The history of mathematics : an *essential* component of the mathematics curriculum at all levels

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Abstract. The majority of our school students do not enjoy mathematics and a small minority achieve success. The curriculum has suffered from prolonged government intervention, and teachers and children are faced with the increased pressures of league tables and international comparative studies of results. The new culture replaces the joy of Arithmetic with the utilitarianism of Numeracy.
Yet there is growing evidence that the inclusion of some history of mathematics in the school curriculum increases student motivation and achievement because it (i) helps children to make more sense of mathematics (ii) humanises the subject (iii) emphasises the continuous and continuing development of mathematics, and (iv) fosters an appreciation of the multicultural inheritance and culturally dependent nature of the subject.
This paper will provide classroom examples of how a pervasive influence of the history of mathematics in the school curriculum, at all levels, can achieve these goals.

When I have asked children to tell me what Pythagoras' theorem means to them, the majority respond by quoting " *a* squared plus *b* squared equals *c* squared ", or something similar. The same is frequently true when I raise this, initially, with my student teachers. The fact that Pythagoras would have made no sense of their response has not occurred to them, but serves to illustrate the inadequacy of their mathematical education. That the early Greeks were primarily concerned with geometry and numbers (mainly whole ones), and had little or no notion of what we now call algebra, is rarely, if ever, mentioned in classrooms in England.

Again, the fact that this theorem bears Pythagoras' name, despite being known to several earlier civilisations (including of course the Egyptians!), largely because of the Greeks' concern for proof, is another example of where the mathematics classroom fails to deliver any notion of historical context. Few children or students are shown how to prove the theorem, despite the existence of so many simple and beautiful geometric dissections – such powerful images of mathematics!
Consequently many never really understand the theorem – illustrated by their willingness to add together the squares on **any** two sides of the triangle, and/or to apply the idea to non right angled triangles – and few have the slightest notion of proof nor its importance in mathematics.

The culture which places the utilitarianism of mathematics (algorithmic hoop jumping in this instance) above the importance of ideas and creativity will only result in the greater alienation of the majority of the population from mathematics. Poincare said:

> The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful(1).

But who (in the classroom) has heard of him?
Or of **her**? Many girls and young women reject mathematics early in their lives. Despite out-performing boys at age 16, only half as many girls as boys in England choose to study the subject further. Many females complain that mathematics is about things, not people. And if people are met they are invariably male Europeans. Pythagoras, Fibonacci and Pascal appear to be the most likely entrants into the 5 to 16 mathematics curriculum (although a name, nationality and date are all the learner will probably receive). The otherwise attractive and informative IBM poster 'Men (*sic*) of Modern Mathematics'(2) compounds this view. The more gender and culturally balanced posters produced by Frank Swetz (3) are on few classroom walls in the UK.

Yet humanising the mathematics curriculum may be so readily achieved. The subject is littered with colourful characters, intrigue, romance, passion, corruption, betrayal, adventure and more! For example, Sophie Germain was undoubtedly one of the leading French mathematicians of her time, whilst Carl Friedrich Gauss is considered to be one of the greatest mathematicians of all time. The relationship that developed between these two, who never met, is a fascinating story of courage, friendship and collaboration. Sophie's determination to succeed as a woman in a man's world, vividly portrayed in the Maths Miscellany series (4), remains as an inspirational example to many girls and young women studying mathematics today. This leads nicely to Germain and Gauss's work on the Two test-theorem (5) which in turn provides an excellent opportunity to introduce the use and value of Excel spreadsheets to, say, 14 year olds.

Rather than patronise children with spurious and unimaginative problems on which to practise and consolidate their spreadsheet skills (purchasing Coca-Cola and crisps for a school disco seems popular with teachers at the moment), why not use some of the hundreds of examples of problems from the history of mathematics that real men and women have posed, solved, struggled with or even devoted their life to. Famous texts from Diophantus' *Arithmetica* to Fibonacci's *Liber Abbaci* (or *Liber Quadratorum*) together with the Ahmes and Moscow papyri, the largely ignored (at least in the West) *Nine chapters on the Mathematical Art (Jiuzhang suanshu)* and the subsequent commentaries, notably Liu Hui's, abound with possibilities. Only three weeks ago, one of my mature student teachers, discovering the Egyptians' derivation for the volume of a truncated pyramid, exclaimed: "Why didn't we ever cover topics like this at school ?" Another student wrote: "When I was at school I loved History and I loved Mathematics, but the two never came together."
Comments like this are not uncommon and I have documented elsewhere(6) the very positive views of many student teachers when studying the history of mathematics for the first time.

