Attitude and Achievement in Euclidean Geometry

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The study investigates the relationship between the attitude of a sample of grade ten pupils working in a rural environment towards Euclidean geometry and their achievement in Euclidean geometry. A great concern amongst South African mathematics educationists is the poor performance of pupils in Euclidean geometry. Similarly, despite its importance, mathematics seems to be an unpopular subject among most of South African pupils. According to the South African education system, every pupil has to take mathematics up to grade nine thereafter one has an option of either continuing or discontinuing with mathematics. Furthermore, pupils electing to continue with mathematics are normally those who have shown some aptitude for mathematics. Four dimensions of attitude namely: enjoyment, motivation, perception of the importance of geometry and freedom from fear of geometry are looked at in the study. Whereas achievement refers to the ability by pupils to solve riders and construct proofs of theorems. The analysis of data was done using SAS. Two dimensions of the attitude scale, namely, enjoyment and importance showed no significant relationship to achievement. A weak but significant relationship was shown between motivation and achievement, whereas a negative relationship was obtained between freedom from fear and achievement. In general, these pupils showed a better attitude towards Euclidean geometry than one would expect from their performance.

1. Background

There is some indication as Nkwe (1985) argues that pupils who are positively inclined towards a subject tend to do well in that subject. It cannot however be concluded that positive attitude will always effect good performance. For example, the study by Kiely (1990) showed that on average a small number of pupils who were not good enough in mathematics obtained high scores in the attitude test. Some of the causes of problems encountered by pupils in mathematics can be attributed to the teaching methods employed by teachers (Peterson, 1973; de Villiers, 1986; Van Engen, 1973) and teachers themselves (Aiken, 1979; Kiely, 1990). In the sense that there is a tendency among teachers especially novice teachers to teach mathematics the way it was taught to them during their days as pupils. In some cases, such a problem is caused by the fact that when such teachers join their professions, they come determined and enthusiastic. To the extent that they are eager to implement the theory and skills they have acquired.

Unfortunately, conditions in the schools are such that it becomes difficult for such teachers to work effectively. Eventually these teachers end up frustrated and find themselves resorting to using the traditional approach. For example, a typical mathematics teacher takes a piece of chalk and copy of a textbook from which he/she will take example problems and solve for the class on the board. Thereafter, he/she gives pupils a set of problems based on what he/she was doing earlier on the board to solve. Solutions to these problems will then be given in the subsequent lesson before proceeding with a new a topic. Most teachers prefer this routine approach because it enables them to complete the prescribed syllabus quicker. Possibly, there is no effective learning that can take place when using such an approach. In some instances as Visser (1991) points out an approach such as this can make pupils think that the content is complex and abstract for them and as a result pupils may develop a phobia towards it.

In some instances teachers find themselves teaching subjects they have limited knowledge in. Such a shortcoming can be mainly attributed to the high failure rate in mathematics in grade twelve which leads to fewer pupils pursuing mathematics at tertiary level. In most cases the few that obtain a good pass in mathematics in grade twelve prefer to pursue careers in engineering, natural sciences, health sciences, computer sciences, technology and so on. The desperate situation in schools compels teachers with a limited knowledge of mathematics to teach grade twelve classes. Their knowledge is limited because they obtained poor results in the subject they only did up to grade twelve and even their pre-service teacher education programme did not include anything on mathematics methodology. It should be noted that in learning a lot depends on the teacher and what he/she teaches. In the sense that the mechanism through which content is learned depends to the greater extent on the quality and depth of the teacher's knowledge of content, which seem to be lacking in those teachers who only went up to grade twelve with their mathematics.

Hence Moja (1982) concluded that lack of appropriate teachers often makes some teachers who are not interested in the subject and also having little knowledge of the subject be forced to teach the subject. It may happen that the lack of interest teachers display can end up being transferred to pupils. Precisely because teachers are expected to persistently motivate and instill confidence in their pupils. In a study that was done by Kiely (1990), pupils in one school displayed negative attitude simply because the teacher concerned was always disgruntled and dejected due to continuous interference with her work by the departmental head. Pupils noted the resulting disagreements which subconsciously affected their attitudes. Thus, the attitude of the pupil to geometry developed through styles and personalities of teachers, affects achievement.

In conclusion, it would appear from the above discussion that there is some relationship between attitude and achievement. Thus it is this which is intended to be researched.

