A theoretical-experimental model for research into epistemological obstacles

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Introduction:

In the theory of situations of G. Brousseau and in general of the French school, the study of the epistemological obstacles has had a secondary role. Initially the study of the obstacles was tied to questions of an ergonomic nature. How can we distinguish in the shortest possible time the didactic and/or epistemological obstacles relative to the specific content to be taught?

The previous modelling such as that of Dourux-Brousseau derived from the works of Bachelard, allow us to recognise if a notion is an obstacle. That model however does not provide strong *a-priori* deductive instruments for the analysis of obstacles.

In the model of Douroux-Brousseau the research instruments were substantially of a didactic type. Previously, other research instruments for epistemological obstacles have been distinguished of historical type without any consideration of the obstacles of didactic type.

The approach followed by me is closely tied to an investigation of linguistic/communicative type. The instruments of analisi were semiotic in nature[2]. In the elaboration of the model time after time the instruments of investigation necessary for the "**falsification**" of the model were indicated. Sometimes of experiemental type and sometimes of theoretical/experimental type.

The topic used in the model is that of the Postulate of Eudoxus-Archimedes. The postulate of Eudoxus-Archimedes is an item of knowledge that constitutes an obstacle to the preliminary introduction of the Hypereals and may be an obstacle to non standard analysis[3].

1.0 What to use in the didactics of obstacles.

The epistemological obstacles and in general the epistemological research give the possibility to the researcher in the Didactics of mathematics to:

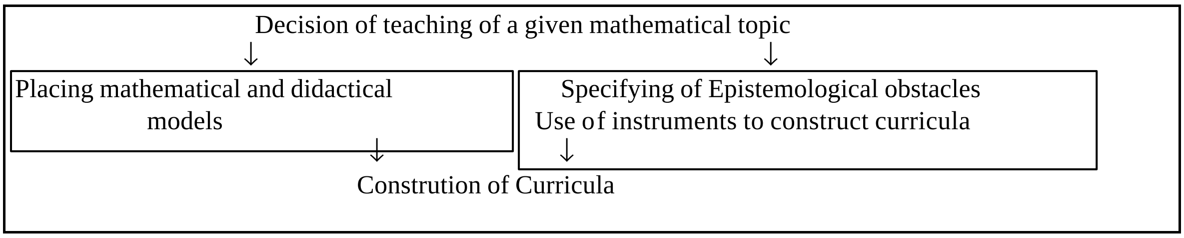
- check the epistemological conceptions induced by mathematicians;
- check the epistemological conceptions derived from the teacher;
- check the epistemological conceptions that intervene in the didactic transpositions;
- allow them to look at the system of teaching from an external point of view;
- differentiate a large number of conceptions on a given topic and allow the regrouping in classes for a didactic analysis.

It seems opportune to underline the two different levels in the communication of the mathematics: the didactic transposition (the putting in place of strategies and curricula for teaching) and the theoretical research for which one must avail oneself of instruments such as the epistemological obstacle.

As far as concerns the didactic transposition, the researcher in Didactics must make evident two approaches :

1) The knowledge of the obstacles and therefore of the conceptions of the pupils allows a possible anticipatory intervention on the part of the teacher in the acquisition of specific concepts;

2) The putting in place of the curriculum must follow a schema of the type :



From micro-didactic research (didactic situations, obstacles, etc..) one unearths quite a precise knowledge of the difficulty of the pupils and instruments to overcome them.

All this allows us to construct the curriculum :

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[2] The model presented in this work is closely tied to the work of my doctorate thesis presented in Bordeaux in 1995 under the guidance of G. Brousseau. The work, from a theoretical systematisation point of view, was then developed in Spain (Insegnare le matematiche nella scuola secondaria, The Nuova Italia, Firenze, 1998).

[3] The Postulate of Eudoxus-Archimedes (PEA) is presented always in the teaching of mathematics:

- Explicitely: For example in the construction of the numerical sets and properties relative to the introduction of classical analysis.
- Implicitly: For example in all activities concerning approximation.

a) correct from a mathematical point of view;
b) correct from a didactical point of view.