Many children regard mathematics as a body of knowledge, and a completed one at that. It exists in books and teachers' heads, and has been there for years. There is no new mathematics; it is a dead subject. Yet the fact that mathematics is alive and well and far from complete has been so wonderfully illustrated in recent years by Wiles' proof of Fermat's last theorem. What fascinated me here was the coverage that this received in the media around the globe and in particular by the popular press. The BBC TV Horizon programme(7) was rightly acclaimed (how often do you see a mathematician in tears?), and Singh's book(8) has been on the best seller list for more than two years. Adults who hated, ignored or failed at mathematics at school were queuing up to read about one man's obsession with one of the world's greatest mathematical problems!

How well too do both the book and the programme reinforce Swetz and Kao's view that :

> Seldom are mathematical discoveries the product of a single individual's genius. Often centuries and thousands of miles separate the appearance and isolated reappearance of the same mathematical or scientific theory(9).

What a supportive statement for those who wish to promote a collaborative approach to mathematics in their classroom and wish to emphasise the continual development of their subject.

Another seemingly unlikely best seller in this country, '*An Instance of the Fingerpost*' (10) is a historical novel cum murder mystery centred around the life of the English Oxford mathematician, John Wallis – regarded by many as Newton's predecessor. Books by Hoeg (11) and Sobel (12) are further examples. There is, it seems, an appetite for mathematical stories; can we not feed this at school?

Of course we can, if we choose to. One of the most exciting school mathematics lessons that I can recall was in 1977 when I entered my classroom of 15 year olds clutching the latest copy of '*Scientific American*' containing the article about Appel and Haken's proof of the Four-colour map problem (13), a problem which the pupils had been introduced to, and done a little work on, coincidentally, only months beforehand!

Again, there are more than enough examples to choose from which help children to appreciate that (a) mathematics is still going on out there (b) some mathematics is hard and it can sometimes take mathematicians hundreds of years to solve certain problems, and (c) some problems are still unsolved and some theorems unproven. The Goldbach conjecture provides a good example of the latter (see, for example, (14)). Not only does this allow children to consolidate their work on primes but it also gives the teacher an opportunity to raise issues about proof and the infinite. The existence of **almost** primes, recounted in a reasonably accessible manner in Hoffman's popular work(15), may have children bemused, or at best curious, but at least it is **interesting** and gives some insight into the world of the professional mathematician.

I have seen children enchanted by the story of Apollo and the Delians and the doubling of the cube. Apart from a consideration of what we mean by doubling, this problem could lead appropriately into Heron's method for approximating the cube root of a number, which may well give children a greater insight into that process by comparison to the nowadays routine pressing of a calculator button.

If the examples that I have given so far seem more appropriate for the secondary classroom (11 to 18 year olds), this is largely a reflection of my own teaching experience. But the history of mathematics fits equally well into the curriculum for younger children. Last week a group of my primary students were exploring how to introduce some aspects of Ramanujan's work on the decomposition and partition of numbers, using Cuisenaire rods. There are some excellent opportunities for pattern spotting here plus the bonus of 'discovering' Pascal's triangle. (The knowledge of this triangle in India, at least eighteen hundred years before the birth of Pascal(16) gives added poignancy). Further delightful classroom activities based upon Ramanujan's work are detailed in three centenary publications(17).

Sometimes I think that children believe that pattern spotting is a game played by teachers (perhaps to keep children occupied ?) and has nothing to do with 'real' mathematics. All the more refreshing then to hear Andrew Wiles :

> I never use a computer. I sometimes might scribble; I do doodles; I start trying to find patterns, really.(7).

In an increasingly multicultural society the English curriculum still has a long way to go to cast off its Eurocentric bias, and ironically (for surely it is one of the most culturally divergent subjects) nowhere is this more urgent than in mathematics. Leone Burton(18) reminds us that :

> History is a dicey business. We tend to think that history books tell us about the best and most important things from the past. But at the same time we know that history books tell us about war, not peace; kings, not farmers; extraordinary events, not everyday life; men, not women.

To which she might well have added: 'Europe, not Africa; …….'

At the risk of contradiction, I would also want to make the case for **local** mathematics. One thing that has encouraged a number of my student teachers has been the discovery that important and interesting things have happened on their doorstep. Four years ago one of them wrote:

> For my personal study I chose a local mathematician (George Boole) as I was astounded to discover that he had been teaching in the place that I had lived all my life and I had no idea. This gave me more motivation with my assignment.(6).