2. Theoretical Rationale

Euclidean geometry affords pupils an opportunity to learn about the formal axiomatic systems with the aim of developing their deductive thinking (de Villiers, 1986). It therefore fosters a creative and imaginative thought through the solution of riders and the writing of proofs. Such skills are important requisites for technologically inclined careers that a developing country like South Africa needs. For pupils to acquire a sound deductive thinking capacity the teacher has to follow acceptable and recommended teaching methods, which according to Nambiar (1997) will make pupils be creative, independent and think critically. These methods should empower the pupils by ensuring that there is proper acquisition of appropriate knowledge and skills. That is to say, the teacher should refrain from confronting pupils with ready-made subject matter, which eventually turns out to be meaningless and irrelevant to their daily needs and challenges. Instead, resort to what de Villiers (1998) calls a genetic or reconstructive approach. He indicates that such approach places emphasis on mathematical processes by which content is developed or constructed.

New knowledge should be built on the knowledge pupils already have. This can happen if pupils are part of the process of developing knowledge. What is crucial in learning is ability for one to think. The process of knowledge development can either be done jointly by the teacher and pupils where the teacher plays the leading role of posing problems or pupils working in groups to solve problems posed. Knowledge in the former will be generated by pupils applying their minds whereas in the latter pupils have to discuss among themselves. In both cases it is not only effective learning that is taking place but pupils also 'attach' ownership to the knowledge they construct. The fact that they are owners of knowledge tends to make them have affection for the knowledge and also makes them want to produce more knowledge. Given the affection of knowledge that develops from attached ownership of the self-constructed knowledge, it can therefore be concluded that there is some relation between attitude due to affection and performance that can be judged from the ability to generate knowledge.

However, there have been few studies looking at the relationship between attitude and achievement. The point of concern raised by various researchers has been the strength of the relationship between attitude and achievement. Haladyna *et al* (1983) state that certain factors, including achievement, are related to some extent with attitude. According to Aiken (1970) assessment of attitude would be of less concern if attitudes were not thought to affect performance in some way. Kulm (1980) concluded that achievement depends on attitudes, hence the ongoing search for a relationship between the two variables. Jackson in Aiken (1970) concludes that attitude scores in the middle range of scores seem to have little relation to achievement. But attitude scores at extremes affect achievement in a significant way. The study by Ernest & Sam (1998) shows that one third of the participants reported a disliking of mathematics. In actual fact this is a group that was mostly recently exposed to mathematics and the group was not intending to continue with mathematics probably because of their poor performance. The definition of attitude in this study deals with aspects of pupils' attitude towards Euclidean geometry which are the enjoyment of, the importance of, pupils' motivation in studying and freedom from fear of Euclidean geometry. Whereas achievement refers to the pupils' ability to solve riders and construct proofs in Euclidean geometry.

In some instances the relationship between attitude and achievement tends out to be causal (see Kulm, 1980; Neale (in Aiken, 1976)). Neale postulates that attitudes are used as predictors of achievement. It can therefore, be deduced that either the direction of causation changes as the child grows older or one reinforces the other. If the latter happens to be case, the point of focus will be, how can the trend be broken? Kulm (1980) indicates that the issue of attitude-achievement causation has a significant relation to the issue of the importance of attitude towards Euclidean geometry.

Furthermore, there are various factors that are related to discrepant achievement. Some of the factors happen to be the integral parts of the four dimensions of attitude mentioned earlier. Among these factors is lack of motivation in the learner. The study by Maqsud (1992) shows that achievement is significantly positively related to task orientation. Which Maqsud describes

> *"as a motivational state in which learner is concerned with primarily evaluating himself and developing mastery .... tasks; he continues to build his belief that greater efforts yield greater competence"(p14).*

It would appear that most teachers and parents believe that when a pupil can appropriately be motivated there would greater possibilities of him/her achieving better. That is why in some schools top-achieving pupils are given prizes as a form of recognition for their efforts. Whereas at home some parents promise their children presents if can perform well at school. This being the case it would therefore be expected of teachers to maintain the positive attitude pupils come with at school. So that pupils have this positive attitude and can arouse some interest in whatever they learn. In this case the assumption would be fostering favourable attitudes towards a school subject like Euclidean geometry is equivalent to encouraging pupils to do that with eagerness and persistence.