The didactic research on putting in place curricula that takes into account the obstacles is still in its early stages. There exists some tentative partial support from the theory of Transposition Didactics of Chevallard, but which is not yet sufficiently tried.

1.1 The Presentation of the Model.

We now give a definition of an Epistemological Obstacle in a linguistic outlook which will permit us to operationally define the Model for distinguishing an Epistemological Obstacle:

When in a certain historical period, the mathematical community seeks to pass from a significant semantic area to a new language relative to a certain class of problems, there enters into the picture some special mathematical "objects".

Definition: The mathematical objects of the previous semantic areas that might serve for the syntactic construction (in the foundations of the new language) are the epistemological obstacles.

And hence it is in the evolution of the semantic areas of the mathematical language that one finds the epistemological obstacles: "The *epistemological obstacles* are defined as the mathematical *objects* of the previous semantic areas that could be used in the syntactic construction of the new language. This new language is then specificed to relate to a certain *class of problems* (supposedly inaccessible in the previous language).

The schema present here constitute a new classification of the obstacles.

After having given an operating definition of an Epistemological Obstacle based on an epistemological investigation of mathematical languages following a semiotic point of view, we present the *Model of the Epistemological Obstacle*[4] *(Theoretical-experimental):*

- The obstacles are to be found amongst the constitutional elements of the mathematical languages that one wishes to study. The analysis is restricted to that which mathematicians call the **fundamentals** of the language.
- An epistemological obstacle is knowledge: that must be verified with historical-epistemological instruments;
- This knowledge produces some replies adapted in a certain frequently confronted context: the experimental verification of investigations that look at how the conceptions accumulate around the questions posted in a specific context with a specific language.
- This knowledge produces some false replies out of context. This is not able to transfer the replies with reagrd to a different context, either because the point of view has changed [$\mathbf{c_1}$] or because it is considered a more general context [$\mathbf{c_2}$] in which the first knowledge was considered a particular case. One can verify it experimentally through the change of context as concerns [$\mathbf{c_1}$]. These two moments, that is **the point of view** and the **generalisation** represent two important instruments for the construction of languages, either in the History of Mathematics, or in the reorganisation of the **fundamentals** of Mathematics. Hence, being two important moments for the putting in place of mathematical ideas, they constitute two significant moments for the characterisation of the epistemological obstacles. It can be verified experimentally by an amplification of the context where one can no longer recognise the role of the knowledge subject to the obstacle with regard to [$\mathbf{c_2}$]. This corresponds to an enlargement of the language where the knowledge, the object of the obstacle, is no longer recognised as a fundamental element (for example an axiom), but will have to be recognised as some property of the language.
- This knowledge holds out in the face of contradictions with which it is confronted. At bottom this aspect, tied to the previous point, consists rather in a fact of procedure than analysis on the **fundamentals** in the sense that the knowledge is presented in the same manner when one reproduces many times the same situation. The contradictions possibly begin with supplementary information, didactic situations constructed **ad hoc** in which there must be made very evident the role of the knowledge/obstacle, of the new extended language.
- This knowledge continues to manifest itself also after it has become conscious. That is, after having become conscious of the role of the knowledge/obstacle in the new language, there are still conceptions relative to the role of the knowledge/obstacle of the initial language. The role of the **fundamentals** of the initial language remains. This can again be verified experimentally in this case with suitable didactic situations.

2.0 The hypotheses of the research

a) Hypotheses supported by theoretical arguments:

- **$\underline{H_1}$: The epistemological obstacles can be characterised as "very profound" modifications of the language and as changes to the axiomatic systems (foundations of the constructive elements of the theory and of the languages of mathematicss).**
- **$\underline{H_{2a}}$: The postulate of Eudoxus-Archimedes[5] one finds in this case: it is clearly identifiable with one of the languages where it intervenes.**

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[4]The reference is the Doctoral Thesis of F.Spagnolo, *Obstacles Epistémologiques: Le Postulat de Eudoxe-Archimede*, Bordeaux, 31.7.1995 (Quaderni G.R.I.M., Supplemento to n.5, 1995).

