ASSESSMENT OF SCHOOL CHILDRENS' NUMBER SENSE

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Abstract

The research reported here is part of a bigger study that aims to look at children's' notion about number sense. This study attempt to assess chil dren's number sense related to five strands of number sense based number sense framework by McIntosh, Reys and Reys (1992). The five strands are: understanding the meaning of size and numbers, understanding and use of equivalent forms and representation of numbers, understanding the meaning and effect of operations, understanding the use of equivalent expressions and computing and counting strategies. We are also interested to find out whether children demonstrate understanding of numerical situations in which they solve computational problems; whether there is a relationship between the strategies children use to solve number problems and number sense. Two tests were administered on 406 ten year old children from four different schools in Malaysia. The first test attempted e test to assess students' understanding of number sense with respect to the five strands of number sense. The second test is a written computation test with similar items designed in number sense format. A series of interviews were also conducted to discover the relationship between understanding of number concepts and the computational abilities. The results obtained indicated there is a wide range of performance. It is discovered that, in general, students seem to experience difficulties with all the strands of number sense except those related to the use of equivalent expressions and computing and counting strategies. This may due to the fact that the other strands require deeper understanding than just mechanical calculations. The result from the written computation test show that generally, students perform better on written computation test than on similar items in number sense format

Background

The primary school mathematics curriculum clearly states that the main of the curriculum is to build and develop children's understanding in the number concept and at the same time attains high facility in the basic skills. Both these are to be applied to problem solving situations, mostly related to everyday problems (Noor Azlan, 1993). Through these, children are expected to appreciate the beauty and importance of mathematics. Mathematics is seen as a branch of knowledge that is useful in dealing with everyday life problems in a disciplined manner parallel with demands of society in a developed nation. (Kementerian Pendidikan, 1989). Hence, the teaching and learning of mathematics, even at the early stages, should attempt at instilling critical and creative thinking, apart from memorizing mathematical algorithms and procedures (Tajul Arrifin and Nor' Aini, 1992).

Number concepts in primary school mathematics

Recent curricular reform documents (such as National Council of Teacher of Mathematics, 1989;Australian Education Council, 1991;Cockcroft,1982, Kementerian Pendidikan, 1991) emphasize the importance of number sense based on the rational that numbers sense will be very helpful to understand numbers in general. Relatively, the focus on term "number" in the mathematics curriculum is quite recent and most has targeted their arguments to the primary school level (Burns,1994; Hiebert,1984; Plunkett,1979; Skemp,1982, Kementerian Pendidikan, 1989).

The understanding of numbers is of fundamental importance in primary school mathematics**.** The understanding and attainment of these basic skills must be constantly applied to real life problems (Noor Azlan, 1993). Attaining number sense becomes more important when they proceed to secondary school as the secondary school curriculum are based on three strands, that is *number, shapes and relation* (Kementerian Pendidikan, 1989).The question then, is have the children mastered number sense well enough so as to be able to grasp the content of the secondary school syllabus? In general, only a limited number of students do really understand number sense while solving problems (Markovits &

Sowder,1994; Noor Azlan and Lui; in preparation). Further, Markovits & Sowder (1994) observed that students who are taught in the "traditional" way do not show understanding of number sense in problem solving situations involving numbers (see also Reys, 1973). A number of mathematics educators seemed to agree that the difficulties experienced by children in solving mathematics exercises is closely related to the development of number sense thinking (Leutzinger & Bertheau,1989; Burns,1989).

What is number sense?

A survey of literature indicated that number sense is difficult to define and that it is not a single entity, but rather has many dimensions. Like 'common sense", number sense is a valued but difficult notion to characterise (McIntosh, Reys, Reys, Bana and Farrell, 1997). Verschaffel and De Corte (1996) emphasize that "this complex, multifaceted, and dispositional nature of number sense suggests that it cannot be compartmentalized into special textbook chapters or instructional units" and that "the development of number sense results from the whole range of activities of mathematics education, rather than a designated subset of specially designed activities." (McIntosh, Reys, Reys, Bana and Farrell, 1997). Nonetheless, various 'indicators" of number sense have been hypothesized (McIntosh, Reys, Reys, Bana and Farrell, 1997). These include well understood number meanings, existence of and reliance on multiple numerical relationships, recognition of relative magnitude of numbers, awareness of the relative effect of operating on numbers, and use of referents for measures of common objects and situations and in their environments (NCTM, 1989). Shull (1998) offers additional indicator of number sense; an intuitive understanding of numbers and the effect of operations and numbers. It is a well-organized conceptual network that enables a person to relate number and operation propitious. Number sense is characterized by an individual's ability to use his/her understanding of mathematics in flexible and creative ways to make mathematical decisions and to develop useful strategies for handling numbers and operations.

