Consistency Between Student-teachers' Conceptions of Mathematics and their Teaching Performance

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Introduction:

The NCTM's Curriculum and Evaluation Standards for School Mathematics (*NCTM, 1989*) tells us that mathematics education practices have to be informed by a vision of what mathematics is; that is a conception of mathematics. A mathematics education practitioner, (e.g., a teacher or curriculum developer), has to have a conception of mathematics. In the mathematics education literature there are a number of such visions or conceptions.

Among these visions is a one that looks at mathematics as a structural integrated whole (*Coxford, 1995*) that encompasses various forms of content (concepts, principles, generalizations, …, etc.) together with the net of relationships interconnecting them. In other words, such a conception of mathematics is based on a vision of what-mathematics-is that goes far beyond content forms to include how such forms are interconnected and interwoven into an integrated structure. That is, mathematics is both content and structure (*Al-Mughirah, 1989 & Hodgson,1995*).

In contrast to the above vision, there is the limited modest conception that looks at mathematics as just a collection of concepts, generalizations, rules and algorithms (*Dean, 1982*). In other words, if the above vision of what-mathematics-is encompasses both content and structure, this vision reduces mathematics only to the content side of the equation ignoring the other side.

In another conception of mathematics, others look at it as a language that has its own vocabulary and syntax (*Rubenstein, 1996, Usiskin, 1996, Vergnaud, 1997 & Krussel, 1998*). Vocabulary here refers to concept names, symbols mathematicians associate with certain mathematical meanings, and figures that we use to express mathematical ideas. Syntax means rules that govern the use of such vocabulary.

Available in the literature, there is a fourth vision of mathematics that looks at it from a different angle. In this vision, mathematics is seen as a way of modeling reality. A mathematical model models a physical phenomenon from which it was initially extracted. However, though such models were initially derived from the physical realm of reality, to it they go back to be applied to more real phenomena similar in their contexts and givens to those from which the models were initially derived (*Kilmister, 1972, Hawkins, 1973, Skemp, 1977 & Abreu, et al, 1997*). Accordingly, mathematical models model and patternize the real world; mathematics "mathematizes" reality (*Freudenthal, 1971*). It may be useful here to give Newton's famous apple as an example. Newton's law of vertical motion under the effect of gravity came initially to model the motion of his apple. However, this law (or model) is well applicable to any other object moving vertically under the effect of gravity. Accordingly, mathematics is a server of other disciplines. Or, rather, it is a tool whereby other disciplines can be simplified and studied. For example, mathematics is used to derive many rules and laws in so many other areas like physics, chemistry, … etc.

Finally, among these conceptions also there is a fifth vision that looks at mathematics as a way of thinking and inquiry (*Shulman, 1970 Dean, 1982, Inder, 1982 & Al-Mughirah, 1989*). That is the way a mathematician approaches and manipulates phenomena regardless of the mode of thinking (logical, inductive, deductive, ..., etc.) his cognitive activity goes along.

Now, It would be reasonable here to question: How are such conceptions being held? For example, are these conceptions of mathematics necessarily held in ones? That is, does a teacher (or student-teacher) necessarily espouse only one of these visions or conceptions? Or, might he hold one, some, or all of these visions? For example, might someone hold the vision of mathematics as a language, but, at the same time espouses its conception as a tool for modeling physical phenomena and studying other disciplines? And, might another person hold a vision of mathematics that looks at it only as a tool of communicating (or a language)?

All these possible conceptions of mathematics were in mind as the researcher set off to give a lecture about the nature of mathematics. The audience was a group of primary mathematics student-teachers who were then at the fourth year of their teacher education program at Kafr El-Sheikh College of Education. The researcher began his lecture with this question: what is mathematics? Following that, the researcher stood in front of his audience listening to their responses to that question. Their answers did not go far beyond the limited modest traditional view of mathematics as a collection of rules and algorithms. One of the student-teachers said that mathematics is "laws and theorems we use to solve problems". Another student said that "mathematics is rules, laws, problems and exercises". As it was not possible, from the practical point of view, to go on listening to students' answers to that question during the lecture, the researcher asked his audience to write their answers (in about two lines) on a piece of paper each. After the lecture, the researcher read the student-teachers' answers to find out that they were almost similar to what their colleagues said during the lecture. Until that point, it was not more than a lecturer's curiosity. I only wanted to know about my students' conceptions of some construct related, in some way or another, to the theme of my lecture; students' repertoire has to be the starting point both for correcting any misconceptions they might have and for crystallize more concepts and conceptions.

