A First Year Mathematics Program.

Anthony Sofo
Faculty of Engineering and Science, Victoria University, Melbourne, Australia.

Abstract.
Students entering courses at tertiary level face many difficulties. In some cases the problems of coping and adjusting may be further compounded by inadequate learning habits, a lack of good teaching and inappropriate assessment methods. Tertiary life in many cases is a new experience, a new culture for students, and it seems appropriate to be sensitive to individual needs. Many programs that are now being developed are considering issues related to student learning, quality of teaching and appropriate assessment methods. These issues are complex and varied, but they are becoming more important in this new era of mass education. With these concepts in mind we outline a new mathematics subject being offered to incoming engineering and science students.

Introduction.

Incoming tertiary students have a greater variety of mathematical backgrounds than they did some five to ten years ago. Clearly this has meant that many Australian, and overseas Universities offering mathematics to first year students have had to reassess their offerings in terms of context, delivery and procedures. Clark [1] from Melbourne University has found that pre-testing of students together with an increased first-year subject offering has helped in coping with this spread of diversity. At Monash University, Varsavsky and Adlem [10] have embraced regular assessment to stimulate the learning, while Lindsay [6] argues that with the assistance of a computer algebra system some students may operate at a higher conceptual level. Remedial help sessions argue Kurz and Hohlock [5], from Germany, are a necessity to make up for deficiencies in both mathematics and physics backgrounds.

In our mathematics subject we, indeed, try and offer a touch of all four ideas to our students and instill in them a sense of worthiness and fun in mathematics. Probably the most essential attribute which education should provide, with regard to a rapidly changing society, is the basis for understanding change. A willingness to anticipate change in the future and a confidence in being able to direct it, to participate actively in the process and not merely be subject to it. Grubb [4] has noted that one real inadequacy of education lies in its failure to teach students to be flexible and adaptable to change. Mathematics classes and syllabi must be sensitive to changes otherwise we may find that a program losses its importance and viability.

The Course and Perceptions.

The mathematics subject offered for our course requires that all students take 110 hours of study per year. The students may go on to study mathematics, computer science, education, engineering, physics and electronics. Next year the enrolment will be in the order of 800 students, entering with varying backgrounds. The cohort ranges from young school leavers to a small number of older aged students. Some have advanced skill levels and others possess a very low standard of manipulation techniques. A large number have English as a second language and therefore some necessary English repairs must be undertaken. The mathematics subject may be studied for its own interest, but it must also be the foundation, which reinforces and complements the other subjects. So we ask a fundamental question, “Why teach mathematics at all?” The answer is best given by Cockcroft [2]. In his report Cockcroft tells us that mathematics is useful for everyday life, for science, for commerce and for industry, because it provides a powerful, concise and unambiguous means of communication and because it provides means to explain and predict. It attains its power through its symbols, which have their own grammar and syntax. Moreover the report claims mathematics has aesthetic appeal and it develops logical thinking. It is possible that the given reasons on why we teach mathematics may strike a harmonious chord with those of us involved in mathematical research or mathematical education. However I have an uneasy feeling that a great majority of incoming tertiary students see mathematics as a necessary evil, forced on them by the compulsory requirement of the course. The great majority of school leavers have a pass in whatever mathematics they may have attempted; however mathematics does not figure prominently in their overall scheme of things. They figure they can get by in their computer science and engineering courses with a minimum amount of mathematics.

Beliefs and Preconceptions.

Students come into tertiary colleges with varying points of view of what mathematics entails. Some of them think of it as nothing more than a set of rules that sometimes work and sometimes don't, and are never quite sure when they work and when they don't. Others think of it as a something with a strange language that does not always make sense: independent variable, differentiate, invert, least upper bound etc. Many students believe their task is to memorize what we teach them, but they do not expect to understand any of it. From talking to many of these students one notices that they see each new item they learn as separate from the rest of mathematics, one more item on the list of facts they have to remember and some enquire, "Will I get this formula on the exam?" Their face lights up with surprise when one points out the relationship between topics, or suggests different approaches to a particular problem; they want one approach only. In other words, some students have an instrumental, rather than a relational approach to learning mathematics. Mathematics is basically right or wrong, black or white with usually one correct method and one correct answer, moreover some students believe that anyone good at mathematics knows immediately how to solve any problem. If they do not immediately see how to begin, they think they will never be able to do it. Some never think that it may be possible to 'doodle' with a problem, to make some sort of educated guess and then check it, to try a few special cases and perhaps generalize it. Basically some students lack the whole repertoire of problem solving techniques which are almost second nature to any mathematician or mathematics teacher. Some have no idea of the role of intuition, imagination or inventiveness in mathematics and therefore have a complete misunderstanding of the nature of mathematics and mathematical activity. To make matters worse, this misunderstanding is sometimes reinforced by the way some tertiary institutions assess and examine courses. Many teachers claim they expect students to understand what they are learning, however very often rote memorization is the only requirement to pass an assessment. We all admit to wanting students to learn to solve problems, but some of us do not help with the problem solving process. Instead we tend to give them problems to solve, and when they cannot do them, we sometimes show them completed solutions, which we may have worked out in advance. This may reinforce the idea that we know instantly what to do when solving a problem and therefore may give the student no opportunity to gain an insight into problem solving processes. As a result we may encourage them in their misconceptions about mathematics. Students sometimes have little idea of how mathematics might be useful to them and certainly pure mathematics for its own sake would be remote from the minds of a great majority of students. As Bertrand Russell once said when speaking about pure mathematics and mathematical theorems, "*The theorems themselves are abstractions that belong in another realm; remote from human passions-remote even from the pitiful facts of nature.*

Confidence.