b) Hypotheses to be tried experimentally:

- **H_{2b}: The Postulate of Eudoxus-Archimedes is knowledge for some pupils. It produces some suitable replies in a certain frequently met context. (The frequently met context is tied to spontaneous conceptions).**
- **H_{2c1}: In a non-Archimedean context, the knowledge of the P.E.A. is manifested with some numerous errors.**
- **H_{2c2}: When the context is generalised, the P.E.A. continues to manifest itself[6].**
- **H_{2d}: The P.E.A. holds out to the contradictions to which it is confronted. In a more general context, the P.E.A. will not be reorganised (and accepted) even having produced some contradictions with respect to the knowledge accepted by the pupils.**
- **H_{2e}: The P.E.A. continues to manifest itself also after it has become conscious.**

3.0 The experimentation and its instruments

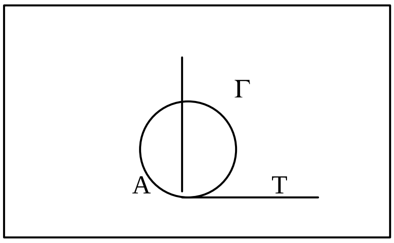
It is proposed to show that the pupils of the 1st year of university in mathematics know the axiom of Archimedes (they state it and they use it) and that part of them continue to use it in circumstances where one does not use it any more. They are therefore presented with a questionaire and some software. The behaviour observed in response to these situations was treated statistically rather than through a clinical analysis of behaviour and content.

The values of the cognitive or didactic variables of this situation will be those that the PEA will appear under different forms (implicit, explicit, direct etc.) as a correct medium of solution in some questions and in others not.

3.1 The straight and curved angles. Presentation of the "mileu".

The examples relative to the non-Archimedean structure come from history and in particular from the Elements of Euclid (prop. XVI of book III): the curved angles (contingent or of contact).

Let us consider the figure formed from an arc of a circle Γ with one extremity the point A, and with the half line AT being tangent at A to the circle.



The portion of the plane between the arc and its tangent constitutes a curvilinear angle. We consider a family of circles all having tangents at A as AT. The curved angles that they determine can be ordered by a sort of inclusion: the angles determined by circles of bigger radii are "included" (as a portion of the plane in a sufficiently small place) in angles determined by circles of small radius. In particular, every curved angle is included in every non empty straight angle of the vertex A and of the side AT.

The measure of all the curved angles is zero.

Does there exist a different "measure" that distinguishes between the curved angles? It will be enough to comply with the linearity: $m(A \cup B) = m(A) + m(B) - m(A \cap B)$.

We admit that, in the set of curved angles in this space, it is possible to define some operations that should correspond to the sum of the measures and to the product of a measure and a non-integral scalar.

Let E be the set formed from all the curved angles generated from A and AT. If it is wished to conserve the relation estabilished with the classical measure of straight line angles, it is enough to increase R to a new numerical structure HR (these will be the Hypereals) where the measure sought for will take its values. The measure of every curvilinear angle will have to remain smaller than that of every rectilinear angle. One can anticipate then that the PEA will be excluded from HR, simply from the fact that n x 0 = 0. Every multiple of a curvilinear angle will not be able to surpass the very smallest non zero rectilinear angle.

To introduce the curvilinear angles, two different approaches are available:

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[5] Euclid in Def. IV of Book V affirms :"One says that they have reason amongst themselves the magnitudes of which may, if multiplied, reciprocatively overtake themselves".

The Initial Formulation is due to Eudoxus in a form translatable in modern terms in the following way: $\forall x,y \ \exists m \ (m \in N : mx > y)$.

a) Archimedes frequently used the postulate of Eudoxus expressly however in the following form: $\forall x,y,z$ [if $x > y$ then $\exists m \ (m \in N : m(x-y) > z]$. This second version is that equivalent to the method of exhaustion.

b) Formulation of the symmetry of the Postulate of Archimedes: $\forall \ x,y \ \exists m \ (m \in N : x/m < y)$.

In current mathematical language, when we refer to the Postulate of Eudoxus-Archimedes one intends the set of the three previous propositions (a), (b), (c).