In short, number sense refers to the ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information (McIntosh, Reys, Reys, Bana and Farrell, 1997). It refers to the understanding about numbers and their related mathematical operations and the ability (tendency) to use this understanding to make decisions about mathematically related situations. Number sense can be seen as carefully arranged concepts that allows one to relate between properties of numbers with that of operations (Sowder, 1992). It can be identified as the ability to synthesize numbers and at the same time able to recognize its representations. Number sense also involves the ability to compare numbers, to sequence numbers in meaningful forms, relate the values the numbers represent, to compute mentally, and be able to sue the appropriate strategy to understand the impact of certain operations. Number sense refers to the ability to understand, operate and understand the result of certain operations on numbers.

Number sense and written computation

NCTM (1989) argues that children must understand number meanings if they are to make sense of the way numbers are used in real life situations. Mathematics educators are concerned that many students demonstrate little understanding of numerical situations in which they solve number problems. The lack of number sense seems to be the result of mindless application of the standard written algorithms which they learned in school (Yang, 1997). Students are good rule followers, however, they do not always understand they procedures they learned (Hiebert, 1984). Students are often better at manipulating and following symbol rule than they are at making sense of the numerical situations. Despite efforts by the mathematics education community to move away from the traditional conception about mathematics, recent findings (Noor Azlan and Lui, in preparation), have strongly indicated that most children have not attained the understanding that demanded by the new curriculum (Noor Azlan Ahmad Zanzali, 1995; Noor Azlan and Lui, in preparation).

Number sense framework

McIntosh, Reys and Reys (1992) developed a framework for examining number sense based on studying and reflecting on the literature associated with number sense, estimation and mental computation. The framework formulated six number sense strands:

1. understanding and use of the meaning and size of numbers
2. understanding and use of equivalent forms and representations of numbers
3. understanding the meaning and effect of operations
4. understanding and use of equivalent expressions
5. computing and counting strategies.
6. measurement benchmarks.

Objectives of the study

Based on the above discussion, there is a need then to:

i. assess childrens' number sense to the first five strands in the number strand number five in the number sense framework. The sixth strand, though is as important if left for further or subsequent study.
ii. assess whether children demonstrate understanding of numerical situations in which they solve number problems, whether there is a relationship between the strategies children use to solve number problems and number sense.
iii. Discover the qualitative relationships between number sense and the ability to compute numerical problems.

Methodology

The sample of study consisted of children in year 4 of the primary school. Two tests were administered. This is further followed by interviews on selected sample.

Research Instrument

All test items and directions were presented in the national language. A 47 item test (adapted from McIntosh et. al.; 1997) containing various aspects of number sense were given to the children to solve. This is followed by a 15 written computation test. All the 15 written computation items are constructed in number sense format and included in the number sense test. For example, question 36 in the number sense test is:

Without calculating an exact answer, circle the best estimate for 5/6 + 8/9

A. 1 B.2 C.19 D.21.

Question 1 on the written computation test is:

Calculate 5/6 + 8/9.

Question 36 which is the number sense item could be solved by thinking that each of the two fractions are slightly less than one, therefore the sum of the two fractions would be almost like the sum of 1 and 1 that is choice B. It is hoped that the children can assess the suitability of their answer based on their understanding of fraction size and the operation involved. Question 1 on the computation test requires student to actually do the calculation in order to find the answer. Table 1 below gives the summary for items in number sense test and items in written computation test.

Table 1. Summary for items in number sense test and items in written computation test

NUMBER SENSE TEST

WRITTEN COMPUTATION TEST

Question Number	***Number sense strand***	***number Of questions***	***Items in con***
S1– S8 **S36, S46**	Number Concepts	10	K1, k15
S9 - S17	Multiple Representations	9	
S27– s31 **S26,S37, S45, S47,**	Effect of operations	9	K5, k2, k11, k
S32,s33,s35- **S34, s38-s44**	Equivalent Expressions	11	K3, k6, k7, k9,
S18 – 21, s23-s25 **S22**	Counting and computation	8	K4

Nb. Bold items have similar items in computation format.

Discussion of the findings

The items in both the number sense and written computation test were given 1 for a correct score and 0 for an incomplete or incorrect answers. The percentage of correct answer for the number sense test is 36.9%. The percentages of correct answers for the various strands of number sense differas indicated in the table below.

Table 2: Percentages of correct answers for the five strands of number sense and the range of marks for each strand.