Closely related to misconceptions is a lack of mathematical confidence. Many students appear to believe that success in mathematics is the sole domain of people with a 'mathematical mind-nerds' and are convinced they do not have such a mind. Some people who hold this belief tend to become anxious if they are placed in a situation where they have to do some mathematics. Indeed, some students display an anxiety towards mathematics similar to that described by Munroe [7]. Extremely anxious students, on the whole, tend to not last the distance and choose an alternative course, which may require only a modicum of mathematics. Lack of confidence is a big issue for many students, some have very little belief in their ability to reason things out for themselves, or to tackle any problem that is at all different from the standard types they have been shown how to do in class. We have all, at some stage, observed such students working on assignments or tutorial problems on material which may be new to them. They do not, generally speaking, begin by trying to understand the theory given in lectures. Instead they tend to look through their notes until they find a worked example resembling the problem they are trying to solve, and then they attempt to 'translate' this into the terms of the new problem. Very often this strategy is successful; a correct answer is obtained and both lecturer and student may feel that learning has been effected. Light questioning sometimes reveals this is not so. In many cases the student is unable to explain what she/he is trying to do or why. It's just a matter of going through a certain sequence of procedures, a sequence which will later have to be memorized to be carried out in an examination. Here we have a vicious cycle, lack of confidence causes many students to resort to rote learning strategies, and conversely, reliance on rote learning, because it places so much strain on the memory, causes increased loss of confidence. How do we try and cover this broad spectrum of student ability, confidence and interest? Is it possible that we may make mathematics appeal aesthetically in a way similar to music or art? Our response to music or art is influenced by personal differences, we cannot expect everyone to enjoy the same sort of music or art, similarly we cannot expect everyone to enjoy the same sort of mathematics. It appears that a basis to our enjoyment of music or art and hopefully mathematics is our response to pattern. Consider, for example the fractal shapes of the von Koch curve or the Sierpinski carpet, they exhibit a wonderful recursive pattern, see Sofo [9]. As G. H. Hardy wrote:

> *A mathematician, like a painter or a poet is a master of pattern.*

It is possible that some students may enjoy mathematics because it is useful. This may be a justification for spending a great deal of time on mathematics, however one should not neglect its appeal based on intellectual or aesthetic responses similar to those of art or music. The aesthetic appeal of mathematics is in my opinion, in the eyes of the beholder. Aesthetic judgements may be transitory and may even change from age to age. I doubt very much that the technical aspects of continuity and differentiability, for example, will send a shiver down one's spine as it may have done in a past era. But it may be the case that an elementary excursion into logic or chaos theory may help to ignite a passion not previously thought possible. Davis and Hersh [3] put it beautifully:

> *Blindness to the aesthetic element in mathematics is widespread and can account for a feeling that mathematics is dry as dust, as exciting as a telephone book, as remote as the laws of Infangthief of fifteenth century Scotland. Contrariwise, appreciation of this element makes the subject live in a wonderful manner and burn, as no other creation of the human mind seems to do!*

Learning theories.

What, if any learning theories should we place to the fore? According to Piaget learning is distinct from cognitive development, it takes place in relation to the relevant stages of cognitive development, but it is achieved through interaction with the environment. Skemp on the other hand, has proposed a theory of learning that takes into account the important question of goals and motivation. Learning, claims Skemp, is a goal directed change of state of a director system towards states which make for possible optimal functioning. Bruner, as a supreme optimist, suggests that any idea or body of knowledge can be presented in a form simple enough so that any particular learner can understand it in recognizable form. Learning, claims Bruner, consists essentially of concept formation, which is the multiple embodiment of an abstract idea in different physical forms. Finally, Dienes suggests that learning is a process of intricate play. While one should be familiar with learning theories and perhaps be aware of their implications in the context of the classroom, one should not consider them as a panacea designed to cure all problems. One should be flexible and adaptable to change, and keep in mind the predicament of the Earl of Rochester: "Before I married I had six theories about bringing up children. Now I have six children and no theories".

According to the sixth edition of the Oxford dictionary, learning is defined as 'possession of knowledge got by study especially of language or literature or history as subjects of systematic investigation'. Learning is inextricably tied up with teaching and like quality teaching the academic's task is to reinforce quality or deep approaches to learning in the students' particular subject area. Deep approaches to learning involve a commitment to understanding. To understand key and basic concepts with a view to translate and utilize ideas across a broad range of activities. Learning for understanding requires detailed information to be organized in a meaningful way. In turn deep approaches to learning should set up more positive attitudes and make learning more rewarding. Pateman [8] argues, in his beautifully written book, that it is not possible to engage in teaching mathematics effectively without some clear notion of the nature of mathematics and how that nature inevitably influences the art of teaching.

