Four revolutions in mathematics and their schooling implications

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Summary

The author argues that a big shake-up in our ideas about mathematics is taking place at the present time. We can distinguish four different recent revolutions in the way we think about, and conceptualize, mathematics. It is an upheaval of unprecedented proportions. The fallout is likely to continue for a number of years. When we add to the four mathematical revolutions the big changes which have taken place in our thinking about psychology, schooling and society during the last few decades, the size of the task facing us---in trying to 'tune' mathematics schooling accurately to current conditions and current knowledge--- becomes apparent.

Let's begin by looking at the four revolutions within mathematics that are either already happening or are about to happen.

The computer revolution First, there is the computer revolution, which began in the 1950s, and has continued through the last five decades. It has happened in a deeply mathsphobic social environment. In the 1960s, after the computer first became both powerful and reliable, the computer industry felt that it had to distance itself at all costs from a perceived 'no hope' mathematical culture. Nothing which has happened since then has led it to change its mind, so its attitudes towards the computer-mathematics gulf have effectively set in concrete.

The word was: "mathematics and computers are entirely different worlds", "you don't have to know any mathematics to use a computer".

The issue here is about how mathematics is publicly perceived. The point was obvious, but the computer industry felt it had to distance itself from mathematics.

Can anything be done about the situation? Isn't the computer industry now so dominant that nothing could ever dent its preferred way of speaking and thinking? Many ordinary people perceive clearly that they and those about them have no leverage of any kind to alter this anti-mathematical bias. But I suggest that we, as mathematicians, can hardly happily accept this fait accompli. To build an anti-mathematical bias into the very way we talk about computers was always a shameless piece of cultural domination. Of course it made a lot of sense in 1960 for the computer industry to steer well clear of any public association with mathematics. But the computer is really only a medium for running automated mathematics. I believe that we need to wake up and insist in schools that it is this: and that both computing and maths are to be regarded by all sensible people as parts of 'greater mathematics'.

Let's face it, what it takes to achieve spectacular new computer applications is a kind of mathematical thinking, or at least a kind of thinking very closely related to mathematical thinking. To pretend that the heart of the computer culture is a celebration of hardware is therefore a social nonsense. Anyone with real knowledge knows this, yet the nonsense persists.

Mathematics is now very much the poor relation to the computer fraternity, not a Big Brother of towering historic intellectual status, as it was in 1960. (The computer revolution, in effect, has now seen off 'historic intellectual status'. The world has become increasingly anti-intellectual, largely as a result of the decline in mathematics standing.)

The computer industry can now afford to be magnanimous in relation to mathematics, a discipline on which it is, in the end, dependent. For mathematics teaching in schools must in the

end provide the mental foundations for all those who work with computers.

Of course the arrival of the computer revolution has had a huge substantive, operating effect on mathematics too. The computer has naturally developed a 1001 ways to handle number-based representations in streams, trickles, bites and bundles. By its sheer existence this opening-up of a myriad new branches of numerical and quasi-classical number-based mathematics has tended to upstage the abstract high mathematical culture which dominated mathematics in 1960.

The arrival of computer-driven algebras hasn't really altered the picture, because it was a symbiosis between the new number-based representations and the needs of business which created the flavour of the last quarter century ---the revival of capitalism, the new shameless materialism and the occlusion of abstract mathematical culture.

Most scientists now work with and through computers. They have largely thrown off their previous dependence on abstruse mathematical know-how ---for which they used to have to go cap-in-hand to their local pure mathematicians. Of course they still consult the mathematicians from time to time, but they (the scientists) are much more mathematically emancipated by their computer skills than their predecessors of forty years ago.

One can argue that the abstract triumphalist 'high mathematical culture' of 1960 never properly recovered from the collapse of 'New Math for Schools' in the early 1970s. The fact that 'New Math' simply wouldn't "go" in schools came (for them) as an awful shock, a calamitous bolt from the blue. I suspect that many today still don't really know what went wrong.

The same people ---i.e. the mathematical establishment--- tend to be sleeping Platonists. Many of them, I'm afraid, have hardly noticed that much of mathematics' thunder (credit for initiating potent applications) has been stolen by the computer industry. The computer industry has implied that the magic displayed in such applications lies in its machines. It has been very tardy in admitting that the magic nearly always depends on the skillful use of mathematical concepts and thinking. To describe applications of mathematics to the real world as 'computer applications' is like describing the Bible as a 'papyrus application' or a Beethoven symphony as an 'orchestral application'.