[6] When the general context is taken into consideration of the Magnitudes where "Archimedean" or the "non-Archimedean" are from particular cases, the Postulate manifests itself as an obstacle.

As a class of homogeneous magnitude. This is the exposition of this paragraph that seeks a mathematical argumentation in Spagnolo (1995, p.239-251);

As an angle formed from two polynomial curves of the same degree and on which is defined a measure that allows us to put in correspondence these angles with the Hypereals (Spagnolo, 1995, p.225-238).

3.2 The variables from experience.

The values of the cognitive and didactic variables (all binary) of the basic situation must take into account that certain pupils know and use the PEA under different forms (explicitly, implicitly…) and in certain circumstances.

The following table sums up the opposing variables used for the experience.

PEA valid (PEA)	versus	PEA not valid
Formulation (F)	vs	Implicit use
formulation "direct" (fd) research of a multiple	vs	formulation "inverse" research of a submultiple
Declaration of possibility (P)	vs	Effective use
Small difference between the (pd) partitioned elements	vs	large difference between the partitioned elements
Relation of order (Ro)	vs	inclusion

3.2 The plane of experience, explicative matrix, method.

Only 22 questions are considered alloted in the following plane of experience as follows.

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	Q22
P.E.A[7]	1	1	x	x	x	x	x	x	1	1	1	1	1	1	0	x	x	1	1	0	0	0
F-E.I	1	1	x	x	x	x	x	x	0	0	0	0	0	0	0	0	0	1	1	1	1	1
F.D.I.	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0
P.-U.E.	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0
PDGD	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0
RO	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0

These questions lead us to the observation of the behaviour of the replies (for example succeeding or not). The behaviour of the replies characteristic of the hypotheses studied must be determined *a priori*. This description takes the form of a set of "variables of behaviour" that determines what is observed.

Only 24 variables of behaviour were used, whose values for every pupil observed made up the contingency matrix (matrix of the observations).

Every behavioural variable was identified in relation to the situations in which they were collected, from an explicative matrix of the questions (or supplementary observations) that may be compared to the results of the analysis of the data. Every line of this matrix can be interpreted as a representation of a characteristic profile of the pupils.

[8]	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	Q22	Q23	Q24
	9 to	10 a	11 a	11 b	12 a	13 a	14 a	15 a	18 a	19 a	20 a	20 b	20 c	21 a	22 a	3a	4a	5a	6a	7a	8a	16 a	17 a	17 b
SO1[9]	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
NSO1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
PEA	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	x	x	x	x	x	x	x	x	x

[7]The value "x" in all the following tables signify that the questions relative to the variables are not pertinent to the conception examined.

- P.E.A. The Postulate of Eudoxus-Archimedes;
- F.-E.I. Formulation of the Postulate: implicit task;
- F.D.-I. Inverse formulation of the Postulate: research of the sub-multiple;
- P.-U.E. Effective use of the Postulate;
- PDGD Small difference between the elements compared, Large difference between the elements compared;
- RO Order Relation.

[8]In the first line it is the indication of the variabile cwith Q_i. In the second line it is the old denomination of the variables that one finds in the thesis (Spagnolo, 1995).

[9]SO1 Profile of pupil who uses always the P.E.A. the same when it has no place and succeeds in questions of order

NSO1 Profile opposed;

PEA Profile of pupil who knows the PEA and how to use it.

In this way it is possible to examine the effect, on the behaviour, of the variables studied and through them the conclusions on the hypotheses utilised.