The researcher's observations during his regular visits to those student-teachers in the mathematics dasses where they have their teaching practice integrate with all that has been pointed out above. During these visits it was noted, for example, that many of the student-teachers were not keen to use the language of mathematics correctly. And, perhaps accordingly, they accepted such incorrect use from their pupils not only in the oral communication about mathematics but also in the mathematical writing activities. All these observations offered the researcher a reasonable rationale to question such lack of keenness. Is it an indication that student-teachers are not aware of the conception of mathematics as a language? Or, are they aware of, but do not believe in, such a conception?

The researcher noted (during the above mentioned visits) also that, a considerable number of student-teachers emphasize products not processes, answers not intellectual mechanisms by which answers were obtained, procedures and algorithms not rationale and logic underlying their steps. That is to say, content was not taken as a medium for teaching thinking. Rather, content was the sole end aimed at. Again, that was a reasonable reason for the researcher to question these instructional practices on the part of student-teachers. Do such practices have something to do with a deficient conception of mathematics as a way of thinking?

Now, based on all that has been mentioned, noted, questioned or pointed out in the proceeding paragraphs, the question is: how do such visions or conceptions affect the way student-teachers teach mathematics in their classes? Does the conception of mathematics a student-teacher holds coincide with his practices inside the classroom during the math lessons? And, in what way? That is what the present study is exploring.

Research Questions:

In the light of what has been pointed out above, the following research questions were formulated:

1) What conceptions of mathematics do student-teachers hold?
2) How are such conceptions being held?
3) To what extent do such conceptions coincide with student-teachers' practices during math lessons?
4) How could the state of the art in this regard be a determiner of our future interventions aiming at these conceptions?

Sample:

The sample for this study was 71 primary mathematics student-teachers in the final year of their four-year teacher education program at Kafr El-Sheikh College of Education. Those prospective teachers had their teaching practice at 16 primary schools in Kafr El-Sheikh Governorate equally distributed between rural and urban communities. Student-teachers were almost equally distributed in terms of sex. Table (1) specifies details of the study sample of prospective teachers.

Table (1): sample of study

Sex of student- teachers	Number of schools	Number of student-teachers		Total number of student-teachers
		Urban areas	Rural areas	
Male	8	21	12	33
Female	8	13	25	38
Total	16	34	37	71

Study Variables:

1) Conception of mathematics:

The word conception here refers to a person's general mental structures that include knowledge, beliefs, views, preferences and understandings (*Wilson, et al, 1998*). By conception of mathematics in this study we mean the vision or view held by a prospective teacher regarding what mathematics is. And, it will be quantified by "a conception profile" presenting the scores a subject gets on the five components of the CMS.

2) Teaching performance:

By this variable we here mean how a student-teacher teaches mathematics in the teaching practice arranged for him during his teacher education program. In this study such performance will be quantified by a performance profile presenting the scores a subject gets on the five components of the TPOS.

Procedure:

1) Developing the Research Tools:

Two tools were prepared for the present study. In what follows, the development process for each of the two tools is described.

a) Conception of Mathematics Scale (CMS):

This scale was developed as follows:

- For each of the five conceptions considered in the study five items were written. Each of these items represented a possible answer to the question: "what is mathematics?" Subjects were to respond to these items on a five-level Likert-type scale extending from 0 (strongly disagree) to 4 (strongly agree).
- To secure a satisfactory level of validity for the scale, five referees were asked to give their opinions regarding the validity of each item as an indicator of the conception of mathematics it was intended to assess. Based on their suggestions, all the necessary modifications were made. Reliability of the scale was established using a pilot sample of 35 student-teachers. The reliability coefficient for each of the five dimensions of the scale was calculated. A reliability coefficient mean value of 0.72 was secured.

b) Teaching Performance Observation Sheet (TPOS):

This tool was prepared in the following way:

- For each of the five mathematics conceptions under consideration in the present study, five instructional behaviors were written to be the teaching skills implied by that conception. The student-teacher's performance level on each of these behaviors (skills) was to be rated on a Likert-type five-level scale ranging from 0 (very poor) to 4 (very good).
- Then, to secure the tool's validity, the same five referees who participated in establishing validity of the CMS were asked to give their views as to whether the items coincide with the dimensions they came to assess. Modifications suggested by those referees were made. The sheet's reliability was then established using the inter-rater agreement coefficient (*Fleiss*, *1981*). Three raters, including the researcher, were involved in this process. Based on the data gathered for this purpose a reliability coefficient of 0.79 was obtained.