Number Sense Strand	Percentage of Correct answers	Range of mark
Number Concept	34.1	90
Multiple Representation	31.7	100
Effect of Operations	29.2	88.9
Equivalent Expressions	45.5	91
Counting and Computing Strategies	41.2	100

The data from this study shows that students perform better on number sense strands 4 (equivalent expression) and on strand 5 (counting and computing strategies). Students seem to have difficulty with the strands number concepts, effect of operations and multiple representations. This may be due to the fact that these three strands require deeper understanding than just mechanical calculations. Despite the relatively "low" percentage of correct answer for each strands of the number sense, the range of mark for each strand is more than 90%. This shows that they are students who have well developed number sense in each of the strand of number sense tested as well and at the same time we as students who have very poor understanding of each strand of number sense.

Evidence from this indicates that the children in this study have the most difficulty with the number sense strand "effect of operations". Understanding the meaning and effect of an operations, either generally or as related to a certain set of numbers (eg. division means breaking a number into a specified number of equivalent subgroups, or multiplying by a number less than 1 produces a product less than the other factor) includes judging the reasonableness of a result based on understanding the numbers and operations employed.

There were nine questions in the number sense test on children's' understanding of effect of operation. The questions focussed on the effect of operation instead of asking children to actually carry out the computation. For example;

Q. 30) Which of the following will give the highest value?
A. $29 \div 0.8$ **B.** 29×0.8 **C**. $29 + 0.8$ **D.** Impossible without calculation.

Question 30 requires the children to choose an operation that will produce the highest value as the answer. The percentage of correct answer for this question is only 4.4%. The researchers wanted to know whether the children in this study have the ability to understand the effect of different operations on the same operand. In this study, 63.5 percent of the children chose B or 29 x 0.8 as an operation that will produce the highest value while only 4.4 percent of the children chose A (29 ÷ 0.8) which is the correct answer. 10.6 percent of the children chose C while 18.5 percent of the children said that it is impossible to get the answer without actually carrying out the calculation. The choice is based on the assumption that the product of multiplication is greater and that the final result of a division will produce a smaller value. One implication from this study is that in order for mathematics to be meaningful to children, they have to understand the effect of numerous operations and calculations that they carry out besides just striving to get the correct answer for the calculations.

The percentage of correct answer for computation test is 61.4%. The percentage of correct answer for computation test is better than the percentage of correct answers for number sense test. Table 3 below gives a comparison of student's performance on items in number sense format and the same item in computation format.

Table 3: Comparison of student's performance on items in number sense format and the same item in computation format.

Item number		Percentage of correct answer		% difference (computation test – number sense test)
Number sense test	Computation test	Number sense test	Computation test	
S36	K1	17.9	70.9	53
S46	K15	29.9	51.8	21.9
S45	K14	30.3	55.9	25.6
S47	K10	53.5	77.1	23.6
S34	K3	74.4	86.2	11.8
S38	K6	6	85.9	79.9
S22	K4	32.1	78.2	46.1
S40	K9	37.1	50.9	13.8
S41	K10	17.4	77.1	58.7
S43	K12	81.2	87.7	6.5
S44	K13	72.4	82.1	9.7
S42	K11	47.7	44.4	-3.3
S26	K5	17.7	0	-17.7
S37	K2	37.9	3.2	-34.7
S39	K7	12.4	6	-5.6

From table 3, it can be seen that children have difficulty solving problems in number sense format. For example the percentage of correct answer for question 36 which is written in number sense format is 17.9%. The percentage of correct answer goes up to 70.9% when the same question is written in computation format. The percentage of correct answers in computation format is higher in 11 out of the 15 questions given. Question 36 in computation format (5/6 + 8/9) is considered a complex addition of fraction for 10 year old children. However, more than 70% of the children correctly computed the exact answer for this item. When the question is presented in the number sense format, only 18% of the children could get the correct answer. The majority of the children (67.9%) thought that the answer to 5/6 + 8/9 is either 19 or 21. This suggests that most of the children were unable to make sense of the relationship between the fraction size and the operation involved. Even though most students can correctly calculate the answer, it is still questionable whether the children demonstrate understanding of the numerical situation.

Conclusion

This study has generated some interesting findings worthy of further probe. We are now in the midst of conducting interview sessions with selected students. The interviews will center on the relationships between the ability to compute numerical problems and number sense. Why does there exist a gap between ability to calculate using established algorithms and attaining number sense. In general children are able to do very well when asked to calculate but do not seemed to show understanding on the concept of numbers.

Current changes in the mathematics curriculum require children to attain higher order thinking. The learning of mathematics should be more focussed on thinking and reasoning about mathematics rather than just memorizing the algorithms mechanically. Data from this study shows that children have difficulty understanding basic number concepts as shown with the low scores in tests of number concepts particularly those related to the effects of operations and multiple representations. The study also shows that children perform better on items that are written in computation format as compared same item written in number sense format.

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