The offer.

Our subject is a basic introduction into mathematics, about half of the time is spent on discrete methods and the other half on continuous processes. Logic, Boolean Algebra and Set theory are all designed to give the students a gentle introduction into mathematics without necessarily relying on a great deal of prerequisite knowledge. Indeed anybody should be able to follow these introductory topics and build their confidence so as to proceed to a more intense program. Combinatorics and an introduction to Linear Algebra make up the remainder of the section on discrete mathematics. The continuous processes section consists of an introduction to the Differential and Integral Calculus. There is available pre-enrolment bridging mathematics for the students, however in practice only a very small portion of students, entering from non traditional means, take advantage of this offer. That is, mainly mature aged students who have not frequented school recently. This is a non-compulsory offer made to students, but in the main the students are not interested. It appears that they seem to think they have the necessary manipulation skills to handle whatever mathematics is thrust upon them. The bridging course is short and intense, designed basically to iron out any major problems and to recall some information of basic functions and algebra, which they may have forgotten. In the bridging course we try to encourage students to be active, to verbalize their ideas with staff or with other students. Current research on the learning of mathematics supports our belief that this approach helps students develop a better understanding of concepts in mathematics. Upon enrolment, diagnostic testing used to be regularly undertaken but over the last few years, because of lack of resources, this has diminished. Also we have found that students were becoming anxious. They were worried that they may fail, rather than considering it for what it was, an indication of their weaknesses.

Once enrolled in the course and throughout the year extra help, mathematics 'drop' in classes, were also offered, but again as a general rule, the drops were infinitesimally small. The drops increased in frequency close to examination time. Even if a staff member recommends a particular student to 'drop' in to these extra mathematics classes they rarely follow the recommendation. The pressure of time and work in other subjects may be a mitigating factor in their non-attendance. The mathematics syllabus is complemented and reinforced with the use of a computer algebra system. We utilize the package *Maple* on a weekly basis to extend the work given in lectures. Experimentation is a feature of the *Maple* work sheets and some students do like to fiddle with problems on the computer. Some students, on the other hand, find it difficult to grasp even some of the simplest procedures that are required for operating the computer algebra system. The nice pictures generated by *Maple* are always an attraction for the students, even if they are not quite sure of the technical procedures, which generate them. I do not believe that de-personalizing learning by using computers will be effective. Computer learning must be kept to a minimum and only used to reinforce ideas and theories already obtained by face to face learning. A single student, left alone with a computer, is likely to achieve very little in learning and maybe increase her/his frustration level. Experience has shown that many of our first year students need, more than anything else, to interact with an academic or a group of other students, who can provide encouragement and just the right amount of help, so they can achieve some success and gradually increase their confidence. It would be nice to change the students' perceptions of the nature of mathematics, and help them to see it as a human endeavor rather than a list of arbitrary and inaccessible rules. I would submit that this is more easily achieved with the help of a sympathetic and supportive academic rather than a computer.

Conclusion.

We believe that we have developed a course in mathematics that should deliver confidence in student learning and achievable outcomes. The theory work is complemented with computer work and a range of options, for the students, are available to help them succeed in this ever shrinking, watered down subject we call mathematics. Many academics may agree that what they did in the equivalent level of study many years ago, was of a markedly higher standard than that which is being taught today. This may indeed be the case, but in educating the masses we must cater for a wider variety of skills: we must be prepared for change, anticipate change and actively participate in the change.

In my talk I will detail more aspects of our course and highlight some successes and failures of our program.

References.

[1] **Clark, J. F.** 2nd Biennial Australian Engineering Mathematics Conference. pp. 553-556, July 1996.

[2] **Cockcroft, M.** Mathematics Counts. HMSO, 1981.

[3] **Davis, P, and Hersh, R.** The Mathematical Experience. Birkhauser, Boston, 1987.

[4] **Grubb, W. N.** Responding to the constancy of change: New technologies and future demands, in Burke, G., and Rumberger, R. W. (Eds.). The future impact of technology on work and education. Lewes, The Falmer Press, pp. 118-140, 1987.

[5] **Kurz, G, and Hohlock, E.** 2nd Biennial Australian Engineering Mathematics Conference. pp. 541-544, July 1996.

[6] **Lindsay, M.** 2nd Biennial Australian Engineering Mathematics Conference. pp. 535-540, July 1996.

[7] **Munroe, J. F.** Mathematical anxiety: its nature and identification. Success, no.2, 1985.

[8] **Pateman, N.** Teaching mathematics: A tantalizing enterprise. Deakin University Press, 1999.

[9] **Sofo, A.** Fractal Shapes. Function, Vol.23, pp. 38-44, 1999.

[10] **Varsavsky, C., and Adlem, R.** 2nd Biennial Australian Engineering Mathematics Conference. pp. 571-575, July 1996.