2) Data Gathering:

Data were gathered using the research tools, CMS and TPOS, as follows:

- At the beginning, the CMS was administered to all the student-teachers involved in the study.
- For each of the 71 student-teachers two visits were made by the researcher. In each visit the researcher observed a mathematics lesson using the TPOS to rate the student-teacher's performance on each of the teaching skills under consideration.
- Student-teachers' responses on the two tools were then scored yielding two profiles for each student-teacher. One of these profiles was for the student-teacher's conception of mathematics and the other was for his teaching performance. A "conception of mathematics profile" looked like this:

	Conceptions of Mathematics				
	Math as a collection of concepts, generalizations, rules and algorithms	**Math as a tool for modeling reality and studying other disciplines**	**Math as a way of thinking**	**Math as a language**	**Math as both content and structure**
Scores					

In the above profile the score under each conception expresses the strength of holding that conception by the student-teacher for whom the profile is compiled. A student-teacher's "teaching performance profile" looked like this:

	Group of teaching skills implied by the conception of mathematics as				
	a collection of concepts, generalizations, rules, and algorithms	a tool for modeling reality and studying other disciplines	a way of thinking	a language	both content and structure
Scores					

In the above profile, each score expresses the performance level of a given student-teacher on the corresponding group of skills. Each of these scores is the mean of the two scores a student-teacher had on the TOPS for the two lessons in which he was observed by the researcher.

Results:

This section presents the study results under three headings, each of which corresponds to one of the first three research questions. In the last section of this report a discussion of the study results for the first three research questions will follow in an attempt to answer the fourth research question concerning future implications of these results.

Table(2): Results of using T-Test (for paired samples) to examine differences between student-teachers' mean scores on the five components of the CMS.

Conceptions of Mathematics		Math as a collection of concepts, generalizations, rules, and algorithms	Math as a tool for modeling reality and studying other disciplines	Math as a way of thinking	Math as a language	Math as both content and structure
	Statistics	MS = 13.746 SD = 4.335 N = 71	MS = 7.408 SD = 2.088 N = 71	MS = 6.254 SD = 3.354 N = 71	MS = 5.901 SD = 2.889 N = 71	MS = 4.859 SD = 3.583 N = 71
Math as a collection of concepts, generalizations, rules and algorithms	MS = 13.746 SD = 4.335 N = 71	_	_	_	_	_
Math as a tool for modeling reality and studying other disciplines	MS = 7.408 SD = 2.088 N = 71	T = 10.81 P = 0.000	_	_	_	_
Math as a way of thinking	MS = 6.254 SD = 3.354 N = 71	T = 11.20 P = 0.000	T = 2.60 P = 0.011	_	_	_
Math as a language	MS = 5.901 SD = 2.889 N = 71	T =11.65 P = 0.000	T = 3.43 P = 0.001	T = 0.70 P = 0.486	_	_
Math as both content and structure	MS = 4.859 SD = 3.385 N = 71	T = 11.84 P = 0.000	T = 5.07 P = 0.000	T = 2.51 P = 0.014	T = 2.45 P = 0.017	_

MS = Mean score; SD = Standard deviation; N = Number of observations; T = T-value; P = 2-tail sig. level of T.

1) Conceptions Held by Student-Teachers:

This research question is concerned with identifying the conceptions of mathematics that are being espoused by the primary mathematics student-teachers. The question states that: "What conceptions of mathematics do student-teachers hold?"

To answer this question, mean score for each of the math conceptions was calculated using the conception profiles compiled from student-teachers' scores on the CMS. Table (2) shows these mean scores together with the results of using T-Test for paired samples to examine the significance of differences between them.

Results in table (2) indicate that though all the five conceptions under consideration in this study are being espoused by the student-teachers involved, these conceptions are not equally strongly held. The most strongly held conception is the narrow view about the nature of mathematics as a collection of concepts, generalizations, rules and algorithms. The most weakly held conception is that of mathematics as both content and structure. However, it seems that the two conceptions of mathematics as a way of thinking and as a language are equally strongly held by the study sample of student-teachers; the difference between the mean scores for these two conceptions is not significant (T = 0.70 with P = 0.484). The conceptions are arranged, in descending order according to the strength of holding, as shown in both the first row and the first column in table (2).