3.3 The questions, the sample.

Summary of the questions and of the variables of behaviour in the following table[10]:

Q1	Knowledge of the Postulate P.E.-A. in operational terms. The need to determine an n that the multiple of the segment **na>b**. (**a<b**). Direct formulation of the Postulate. The representation with hyphens with a link with the measure. Replies expected: n>4.
Q2	Questions similar to the previous, but the segment **b** is very much bigger and the segment **a** was designed to be smaller. Replies expected: n>19. Direct formulation of the Postulate.
Q3	Affirmative response to the existence of n / n**a**>**b**. Direct formulation of the Postulate.
Q4	Giving a justification to the reply given in the previous question. Direct formulation of the Postulate.
Q5	Knowledge of the P.E.-A. in operational terms. The need to determine an n so that (1/n)**to**<**b**). Replies expected: n>3. Inverse formulation of the Postulate.
Q6	Affirmative response to the existence of n / (1/n)**a**<**b**. Inverse formulation of the Postulate.
Q7	Linguistic formulation different to the previous question: "...is always possible...".
Q8	Change of the point of view: "Veronese" model, not Archimedean, in geometry. Reply expected: Affirming the non validity of the Postulate.
Q9	Change of the point of view, the pupil must follow the construction evoked in the proposition X,I[11] of Euclid and concludes with its validity (rectilinear angles).
Q10	Change of the point of view, validity of the proposition X,I (curvilinear angles). Question 17 indicates how to compare curvilinear angles amongst themselves. The pupil must make a construction and conclude with the validity of the P.E.-A..
Q11	Generalised context (comparison of curvilinear and rectilinear angles). The pupil must, in this case, reject the validity of the proposition X,I.
Q12	Resisting the contradictions with the contingency: the pupil gives a justification of the X,I (a context non Archimedean).
Q13	To succeed, the pupil gives an argument for rejecting the procedure of the X,I in a context non Archimedean.
Q14	Confirmed: the obstacle persists: Affirmation of the validity of the proposition X,I for a pupil, who manages to test their interpretative model in a more general context.
Q15	Confirmation of the position of problem Q13: Affirmation of the non validity of the proposition X,I
Q16	The pupils must find an order relation between the three triangles.
Q17	Order relation between the three triangles (other contexts).
Q18	Order relation (R.O.) between the three triangles (other contexts).
Q19	R.O. between rectilinear angles.
Q20	R.O. between curvilinear angles (parabolas).
Q21	R.O. between curvilinear angles (contingency or contact).
Q22	R.O. inclusion between rectilinear angles.
Q23	R.O. inclusion between contingent angles.
Q24	R.O. inclusion between rectilinear and contingent angles.

The sample: is composed of 107 pupils of the 1st year of a Mathematics course at the University of Palermo. The trial was anonomous.

3.4 The manifestations tried out following the hypotheses.

Comparison of marginal values:

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[10] The questions for extension can be seen in Spagnolo, 1995, Doctoral thesis. In the same text there is presented a detailed analysis a priori that takes account:

- *of the epistemological representations (they are the representations of the eventual path of knowledge with regard to a particular concept).*
- *of the historical-epistemological representations (they are the representations of the eventual path of knowledge with regard to the syntactic, semantic,pragmatic reconstrution of a particular concept).*
- *Of the hypothesisable behaviour of the pupil in front of the situation/problem (they are all the possibile risolving strategies whether correct or not).*

[11] Proposition X. I: *Given two unequal magnitudes, if we subtract from the larger a magnitude greater than its half, and from the remaining part a magnitude greater than its half, and proceed onwards there will remain a magnitude that will be less that the originally smaller magnitude chosen.*

If H2b true the % pupils of success observed	If H2c1 true weak % success	If H2c2 true weak % success	If H2d true weak % success	If H2e true weak % success
Q1, Q2, Q3, Q4 Postulate of Archimede	**Q8, Q9, Q10** Change Of context	**Q11** generalisation	**Q12, Q13** Holds out to the contradictions	**Q11, Q15** continues to manifest itself

3.5 Analysis and results

Table of behaviour of success.

	Q 1	**Q 2**	**Q 3**	**Q 4**	**Q 5**	**Q 6**	**Q 7**	**Q 8**	**Q 9**	**Q 10**	**Q 11**	**Q 12**	**Q 13**	**Q 14**	**Q 15**	**Q 16**	**Q 17**	**Q 18**	**Q 19**	**Q 20**	**Q 21**	**Q 22**	**Q 23**	**Q 24**
%	91	90	91	66	89	67	61	29	53	30	03	03	13	00	09	57	57	51	73	68	48	87	29	66

H2a: These marginal results will now be compared with the hypotheses studied with the mean of the elementary tests: we will accept a percentage of success that is significantly bigger than 50% either from knowledge, from ignorance, in the population considered.