The net result of this misdescription is, however, quite serious. It is that the social role of mathematics ---as perceived by the media, politicians, etc.--- is greatly diminished. It is seen as a dry, arcane, reclusive, sub-specialism within programming: itself only a sub-department of computing. This gives mathematics, as viewed from the school classroom, a much less inspiring image than it ought to enjoy.

The modelling revolution This arrived in the 1960s on the back, one has to say, of the computer revolution. It enables us to apply mathematics to the world in a more satisfying, in-depth way than the hit-and-run applications of yesteryear. It is also, incidentally, when done well, a more commercially useful way.

Instead of just 'applying' a given formula to a given situation by substituting numbers into a formula, it is now possible to 'discuss' the situation in a sustained fashion via the 'mathematical model' of the situation.

In the era of unaided paper-and-pencil mathematics modelling was an intrinsically difficult aspiration: it was the arrival of copious computing power which made it an attainable aspiration.

Unfortunately the modelling revolution's impact on schools was largely hi-jacked in the early 1980s by newly converted Thatcherite applied mathematicians who proclaimed that it turned the useful commercial application of mathematics to the real world into the main purposive reason for pursuing maths in schools and universities. Mathematics, they claimed, had a newly acquired potency to "make things happen" in the real world.

(It is convenient to call this the 'practicalist' view of school mathematics.)

Yes, but to throw the main emphasis onto this new operational potency was hardly satisfactory as a 'view of the essence of mathematics'. It led to the introduction of near-contradictory notions such as 'practical mathematics' sessions in schools. (Insofar as they were 'practical' they weren't 'mathematical', and insofar as they were 'mathematical' they weren't 'practical'.) How anyone thought that mathematics, the heartland of pure theory, could be adequately explained as a 'practical' subject, chiefly justified by its utilitarian applications, remains a mystery. It was like saying that the essence of ballet, a mute artform, lies in its *storylines*: or like saying that the central idea in cooking is the preparation of *drinks*!

This practicalist Thatcherite interpretation of the modelling revolution came to exert a huge influence on the maths curriculum in schools in the UK and some other countries for a few years through the Cockcroft Report, SMP 11-16 and the National Curriculum. It is now a paradigm in sad decline, but its decline is, I think, being widely misunderstood. A lot of commentators seem to think that its decline tells us that "application-led maths doesn't work well in the classroom".

What doesn't work well in the classroom is a narrow, gritty, commercially oriented version of the modelling view of mathematics. "Making things happen with maths" turns maths, in effect, into a low-key conjuring trick. (Low key, because what is 'made to happen' is often rather ordinary.) But maths not only *isn't* a conjuring trick: its chief and deepest value lies in the mental transparency and disciplined imagination which it evokes ---the very things which the conjuring trick approach plays down.

So is there a better way of looking at the modelling revolution? Yes, the logical conclusion of the modelling revolution is the 'Peircean interpretation' of maths, which sees its (maths') principal raison d'être as its *projective applicability*. This consists of modelling humanly, socially and technically interesting (innovative) ideas for the future. They are essentially *hypotheses* to the effect that a proposed gadget of some kind might deliver what we want. Peirce said "Mathematics is the science of hypothesis" thus hitting the nail firmly on the head. Unfortunately he didn't develop this interpretation of mathematics sufficiently to grab scholarly attention.

The central point of projective modelling is to *illuminate* the overall predictable implications of interesting (promising) new gadgets in science and development. The work amounts to fascinating 'thought application', not banal actual application.

This perspective in teaching maths was originally developed in the 1970s by the *Mathematics Applicable* project, which I was fortunate enough to lead. It worked extremely well: so much so that the number of students taking the course continued to grow year on year until the London Examination Board summarily announced that it was discontinuing the examination.

The reason why the Peircean approach works extremely well in the classroom is easy to state. By focussing on mathematics as a hypothesis-exploring device, one is able to tie its study to intriguing ideas with great imaginative appeal to the youthful mind. It is very motivational. It is not a case of sugaring the

pill, but of finding exactly the right conditions in which the pill's natural sweetness can be appreciated and shown off.

There is a wide cultural gulf between the Thatcherite and Peircean versions of mathematical modelling in schools. The former prides itself on getting things to happen, especially in gritty contexts: things with an evident cash or hard-nosed practical value. The latter is more interested in the magic produced by illuminating interesting practical possibilities (and incidentally impossibilities). The former pins its faith on 'getting answers'. The latter on the individual's improved mental vision and the satisfaction this brings in its wake.

The logical gap between the two perspectives, however, is quite thin. To close the gap all that is required is a realisation of the electrically motivating effect on students of innovation-targeted mathematics. In my opinion the gap is chiefly a result of an idiosyncratic 'industrial' point of view held by a few leading practicalists. They can't see that their mental perspectives were formed by working in hard-nosed and deeply mathsphobic environments, and that these attitudes so painfully acquired are a huge barrier to finding the kind of maths which goes with a swing in the classroom.