One of my current students is carrying out his own research into the life of Nicholas Saunderson, the blind Cambridge mathematician and Newton's successor in the Lucasian chair, who taught himself to read by fingering the engraved tombstones in the churchyard in Penistone, where the student now lives.

In the pursuit of more utilitarian mathematics, school teachers are once again being required to devote yet more time to fractions, decimals and percentages. It seems unlikely that many children will understand these any better than they did when Hart reported(19) in 1981. Meanwhile the use of unit fractions by the early Egyptians goes largely unnoticed. If ever there was a topic ripe for exploration, with a well documented historical background, which would help many learners to gain a deeper appreciation of the arithmetic of fractions, then this is surely it. As O'Reilly rightly claims:

> Egyptian fractions can be a rich source for investigation in the classroom.(20).

What do we want our children to know about mathematics? What do we want them to **feel** about it? What would we like them to know about mathematicians? "Mathematicians are people, too" write Leuetta and Wilbert Reimer, in their mathematical stories for younger children(21).

But then, what is mathematics ? Torkil Heiede comments that:

> … only the realisation that mathematics has not always been what it is right now, and that in the future it will be something different, in other words that mathematics has a history, gives this question its real perspective.(22).

More recently, one of my student teachers wrote :

> [our history of mathematics course] is making me realise that mathematics is not so clear cut. The things we take for granted took thousands of years to achieve. This should be taught also to children … prior to this unit I was unaware of any female mathematicians or the fascinating elements in the history of mathematics – I will definitely teach this to children when I become a teacher(23).

References.

1. Pappas, T. (1995) *The Music of Reason*, San Carlos : Wide World Publishing / Tetra
2. Eames, C. (1966) *Men of Modern Mathematics : A history chart of mathematicians from 1000 to 1900*, New York : IBM
3. Swetz, F.J. (1997) *Mathematicians and Mathematics History (set of 12 posters)*, Portland : J.Weston Walch
4. Jaworski J. & Corbett G. (1994) *Who is Sophie Germain?* (*audio cassette*), Milton Keynes : BBC / OU
5. Bell, E.T. (1953; originally 1937) *Men of Mathematics (volume 1)*, London : Pelican
6. Lingard, D.W. (1997) The role of the History of Mathematics in the Teaching and Learning of Mathematics (Appendix 3), submission to the ICMI study, Luminy, April, 1998
7. Singh, S (1996) *Fermat's Last Theorem (BBC TV Horizon programme*), London
8. Singh, S (1997) *Fermat's Last Theorem*, London : Fourth Estate

9. Swetz, F. J. & Kao, T.I. (1977) *Was Pythagoras Chinese?*, Pennsylvania : National Council of Teachers of Mathematics
10. Pears, I. (1997) *An Instance of the Fingerpost*, London : Jonathan Cape
11. Hoeg, P. (1993) *Miss Smilla's Feeling for Snow*, London : Harvill
12. Sobel, D. (1999) *Galileo's Daughter: a Drama of Science, Faith and Love,* London : Fourth Estate
!3. Appel K. & Haken W. (1977) The solution of the 4 colour map problem – how the celebrated conjecture was proved with the aid of the computer, in *Scientific American, Volume 237 (4), p 108 – 121, 1977*
14. Hofstadter, D.R. (1979) *Godel, Escher, Bach : an eternal golden braid*, Cambridge : The Harvester Press
15. Hoffman, P. (1991) *Archimedes' Revenge*, London : Penguin
16. Joseph, G.G. (1991) *The Crest of the Peacock : Non-European Roots of Mathematics*, London : I.B.Tauris
17. Srinivasan, P.K. (1987) *An Introduction to the Creativity of Ramanujan (3 Volumes),* Madras : The Association of Mathematics Teachers of India
18. Burton, L. (ed) (1982) *Girls into Maths Can Go*, London : Holt, Rinehart & Winston
19. Hart, K.M. (ed) (1981) *Children's Understanding of Mathematics 11 – 16*, London : John Murray
20. O'Reilly, D. (1995) 'Creating Egyptian Fractions', in *The Australian Mathematics Teacher, Volume 51, number 1, 1995*
21. Reimer L. & W. (1990) *Mathematicians Are People, Too (2 volumes),* Palo Alto : Dale Seymour
22. Heiede,T. (1992) ' Why teach history of mathematics ' in *The Mathematical Gazette, Volume 76, number 475, March 1992*
23. Povey, H., Elliott, S. & Lingard, D.W. (1999) ' How mathematics is made and who makes it : a consideration of the role of the history of mathematics in developing an inclusive epistemology ', paper presented to the Gender and Education conference, Warwick, March 1999.