The questions manifest knowledge of the PEA in a numerical case

Direct are {Q1, Q2, Q3} 90% of success

Inverse are {Q5, Q6, Q7} 70% of success

On the set 80% that differ significantly from 50%.

Conclusion: The hypothesis H2B may not be rejected: the pupils used better the Postulate of Eudoxus-Archimedes in direct cases rather than the inverse. Presumably there will be problems of linguistic formulation. These problems are already present in history. The inverse formulation appears later than the direct, with Pascal in 1600.

H2C1: This hypothesis afferms that when there is a **change of the context** the pupils continue to apply the old knowledge also when this is no longer valid. This behaviour is manifested experimentally with a false response to the following variables.

{Q8, Q9, Q10} {29%, 53%, 30%} 34% of success

The change of point of view considerably lowers the percentage of correct replies. It remains high, however, due to the fact that the change of the point of view is tied to the direct formulation of the postulate (the more solid conception).

H2C2: This hypothesis that concerns the **generalisation** is based on the replies to question Q11. It is asked to verify the proposition X,1 in a context of non Archimedean magnitude. To be able to reply to this question it will be needed to relook at the Archimedean and non Archimedean magnitude as classes of homogeneous magnitude. The percentage of correct replies is 1.3%. This result signifies that the generalisation is not produced.

H2d: The hypothesis affirms that the pupils conserve the hypothesis of the PEA also if it is contradicted by facts to which that hypothesis is confronted. It will be contradicted by pupils who correctly reply to questions Q12 and Q13 (Comparison of a contingent and rectilinear angle). It evokes the information, known by all the pupils, that the tangent meets a circle in a unique point.

The percentage of success to the two questions is <1%.

H2e: The hypothesis relative to the persistence of the knowledge/obstacle is tested in questions Q14 and Q15. The two questions bring up the problems of question Q11, how to be conscious of the problem; but the pupils were not able to significantly compare their interpretive models with the generalised context. The percentage of the replies to the question Q9 (the relation between the angle of contingency and the radius) was 53.3%, this proved that the pupils had taken charge of the problem.

The percentages of the two questions are <1% with a gap of <0.3.

Any of the comparisons of the percentages contradict the hypothesis of the model concerning the obstacle.

4.0 Analysis of the data

One now wishes to know

• in what measure the results that appear in the margins are confirmed and important in relation to the others arising from the variation. Do the variables studied determine the principal axes? Are the variables connected to each other or are they independent?

• Can we identify groups of pupils whose behaviour follows the same profile but correspond to two conceptions: PEA dominant (generalised) or PEA/obstacle? If not, the marginal values result from incoherent (erratic) individual behaviour and strongly determined from the situation. The obstacle will not be "knowledge of the pupils" but rather a "difficulty tied to the situations and to what they know".

The situation of the data is as follows: Matrix of contingency (active matrix): sample 107 pupils and 24 variables, Explicative matrix of 5 supplementary variables, and explicative matrix of 2 profiles of pupils.

Supplementary variables: sum of values of the characteristic variables

FT Success of the pupils (sum of all the variables)

PAD Profile "Postulate of Eudoxus-Archimedes (direct)" Q1,Q2,Q3,Q4
PAI Profile "PEA inverse"
NPA Profile of good response to "non Archimedean contexts"
RO Order relation
The profiles of the pupils are:
So1 Profile of epistemological obstacle of the PEA
So2 Profile of the negation of So1

The analysis of principal components (ACP) is used and factorial analysis of the simple correspondences (AFC)[12].

4.1 The analysis of principal components (ACP) The percentages of inertia carried by the principal axes are:

axis 1	axis 2	axis 3	axis 4	axis 5	Σ
16,8%	11%	9,9%	7,5%	6,1%	51,4%

If every question was independent of the others and carried the same information, this should account for 100/24=4.1% of the total inertia. The four first axes are hence significant.