In some instances the use of everyday examples in problem situations can encourage pupils.

Precisely because pupils can see the relevance of whatever they are being taught in their everyday situation.

Freedom from fear of Euclidean geometry is another aspect of attitude. There seems to be a link between anxiety, self-confidence and fear. Fearlessness of a subject can also induce good performance. According to Maqsud (1992) anxiety and self-confidence are related to achievement. Another aspect of attitude being looked at in this study is that of perception of the importance of geometry. Wigfield & Meece (1988) perceive that the value pupils attach to a subject could moderate or augment the effects of poor performance on pupils' anxiety. For example, a pupil who performs poorly in a subject but attaches little importance to it may not be anxious about it. Also, there is a tendency among pupils to attach value to the subject they perform well in.

The last aspect of attitude to be considerd here is that of enjoyment of Euclidean geometry. Enjoyment of a subject is influenced by various factors. Among these factors, are classroom activities and the subject teacher. The nature of activities that happen during the lesson have a bearing on the overall enjoyment of the subject. If there are exciting and fascinating activities during the lesson, pupils may tend to associate that with the subject. It should therefore be incumbent upon teachers to formulate activity-oriented exercises that are both suitable for and of interest to pupils.

In conclusion, from the preceding discussion, it is evident that there seems to be a relationship between attitude and achievement. Most of the studies present the relationship between attitude and achievement in mathematics as whole. Little if anything focusing on Euclidean geometry is mentioned. Also, in spite of geometry being removed from mathematics curriculum in most countries, it still remains important in a variety of ways. Hence, it is the purpose of the study to look at the relationship between attitude and achievement.

3. Methodology

Sample

The sample was taken from eight of the eleven high schools in the Makapanstad Circuit, Northwest Province. The three remote high schools were left out for logistic reasons. The remoteness of these three schools could have hindered the proper management of the research. The initial size of the randomly selected sample for the study was ninety-seven. For some reasons one school failed to administer the tests. The resulting sample size was seventy-four which was still however more than adequate in line with the aims of the study.

Research Procedure

The two tests, attitude and achievement tests, were administered to the subjects. It was assumed that the subjects had similar background knowledge of Euclidean geometry given that they had just passed an externally examined level. Hence, they were expected to use their background knowledge of Euclidean geometry in answering the achievement test. Administering the test before none of the subjects had been exposed to any Euclidean geometry in the new level did this. Otherwise the scores obtained would not be attributed to their individual achievement capacities instead to the fact that they had been exposed to more geometry than others.

When answering the tests the subjects used codes instead of their names on their scripts. The five-point Likert-type attitude test was written first to avoid any possible influence arising from the subjects' performance in the achievement test. Poor performance in the achievement test might have led to false responses in the attitude test.

4. **Results**

Despite the fact that the attitude scale was adapted for this study, an internal consistency reliability of the instrument was determined using the alpha coefficient of Cronbach. The alpha coefficient of Cronbach was computed independently using SAS for the four attitude dimensions namely, motivation; enjoyment, fear and importance of Euclidean geometry as follows:

Variable	Alpha Coefficients
Enjoyment	0.73
Motivation	0.68
Importance	0.69
Fear	0.76
Total Attitude	0.77

Table 1: Cronbach Alpha Coefficients of reliability for the four variables in the Attitude Scale

The estimate of the internal consistency of the achievement test was determined through the application of the Kuder-Richardson 21 (KR-21) formula. KR-21 yielded the value of 0.64 as the internal consistency coefficient for the achievement test.

The descriptive statistics of the data was as follows:

Variable	Mean	SD	Minimum Score	Maximum Score	Highest Possible Score
Enjoyment	20.86	3.65	12	28	30
Motivation	21.34	4.77	9	29	30
Importance	20.32	4.57	11	28	30
Fear	19.32	3.83	12	30	30
Total Attitude	81.78	12.95	50	103	120
Achievement	3.08	1.79	0	10	57

Table 2: Means, Standard Deviation (SD) and Ranges of scores.

