction of graphs that correspond to mathematical equations. This problem offers to the students the opportunity of finding out by themselves how certain functions come out in a natural way, by analysing the relationship between two variables and by constructing the corresponding graph.

The topic of the properties of gases was employed with this aim, because the laws that represent the relationships between the different variables defining the state of a gas (volume, pressure, temperature) are expressed in terms of equations. In consequence, some types of mathematical graphs -straight line and equilateral hyperbola- can be introduced by means of studying the behaviour of gases.

Incorporation of informatic media:

This procedure involves in many cases a disadvantage: that the time required is too long. But this time can be reduced by incorporating informatics, that also contribute to increase the interaction and, in turn, the self-learning of the students.
The new technologies become a powerful teaching aid in science education. They help the students to better understand these disciplines by means of allowing them to do a more interactive study. In addition, they are an important motivating element, enhancing the interest of the students in their learning.
In this way, the incorporation of mathematical assistants in sciences teaching is very effective. One of the most remarkable applications is to construct graphs. A mathematical assistant allows the students to plot curves by themselves in an easy, rapid and attractive way. Then, they will stop being a mere passive observer of graphs in a book, in accordance with the meaning learning theories. On the contrary, they will participate in a more personal and active learning process.
The mathematical assistant employed in our study is DERIVE. The reasons for this option are various: to work with it is very easy, it has a very wide range of applications and the requirements of hardware are very simple.

In sum, in this paper we discuss the description of some examples, in which graphs corresponding to physical-chemistry problems are constructed with the help of a mathematical assistant, DERIVE, the topic being selected that of gas properties for that purpose.