I wrote a book in 1997 systematically explaining the basis of the 'Peircean Interpretation', giving lots of tried-and-tested examples, and extolling the advantages of the Peircean approach in schools.

But the new emphasis on Peircean illuminative applications has emerged into a doubly hostile environment. It is flatly opposed on the one hand by distinguished sleeping Platonists in the mathematical hierarchy who desperately want to think that the emphasis of the last two decades on applications has all been a bad dream, and that now at last it is over ---discredited by its dire effects in schools. On the other hand, it is opposed by obstinate defenders of the failing status quo ---Thatcherites who are not interested in illuminative modelling for its own sake (with its great educational potential) but only interested in trying to get punchy algorithms, answers and results 'across' to students.

The constructivist revolution This is a new mode of thinking about the ontological status of mathematics. Some leery, 'radical', 'post-modern' versions of constructivism do little to recommend it (constructivism) to scrupulous commentators. Constructivism of this kind seems to be claiming that mathematics was constructed for dubious hidden social, psychological and other arbitrary reasons. This naturally infuriates anyone who is strongly subject-dedicated, because it implies that a kind of subversive dross has crept into their reasoning.

In my opinion the outline claim that mathematics is a human artifact which was progressively 'constructed' is, however, simply a plain reading of the historical record. The important question is not whether mathematics was 'constructed' (of course it was) but *why* it was constructed in the way it was. In my opinion the *reasons* for the 'construction' were chiefly (a) to find models which would faithfully reflect physical phenomena (which had been previously discovered by careful, patient inquiry), (b) to solve challenging logical problems thrown up within mathematics itself, and (c) to model potential developments in building, canal construction, military methodology, industry, etc. The insinuation of hidden social values behind mathematics is just that ---an insinuation. Any actual example, once brought out into the light of day, generally turns out to be either a mistake, or else more commonly an honest attempt to achieve explicit aims under heading (c) above.

Here, too, there is a distinction to be drawn between an acceptable and unacceptable version of the revolution.

There is a new lean, but very uncompromising, version of constructivism which I have outlined in four monographs during the last decade. It is 'quasi-platonic' because it has the operational effect of retaining most of the working assumptions used by honest practitioners, but it *also* has the important side-effect of drying out the quagmire in the foundations --- something which has been zapping confidence in the subject's rationality since 1901. (The year Bertrand Russell discovered his famous paradox, which however neither he nor anyone else was ever able to explain satisfactorily.)

The new quasi-platonic constructivism claims that mathematical objects are 'honorific existents' created and secured by social convention. They are based in the last analysis on 'tally conglomerates' onto which we *impose* a set of judgments. For example the fraction ¾ can be regarded as shorthand for ///\\\\. When we originally set up something like this we decided to treat the tally conglomerate //////\\\\\\\\ as being equivalent to the tally conglomerate ///\\\\, even though they are, of course, quite different conglomerates.

By thinking of mathematics in these terms we can encompass the most bizarre, abstruse reaches of modern abstract theory as well as arithmetic and its derivative classical algebra. Technically it is an 'object-centred' formalism. However when we speak *about* the symbol conglomerates we naturally use meaningful ordinary language, as when we say that it is "true" that the square root of 169 is 13. (This U-turns a lot of nonsense associated with formalist attitudes in metamathematics.)

The contradictions of self-referential set theory can be explained as examples of contradictions of a fundamentally new *serial* kind. (Classical contradictions are then re-described as 'parallel contradictions'.) For example, when the Liar says "I am lying" (p), what he is saying implies that p is false, which tells us that p is true, which tells us that p is false, which tells us that p is true... etc. ad inf. If we assume that the current interpretation of 'what p is telling us' supercedes previous interpretations, at no stage does a 'contradiction' literally occur. However, over time, the statement clearly in some sense 'contradicts itself'. This is what I call a 'serial' or 'dynamic' contradiction.

There are thus great advantages in adopting the new perspective, but in general progress in disseminating this new lean version of the constructivist revolution has been very slow. Radical constructivists are against the status quo at all costs: this, in effect, is their defining characteristic. The quasi-platonic version of constructivism looks (to them) to be too close to Platonism for comfort. Actually it isn't, because it leads us, among other things, to reject all transfinite sets beyond aleph one. It also leads us to the idea of showdown sequences, a form of active process, which can't be represented by any present or future mathematical structure.

But most radicals are not really tuned to the problem of the foundations: they are much more bothered by what they see as the 'no hope' official culture of Platonist mathematics in schools. So one can hardly expect them to notice these departures from the official story, nor to see the immense benefits of draining the quagmire.