Reflections on the scores of the Achievement Test

The low scores indicate that pupils were not able to solve riders and complete proofs of theorems. The following points were noted from the scripts:

1. Pupils were not able to name angles correctly. For example, $\angle$ CD instead of $\angle$QCD or $\angle$ DCQ.
2. No logic in the proof layout. For example, a pupil would say $\angle$QRS = 180^0. Reason given for this statement was "angles of triangle".
3. Lack of structure and sense when writing proofs. For example, a pupil would write as follows: $\angle$AEF = 40º
   180º = angles of triangle
   $\angle$ AEF 40º + angle EPG = 40º
   40º + 40º = 80º
   180º – 80º = 100º.
4. Some pupils seem to show no understanding of geometric concepts. For example, a particular pupil made the following statements:
   a) alternative angles in a triangle
   b) $\angle$ QRS = 180º (the sum of isosceles triangle = 180º)
   c) $\angle$RQP = 360º (the sum of straight line)
5. Pupils could not distinguish between a line and an angle. For example,
   a) $\angle$ A_1 = BC + DE = 180º (alternating angles)
   b) $\angle G_1$ = HE + CF + 180º (straight lines)

Although the test was based on what they had done in the previous class, it appears like pupils still found the test difficult given the descriptive statistics of their achievement test scores in table 2. It should also be taken into consideration that the pupils had not been exposed to any

Euclidean geometry in their current class. There is a tendency among teachers to leave geometry for late in the year and begin with algebra. Therefore, there are possibilities that the pupils had forgotten geometry content and their minds were still focused on whatever section of the syllabus they were busy with.

In conclusion, at least one of the points mentioned above appeared in each script. Some of the scripts even had almost all of the points. In certain instances scripts had a lot of blank space in the place of answers. Thus, the low scores obtained in the achievement test can be attributed to the above points. However, the low scores and the narrow range of scores do create difficulties for the statistical analysis of data. Since the data for achievement can now with difficulty be considered as being on an equally divided interval side. The effect of the narrow range is to alter the correlation between the variables from what it would have been if the range had not been narrow. In order to circumvent the problem posed, the scores of the variables are then expressed as ranks. The appropriate correlation coefficient is the Spearman's correlation coefficient (also referred to as the Spearman rho) for ranked data.

Correlation Coefficients

SAS was also used to rank the scores and determine the Spearman's correlation coefficient for ranked data (r_s). Table 3 summarizes the quantitative relationship of ranked scores between achievement and each of the four variables of attitude and attitude itself.

Attitude Variable	r_s
Enjoyment	0.18
Motivation	0.26*
Importance	-0.01
Fear	-0.11
Total Attitude	.0

Table 3: Spearman Correlation Coefficients (r_s)

[significant; *p < 0.05]

Pearson's product moment correlation (r) was computed because the variables correlated are expressed as interval data. Furthermore, Pearson's product moment was determined using SAS as follows:

Attitude Variable	r
Enjoyment	0.22
Motivation	0.26*
Importance	0.06
Fear	-0.09
Total Attitude	0.14

Table 4: Pearson's product moment correlation coefficient (r).

[significant; *p < 0.05]

Since Pearson's product moment correlation coefficient is more often used in literature, the interpretations were done from its values.

Overview of Results

The results indicate that there were very weak relationships between achievement and each of the four variables including attitude itself despite the fact that determining their validity and reliability standardized the instruments. And this enabled a confident and credible use of instruments. The relationship between achievement and all the variables was not statistically significant, except for motivation which was statistically significant at 0.05 level. Thus, the weak relationship between attitude and achievement is due to the weak relationships between achievement and each of the four variables. Furthermore, lack of strong correlation coefficient between attitude and achievement can be attributed to various factors.

5. Discussion

In general, pupils show a better attitude towards Euclidean geometry than what one would expect from their achievement in Euclidean geometry. In most cases those pupils who choose to continue learning mathematics in grade ten are normally those with high mathematical (arguably Euclidean geometry included) ability and would intend following mathematics-related careers. According to Kulm (1980) careers preference is amongst various factors that may mediate the way for attitude to change. That is why pupils in this study display positive attitude towards Euclidean geometry. They see geometry as a useful and important topic to learn and see it relevant to their aspired careers. Possibly, they see geometry as that topic which provides them with certain skills and background which are directly related to their future careers.

Another possible explanation for high scores in attitude is that formal geometry, which involves deductive reasoning, has just been introduced to pupils in grade nine. It may be that pupils are still struggling with the axiomatic deductive system. It should also be noted that although geometry is being taught in lower classes, it mostly deals with properties, constructions and measurements. Constructions and measurements are activities that are closely attached to the pupils' everyday lives. It may be that they therefore attach some value to the geometry they do in class.