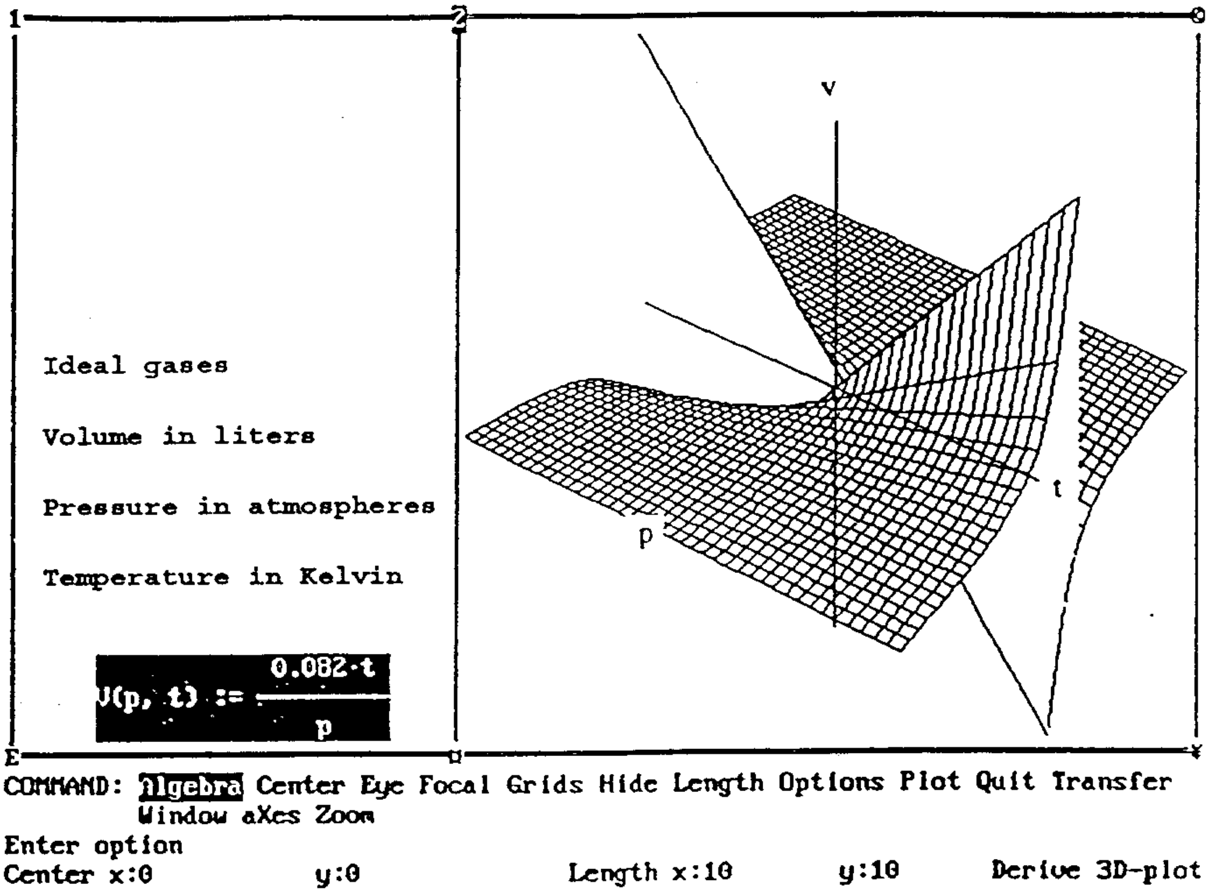

Figure 1.- Three-dimensional representation of the ideal-gas equation with DERIVE..

Results and discussion

Why was the subject of gas properties selected? First, because it belongs to a basic chemistry of general concepts. Second, because it very clearly shows the relationship between a "real" behaviour and the formulation of mathematical equations to express this behaviour. That is to say, between experimental phenomena and theory.

Thus, in this paper we will present the construction of the following graphs:

- equilateral hyperbola, corresponding to Boyle´s law.
- straight line, corresponding to Charles´ law.

Both laws are formulated by mathematical equations that were obtained through an experimental study. After analysing a great amount of experiments, a regularity in the dependence of volume upon pressure (Boyle´s law) and of volume upon temperature (Charles´law) was always observed. That led to the expression of each regularity by means of a mathematical equation: that is to say, the formulation of the corresponding laws.

Besides, other mathematical contents could also be discussed through the gases study. For instance, the concepts of first and second derivatives, as well as those of maximum, minimum values and inflection point.