2) How Conceptions Are Being Held:

The second research question of the present study is concerned with how the conceptions of mathematics are being held by the student-teachers involved in the study. The question as specified earlier in the research questions states that: "How are such conceptions being held?".

To answer this question using the data gathered, the student-teachers' conceptions of mathematics profiles were analyzed. A count of the numbers of student-teachers holding only one, two, three, four and all five conceptions of mathematics was undertaken. Table (3) presents the results of that count and the corresponding percentages.

Table (3): Numbers and percentages of student-teachers with the associated numbers of conceptions held

Classes	Numbers	Percentages
Student-teachers holding no conception at all	**0**	**00**
Student-teachers holding only one conception	**7**	**10**
Student-teachers holding only two conceptions	**9**	**13**
Student-teachers holding only three conceptions	**26**	**36**
Student-teachers holding only four conceptions	**17**	**24**
Student-teachers holding all five conceptions at the same time	**12**	**17**
Total	**71**	**100**

* **In this table a student-teacher is considered to be holding a given conception if his score on that conception is larger than 0.**

The results in table (3) indicate that all student-teachers do have a conception of mathematics. And, such conceptions are not necessarily held in ones; a student-teacher might espouse more than one conception at the same time though a small proportion of student-teachers (7 out of 71) hold only one conception. In fact, the largest percentage of the student teachers involved (36%, i.e. 26 out of 71) held three conceptions at the same time. The percentage of students holding all five conceptions at the same time is 17% (12 out of 71). However, as the results of the first research question presented earlier show, these conceptions are not equally strongly held.

3) Consistency Between Conceptions and Instructional Performance:

This is the main research question informing this study. It is concerned with consistency between student-teachers' conceptions of mathematics and their instructional performance. The question as specified earlier states that: "To what extent do such conceptions coincide with the student-teachers' practices during math lessons?"

To answer this question, the student-teachers' conceptions of mathematics profiles were matched to their teaching performance profiles. Then, the correlation coefficients between scores on the dimensions of the CMS and scores on their corresponding dimensions of the TPOS were calculated. Table (4) presents the correlation coefficients' matrix. Each correlation value in the matrix is accompanied by its significance level.

Table (4): Correlation coefficients between student-teachers' scores on dimensions of the CMS and on the corresponding dimensions of the TPOS.

Dimensions of the CMS / **Dimensions of the TPOS** (teaching skills implied by the conception of math as …)	**Math as a collection of concepts, , generalizations, rules, and algorithms**	**Math as a tool for modeling reality and studying other disciplines**	**Math as a way of thinking**	**Math as a language**	**Math as both content and structure**
a collection of concepts, generalizations, rules and algorithms	r = 0.430 p = 0.000	r = - 0.046 p = 0.703	r = 0.036 p = 765	r = 0.067 p =0.575	r = - 0.085 p = 0.479
a tool for modeling reality and studying other disciplines	r = 0.299 p = 0.011	r = 0.541 p = 0.000	r = 0.145 p =0.226	r = -0.128 p = 0.286	r = - 0.065 p = 0.590
a way of thinking	r = - 0.169 p = 0.159	r = 0.089 p = 0.457	r = 0.940 p =0.000	r = 0.072 p =0.546	r = 0.122 p = 0.311
a language	r = - 0.204 p = 0.086	r = - 0.034 p = 0.773	r = 0.231 p =0.052	r = 0.750 p =0.000	r = 0.270 p = 0.022
both content and structure	r = 0.042 p = 0.725	r = - 0.117 p = 0.333	r = 0.110 p =0.362	r = 0.161 p =0.182	r = 0.447 p = 0.000

r = Correlation coefficient
p = Significance level of r

Results presented in table (4) indicate that the correlation value laying in each of the diagonal cells is the largest both in its row and column. And, all these diagonal correlations are positive and statistically significant at the 0.000 level; other correlations in that table are either weak or insignificant. These results imply that scores on a given conception of mathematics increase as scores on the corresponding group of teaching skills increase and vice versa. Accordingly, one can deduce that the stronger a student-teacher holds a given conception of mathematics, the more competently he will perform the instructional skills implied by that conception. Within these limits, to conclude, a student-teacher's instructional performance coincides with his conception of mathematics.