However, the pupils' answers in the achievement test show lack of problem solving skills and poor understanding of geometry concepts. This may possibly be attributed to the fact that geometry is normally being taught late in the year by most teachers. As a result pupils are not afforded enough time to understand geometry content and develop their problem solving skills. It may be that when teaching the deductive system the activity-oriented approach that presumably has been used in lower classes is abandoned. Instead, teachers begin to teach the axiomatic deductive system through the use of ineffective teaching methods, possibly the transmission method. Precisely because teachers have a tendency to exert themselves whenever they teach an externally examined class. They therefore drill pupils and in some cases teachers even avoid demanding topics like geometry. Such an approach encourages memorization. That is why pupils perform poorly simply because they have forgotten whatever they were taught.

Furthermore, as determined from pupils' answers, problem-solving skills seems lacking or at least, not well developed. Pupils cannot apply definitions, axioms and theorems appropriately. Hence there is no logic and well developed structure in proof layout. Such discrepancy is attributable to the fact that pupils are not adept in setting a plan to determine a solution to a problem. A well-set up plan forms a sound basis for a solution to a problem. According to Polya (1973) for a well-structured plan to be drawn up there has to be an understanding of basic concepts. Lack of understanding of basic concepts manifests itself in the answers pupils give and can arguably be attributed to a tendency to memorize. Where emphasis is placed on applying procedures by rote and approaching all problems in a mechanical fashion. The latter refers to being able to recall teacher methods for solving problems.

That is why there is a tendency among pupils not to go beyond what the teacher has taught in class. In some instances when teachers set examinations they only concentrate on those riders they solved in class. Pupils would simply recall how the riders were solved and give the correct answers. They normally concentrate on what the teacher said is important or examinable. Some would even resort to solving problems as in the textbook. Otherwise, any rider taken from somewhere else becomes difficult to pupils. Pupils performed poorly in the achievement test of this study because problems used were not taken from their prescribed textbook. Their poor performance is precisely because of the fact that they do not solve as many riders as possible to develop their problem skills. It should be noted in order for one to develop his/her problem solving skills he/she has to solve as many riders as possible. As a result pupils lack ability to explore various methods and never show the ability to ponder the problem until a solution is found. This possibly explains the low scores pupils obtained in the achievement test.

6. Conclusion

From the study it is evident that pupils have a positive attitude towards Euclidean geometry in spite of their poor performance. Poor performance is mainly caused by a lack of problem solving skills. Lack of problem solving skills can inhibit pupils' curiosity and inventiveness which stem from the interest they have in Euclidean geometry (Schamlz, 1989). Such interest should make them persevere for a lengthy time at a problem and a variety of strategies. This poses a challenge on teachers, as there is willingness to learn geometry. Teachers should therefore think of better strategies of improving the problem solving skills of pupils. It should be noted problem solving is a fundamental mathematical skill of all and also relates very much to real life applications of mathematics (geometry included). Learning to solve problems in mathematics mainly comes from experiences of solving problems and using them in daily situations. The relevance of mathematics in daily situations then becomes discernible. This can build the confidence of the pupils in learning and using mathematics widely. Thus pupils become more creative and innovative.

Furthermore, the problems solved provide a meaningful context for developing learning skills. Abstract problems afford an opportunity for exercising logic and also make pupils think abstractly and critically (1973). Therefore the mathematics curriculum should be designed in such a way that it encourages pupils to adopt such an approach. Also, pupils should be encouraged to work on their own more. They should refrain from relying too much on their teachers instead have a propensity to apply his/her mind to solve problems encountered.

It can also be concluded that teachers should capitalize on the fact that pupils show a positive attitude towards Euclidean geometry when teaching. There should be a move away from the traditional teacher-centred approach, instead pupil-centred should be used so as to facilitate learning. Pupils' own experiences should also be used to enhance problem solving skills. This can be done by refraining from channeling pupils into standard approaches but rather encourage them to arrive at their own conclusions. Teachers should also refrain from teaching exonerate themselves from blame of completing the prescribed syllabus. Instead, they should organize their work so that the syllabus is well completed and content is well understood and assimilated. Effective learning is a factor of good performance.

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