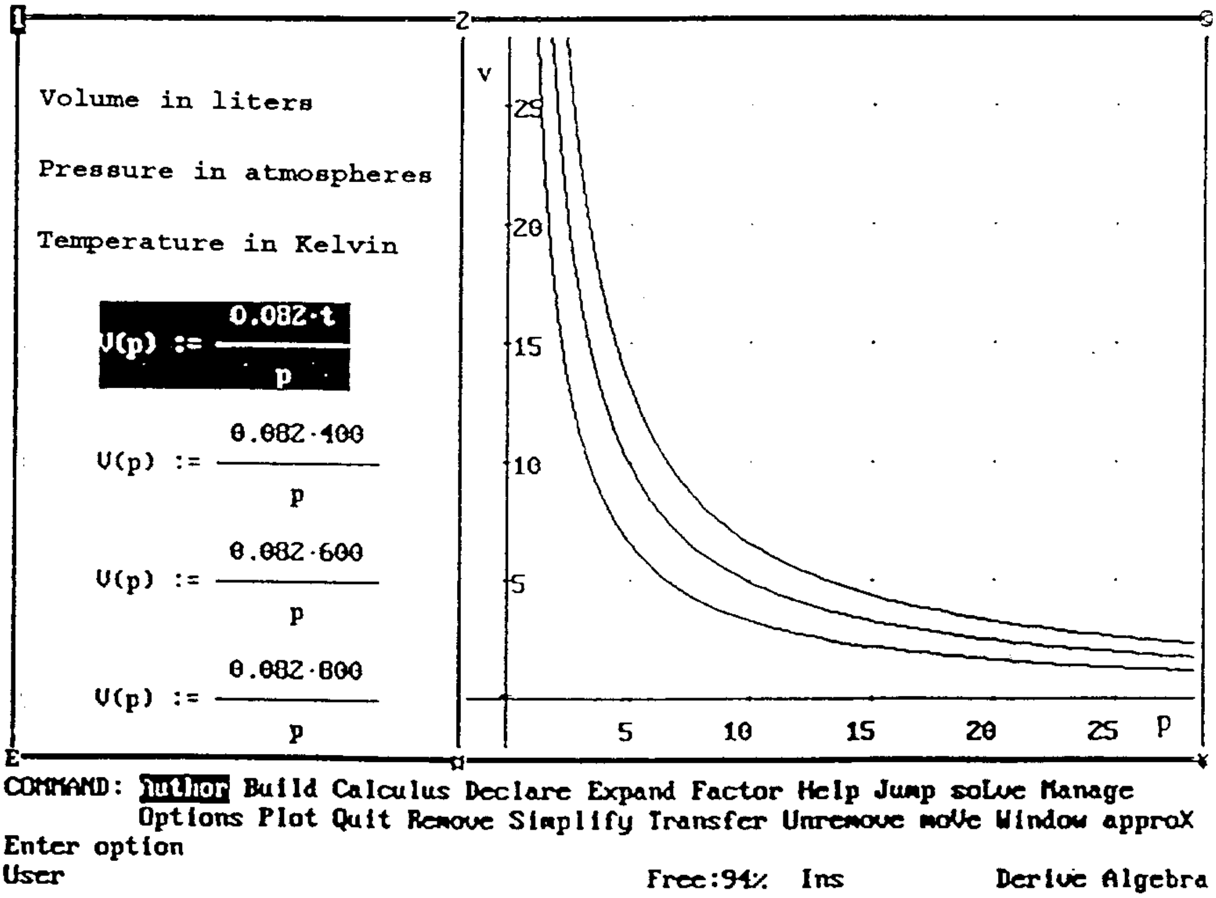

Figure 2.- Graph of Boyle´s law obtained with DERIVE.

Procedure:

The equations that define the gas state are functions of two independent variables. So, the representation of these equations requires a three-dimensional space, being not easy at all to build this graph without the help of a mathematical assistant. By means of employing DERIVE this work becomes very simple. Thus, in order to represent the ideal-gas law (Fig. 1), two windows have been opened. The equation to be plotted is written in the left window, as well as the units of each variable. In the right window the corresponding graph appears, in which the volume is function of both independent variables, pressure and temperature.

The two types of curves that will be studied here –hyperbola and straight line- are easily induced from the ideal-gas law graph. By fixing the temperature, the volume is a function of the pressure: the volume of a gas is inversely proportional to the pressure, when t=constant (*Boyle´s law*). According to this, for a determined value of temperature an equilateral hyperbola is obtained and for different values of temperature a group of "isotherms" appears (Fig. 2).

On the other hand, the volume is a function of the temperature when the pressure is fixed: when pressure is constant, the volume of a gas is directly proportional to temperature (*Charles´ law*), and so a straight line is plotted. Three "isobars" are represented in Fig. 3, corresponding to three fixed values of pressure.