Possible Implications - Looking Ahead:

The present study explored a number of issues concerning student-teachers' conceptions of mathematics. One of these issues was the question of future implications results of this study may have regarding our possible future interventions aiming at such conceptions. To this question we no turn, but before that let us first have a quick look at the results. The study results, to sum up, indicated that:

- Every student-teacher does have a conception of mathematics. Such conceptions range from a narrow limited modest view of mathematics as a collection of concepts, generalizations rules and algorithms, to a more elaborate conception such as the one that looks at mathematics as a language. However, these conceptions are not equally strongly held.
- Such conceptions are not being held on a single basis; a student-teacher might hold more than one single conception at the same time. In fact, only 10% of the student teachers hold a single conception and, accordingly, 90% hold more than one conception at the same time. And, about 17% of the student-teacher hold all the five conceptions under consideration.
- A student-teacher's conception of "what-mathematics-is", is generally consistent with his instructional performance. That is, the stronger a student-teacher adheres to a specific conception of mathematics, the more competent he will be in the teaching skills implied by that conception.

These results corroborate those of previous studies (*Thompson, A., 1992, Benken, et al, 1996 & Stein, et al, 1990*) to suggest that teachers' conceptions of mathematics have a considerable impact on how they teach. Moreover, findings of the present study elaborate and extend previous findings by providing specific examples of the conceptions being held and how they might influence instruction. For example, in this study, student-teachers who hold a conception of mathematics as a language emphasized their students' mathematical communication skills. Not only this, but also that those student-teachers were more competent in the teaching skills that may contribute to the promotion of such communication skills.

Now, based on these results and assuming that such conceptions could be modified through convenient significant interventions (*Lappan, , et al, 1988 & Benken, , et al, 1996*), it seems reasonable to suggest that programs should be developed to reveal and enrich student-teachers' conceptions of mathematics. Enrichment of the espoused conceptions and modification of misconceptions may result in improved instructional performance. This seems understandable in view of what the present study results indicated concerning the consistency that exists between such conceptions and instructional performance. A reasonable rationale for this may be that, an environment dominated by elaborate broad rich views of mathematics may broaden the range of mathematical outcomes as recommended in recent calls for reform in mathematics education (*Cooney, et al, 1998*). And, accordingly, activities would be provided in mathematics teaching to promote such outcomes. Such enrichment programs may be arranged as part of the teacher education programs within the methods courses. However, this may need some more studies aiming at planning such programs and at examining their effectiveness.

As far as teachers' in service are concerned, there seems to be a need for more studies in this regard. Such studies should be undertaken to:

- reveal the conceptions of mathematics espoused by those teachers and how such conceptions are being espoused.
- explore how to broaden and enrich their existing conceptions.
- examine the influence of such enrichment on teachers' instructional performance.

The results of all such studies may, then, be translated into in-service teacher education programs aiming at such conceptions.

References:

(1) *Abreu,G., Bishop, A. & Pompeu, G. (1997), "What children and Teachers Count as Mathematics". In Nunes, T. & Bryant, P. (Eds.), Learning and Teaching Mathematics: An International Perspective, Psychology Press, Hove, East Sussex, UK, pp. 233-263.*

(2) *Al-Mughirah, A. (1989), Methods of Teaching Mathematics, Library Affairs Deanery, King Saud University, Al-Riyad, Saudia Arabia, p. 21. (In Arabic).*

(3) *Benken, B. & Wilson, M. (1996), "Developing Problem Solving Conceptions of Mathematics: A Preservice Teacher's Experiences", ERIC Document Reproduction Service No. ED 401 133, ERIC Clearinhouse for Science, Mathematics and Environmental Education, Columbus, OH, USA.*

(4) *Connell, M. (1992), "How do They Know?: An Investigation into Student Mathematical Conceptions and Beliefs", ERIC Document Reproduction Service No. ED 346 143, ERIC Clearinghouse for Science, Mathematics and Environmental Education, Columbus, OH, USA.*

(5) *Cooney, T., Shealy, B. & Arvold, B. (1998), "Conceptualizing Belief Structures of Preservice Secondary Mathematics Teachers", Journal for Research in Mathematics Education, Vol. 29, p. 306.*

(6) *Coxford, A.F. (1995), "The Case for Connections". In House, P.A. & Coxford, A.F. (Eds.), Connecting Mathematics Across the Curriculum, the 1995 Yearbook of the National Council of Teachers of Mathematics, The Council, Reston, Va., USA, pp. 3-13.*

(7) *Dean, P.G. (1982), Teaching and Learning Mathematics, Woburn Press, London, England, pp. 97-100.*

(8) *Fleiss, J. (1981), Statistical Methods for Rates and Proportions, 2nd Edition, John Wiley, NY, USA, p. 212.*