Conventional opinion in the foundations of mathematics, on the other hand, seems to be unable to tear itself away from its blind adherence to the platonic norms of yesteryear. It can't see the lucidity implicit in the new paradigm because it is clinging for dear life, like a cragfast rock climber, to what it thinks it knows.

Mondimathics The general epistemological problem of formulating a replacement for Descartes' research programme for science has led recently to a revolutionary new post-Cartesian research

programme. The key to this is that we can now use the constructivist point of view to justify the construction of a new trans-mathematical discipline much closer to the basic needs of both physics and biology than ordinary mathematics.

'Mondimathics' is the new trans-mathematical discipline. It is based on the same principles as mathematics, but applied fundamentally to 'randomly active sequences' rather than inert tallies.

Technically it is 'an inquiry into relationships between randomly active sequences'. This doesn't sound very exciting, but it has a trump card compared with ordinary mathematics: it gives us *active* models with which to match the phenomena we find in the real world. It has, as it were, a life of its own.

This is a greater earthquake potentially than the previous three, since it removes ordinary classical and modern mathematics from their epistemologically privileged perches. The sheer positivity and hope behind the new mondimathic research programme is, however, likely eventually to make a big difference to schooling. Its main effect should be to disarm the tendency to self-robotification which is sadly now dominant in many areas of commerce and industry. Self-robotification is also badly affecting the schools, where it leads towards ever more pressure for behaviouristic training. So self-robotification, unopposed, is likely to kill a 'genuine education'. The new post-Cartesian epistemology attacks it at its root, which may be identified as Cartesian triumphalism in science.

Summary of the revolutions The computer revolution came first. It led to huge changes in the way mathematics is predominantly done 'out there'. Among the changes it introduced was the possibility of doing applications synoptically, i.e. by adopting a 'modelling' approach. The arrival of a 1001 new methods in computer-based mathematics, and the creativity opened up by the availability of mathematical programming languages, led commentators gradually to emphasise the central importance of *construction* in mathematics. This in its turn has finally led to an unexpected new construction: a new trans-mathematical science (mondimathics) whose specification is designed precisely to avoid the woodenness and rigidity (static timelessness) of classical mathematics.

Post-revolutionary mathematics in schools We remarked at the beginning that in addition to these changes in our thinking about mathematics, there have been profound changes too in psychology, education and social thinking generally.

In psychology alone a host of changes have occurred, with the virtual disappearance of simple behaviorism and the rise of 'cognitive science' based loosely on a computer analogy to the human brain.

The practical working 'psychology' of pupils in schools has been greatly affected by the partial collapse of all kinds of social and traditional authority. This is a social change of momentous proportions.

In this new low-authority, postmodern environment the only strong source of educational values and educational drive is the local parental constituency of a school. They can insist on the kind of strong values and dedication needed in their classrooms. It would be impossible under modern conditions to get nationwide agreement for such convictions.

Education today is widely seen as an active process, in which we should be encouraging youngsters to build their skills and initiative awareness: in which we should be constantly challenging the pupil to show what he or she can *do*. (And in contexts recognisably *meaningful* to both her/him and to us as teachers.)

The general effect of these changes seems to be to make all traditional methods of teaching maths more difficult to operate than before. Children today are used to a much richer cognitive environment than their predecessors of half a century ago. There is less emphasis in society at all levels on 'purity' of formulation. When it is tried it is apt nowadays, especially in low prestige surroundings, to be mistaken for threadbare formulation.

This is a measure of the materialism of the present age.

Children today are more 'uppity' than ever before. They look at the adults around them and see a jaded, mentally burnt out picture. They won't just listen to maths being taught 'from authority'. They need to explore for themselves in mathematics, but crucially they need to do this in what are, for them, thoroughly meaningful contexts.

Recent changes in psychology, education and social aims all point towards the necessity of using a rich application-based, discussive regime of some sort in mathematics teaching in schools. To carry the greatest overall meaning it will have to be a modelling approach. Within the modelling approach, though, the sterility of a narrow Thatcherite (practicalist) perspective will have to be overcome. We need the kind of fascinating, clear, lucid, innovative mathematical scenarios which are the hallmark of a Peircean approach to teaching, learning and enjoying mathematics.

You can read about the Peircean interpretation of mathematics and the new quasi-platonic constructivism by joining the MAG. (P.O. Box 16916, London SE3 7WS, UK: joining fee £3)

You can read about the new post-Cartesian epistemology and Mondimathics by visiting: http://ourworld.compuserve.com/homepages/chrisormell

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