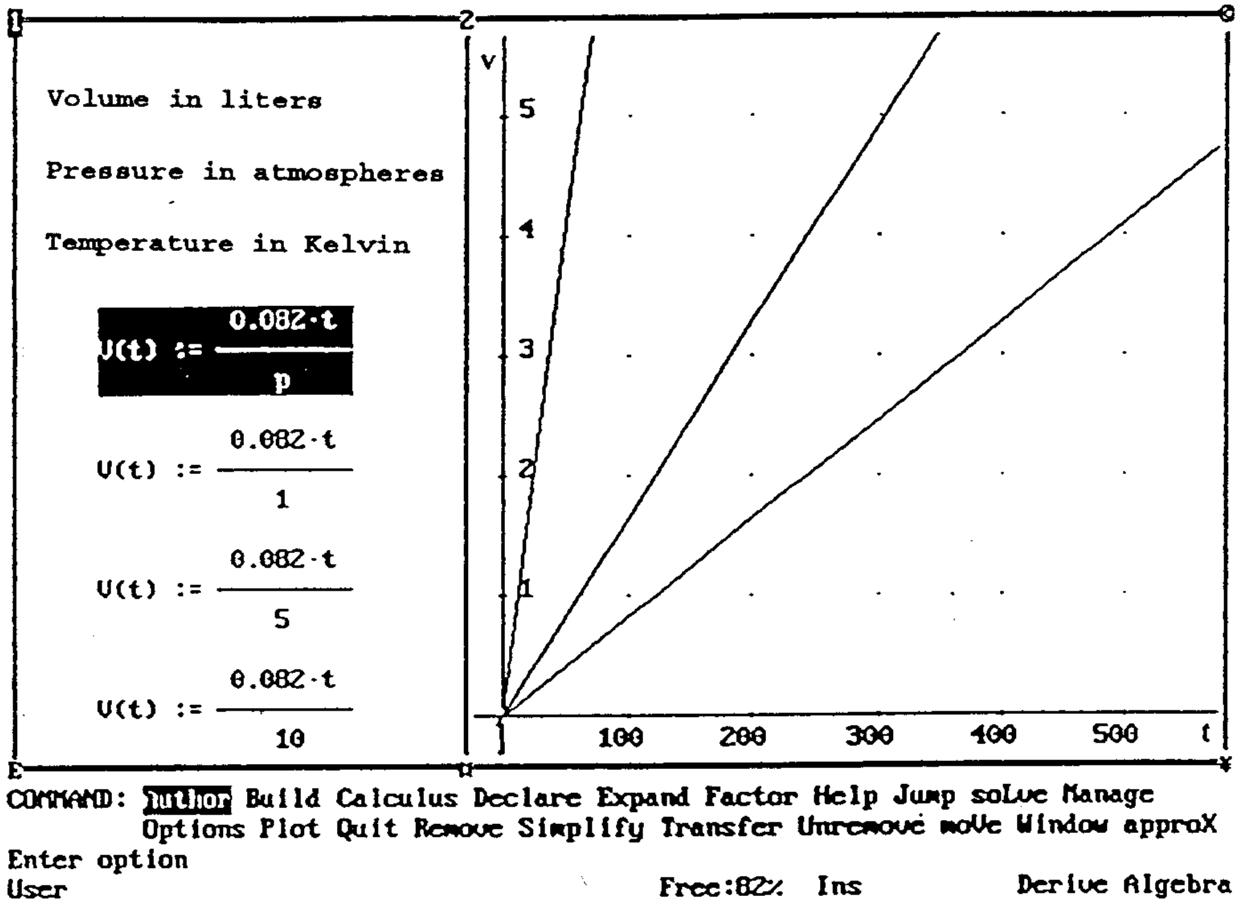

Figure 3.- Graph of Charles´law obtained with DERIVE.

In the same way, the state of a "real" gas can be represented, starting from another equation, such as the Van der Waals equation:

$$(p+a/v^2)\,(v-b) = RT$$

where **a** and **b** are two constants whose values depend upon the particular gas. In the case of CO_2, the values of **a** and **b** are 3.6 and 0.0428, respectively. To represent this equation graphically without any help is much more complicated than in the case of an ideal gas. However, this task becomes very simple by using DERIVE. As an example, three isotherms are plotted for three values of temperature (Fig. 4).

These three isotherms have a different shape. The one at 320 K is very similar to the isotherms of an ideal gas, whereas the isotherm at 280 K shows a maximum and a minimum point and the one at 300 K is almost an intermediate case between the others. In order to know the maximum and the minimum it is necessary to calculate the first and the second derivatives. In the same way, the temperature of the isotherm that has an inflection point with a horizontal slope can also be determined (the first and the second derivatives become zero). This temperature, called *critical temperature*, is very important: a gas above its critical temperature cannot be liquefied no matter how high a pressure is applied.

To do all these operations is rather complicated, but the students can do them very easily with DERIVE. They will find out by themselves the values of the maximum, the minimum and the inflection point and realize the significance of all these mathematical concepts.