(9) *Freudenthal, H. (1971), "Geometry Between the Devil and the Deep Sea", Educational Studies in Mathematics, Vol. 3, pp.413-435.*

(10) *Hawkins, D. (1973), "Nature, Man and Mathematics". In Howson, A.(Ed.), Developments in Mathematical Education, Proceedings of the Second International Congress on Mathematical Education, Cambridge University Press, Cambridge, UK, pp. 115-135.*

(11) *Hodgson, T.R. (1995), Connection as Problem Solving Tool. In House, P.A. & Coxford, A.F. (Eds.), Connecting Mathematics Across the Curriculum, the 1995 Yearbook of the National Council of Teachers of Mathematics, The Council, Reston, Va., USA, pp. 13-21.*

(12) *Inder, W. (1982), "The Nature of elementary Mathematics and Teaching it While Emphasizing Mathematics as a language", Journal of Science and Mathematics Education in Southeast Asia, Vol. 5, pp. 37-40.*

(13) *Kilmister, C.W. (1972), The Nature of Mathematics. In Chapman, L.R. (Ed.), The Process of Learning Mathematics, Pergamon, Oxford, UK, pp. 1-20.*

(14) *Krussel, L. (1998), "Teaching the Language of Mathematics", The Mathematics Teacher, Vol. 91, pp. 436-441.*

(15) *Lappan, G., Fitzgerald, W., Phillips, E., Winter, M., Lanier, P., Madsen-Nason, A., Even, R., Lee, B., Smith, J. & Weinberg, D. (1988), The Middle Grades Mathematics Project, The Challenge: Good Mathematics - Taught Well, (Final Report to the National Science Foundation for Grant #MDR8318218), Michigan State University, East Lansing, Michigan, USA.*

(16) *Nowosad, S.N. (1998), Children's Perceptions of Mathematics, Dissertation Abstracts International, Vol.58, p. 4215-A.*

(17) *Oaks, A.B. (1988), The effects of the Interaction of Conception of Mathematics and Affective Constructs on College Students in Remedial Mathematics, Dissertation Abstracts International, Vol.49, p. 54-A.*
(18) *Rubenstein, R. (1996), "Strategies to Support the Learning of the Language of Mathematics". In Elliott, P.C. & Kenney, M. (Eds.), Communication in Mathematics: K-12 and Beyond, National Council of Teachers of Mathematics, Reston, Va., USA, pp. 214-218.*
(19) *Schram, P., et al(1988), "Changing Mathematical Conceptions of Preservice Teachers: A Content and Pedagogical Intervention", ERIC Document Reproduction Service No. ED 302 549, ERIC Clearinghouse for Science, Mathematics and Environmental Education, Columbus, OH, USA.*
(20) *Sfard, A. (1991), "On the Dual Nature of Mathematical Conceptions: Reflections on Processes and Objects as Different Sides of the Same Coin", Educational Studies in Mathematics, Vol. 22, pp. 1-36.*
(21) *Skemp, R. (1977), The Psychology of Learning Mathematics, Penguin Books, Harmondsworth, Middlesex, UK, pp. 178-179 & 202.*
(22) *Stein, M., Baxter, J. & Leinhardt, G. (1990), Subject Matter Knowledge and Elementary Instruction: A Case from Functions and Graphing, American Educational Research Journal, Vol. 27, pp. 639-663.*
(23) *Thompson, A. (1992), "Teachers' Beliefs and Conceptions: A Synthesis of the Research". In Grouws, D.(Ed.), Handbook of Research on Mathematics Teaching and Learning, Macmillan, NY, USA, pp. 127-146.*
(24) *Usiskin, Z. (1996), "Mathematics as a Language". In Elliott, P.C. & Kenney, M.(Eds.), Communication in Mathematics: K-12 and Beyond, National Council of Teachers of Mathematics, Reston, Va., USA, pp. 231-243.*
(25) *Vergnaud, G. (1997), "The Nature of Mathematical Concepts". In Nunes, T. & Bryant, P. (Eds.), Learning and Teaching Mathematics : An International Perspective, Psychology Press, Hove, East Sussex, UK, p. 7.*
(26) *Wilson, M. & Gwendolyn, M. (1998), "Supporting Innovations: The Impact of a Teacher's Conceptions of Functions on his Implementation of a Reform Curriculum", Journal for Research in Mathematics Education, Vol. 29, p. 249.*