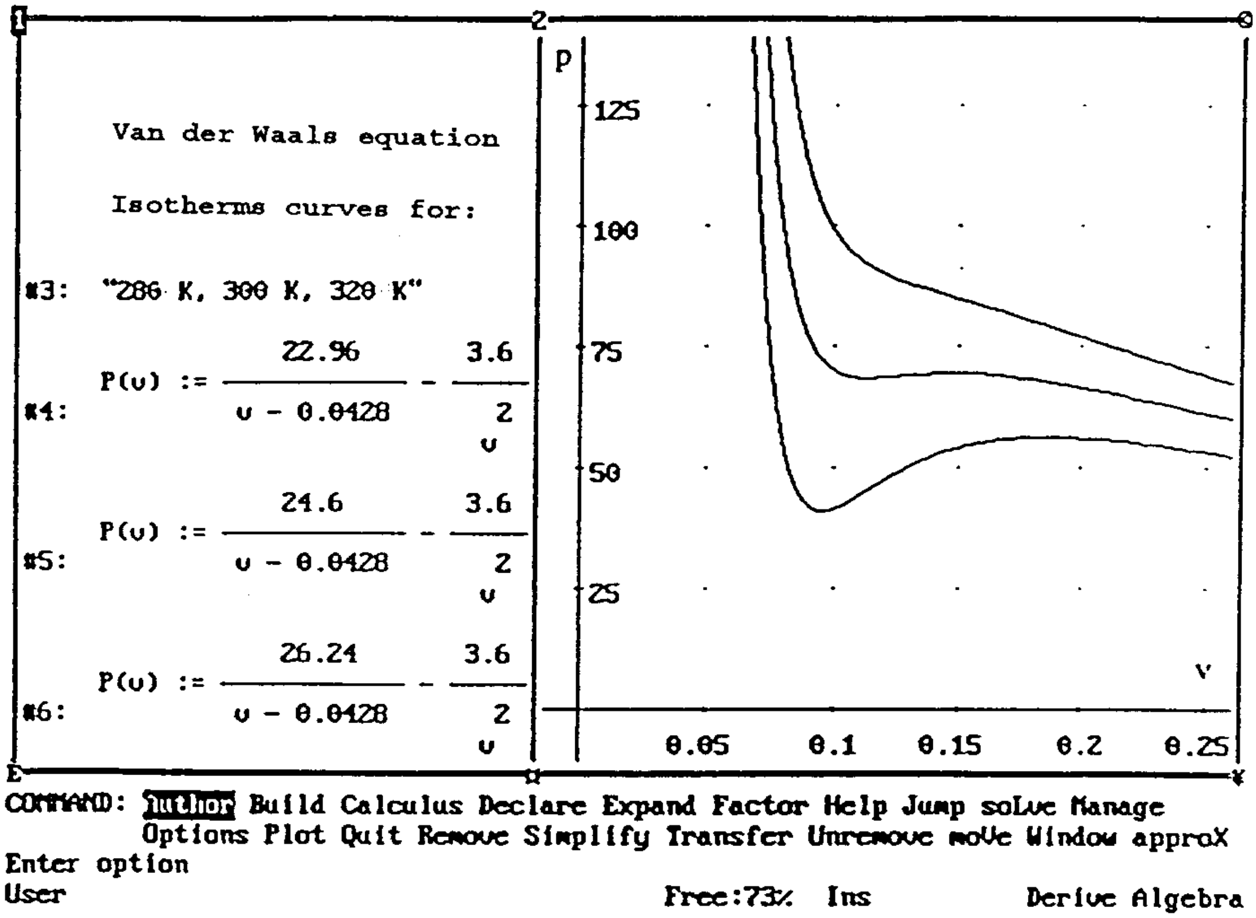

Figure 4.- Isotherms of CO_2 from the Van der Waals equation, obtained with DERIVE.

Conclusions

These examples can familiarize the students more with the mathematical significance of simple graphs and maths become closer to our environment. Thus, an improvement in mathematics education can be reached.

Besides, this teaching methodology allows us to show students that:

- Sciences have an interdiciplinary character: mathematics, chemistry and physics have a deep relationship.
- The matter has a behaviour that very often can be synthesized in terms of mathematical formulas.
- Informatics is also interrelated with other sciences and constitutes a powerful help for calculus.

With all that a more active and personal learning will be reached and the interest of students for sciences can be increased.

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