The first principal plane (28% of the inertia) shows a strong auto correlation of the variables (all in an angle of 120°), factor G that expresses the tendency of certain pupils to have more success than all the others. The axis of the general succeses is indicated by FT, correlated with NPA.

The only variables that are accounted for in this plane (important contributions and correlations) are

•Q18, Q17 and Q16, determine the axis 1 that may be interpreted as the success with the questions in order;

•and Q1 and Q2 that determine axis 2, which must explain itself as determined by success in the questions where the direct PEA must be acquired. The questions where the PEA must be accepted are partially correlated with this plane.

The two supplementary variables RO and PAD are well placed in the two first principal axes.

The profiles with regard to So1 (a pupil who always uses the PEA and succeeds in the questions of order) and Nso1 (opposed profile) are discriminated by the 2nd axis and are found in the midst of two consistent groups of pupils who therefore could be determined by the conceptions.

4.2 Factorial analysis of the correspondence (AFC).

axis 1	axis 2	axis 3	axis 4	axis 5	Σ
13.8%	12.5%	10.4%	8.2%	7.9%	52,8%

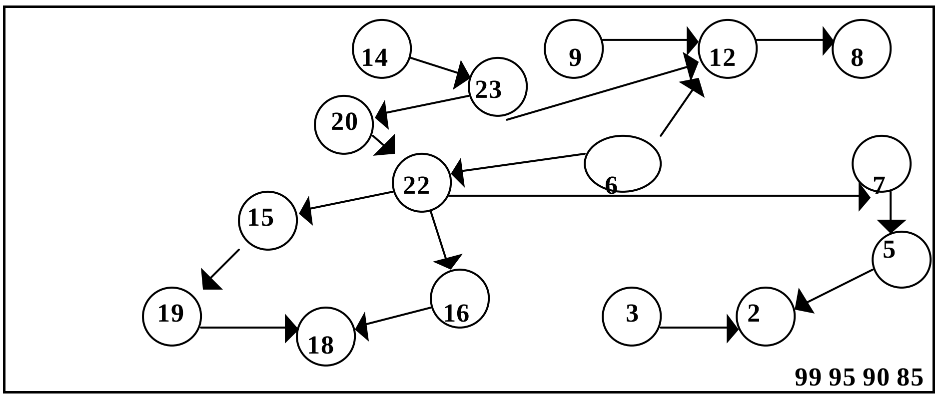
As always the factor G vanishes, FT is at the centre. Almost all the pupils are regrouped on this variable.

The profile So1 (obstacle) is almost to the centre with more than 87.8% of pupils; the profile So2 is a long way from the centre and does not attract pupils. We may therefore conclude that the Postulate of Eudoxus-Archimedes represents an epistemological obstacle. Another strong conception is that relative ti the universal use of the PEA (percentage of pupils >87.8%) in the 5 axes, 98% on the first two).

4.3 Implicative Analysis.

The same sample for the factorial analysis is used for the implication of variables, the constrution of an implicative graph and the implicative hierarchy of classes[13].

The complete sample gives the following implicative graph up to an intensity of 0.85:



The information that may provide the implicative graph is almost all bound to the historical path:

[12]The analysis was completed with the software STATITCF of the LADIST of Bordeaux.
[13]The software used is the CHIC of IRMAR of Rennes, the implicative analysis is of R. Gras.

- 9→8, the change of point of view on X,1 with the rectilinear angles implies the change of point of view with the Veronese Model in accord with the historical development.
- 12→8, the resistance to the contradictions in a context non Archimedean (curvilinear and rectilinear angles) implies the change of point of view (Veronese Model). The history of the contingent angle is in accord with this implication (Spagnolo, 1995, p.115-131, p.215-217).
- 8→23 and 22→8, the model of the Veronese implies the relation of inclusion between curvilinear angles up to when the order relation between rectilinear angles implies the model of the Veronese.
- 6→5→2, the inverse to the Postulate of Archimedes implies the direct formulation of the Postulate, confirming the historical path.
- The persistence of the obstacle implies the change of the point of view with the Model of the Veronese and the verification of X,1 with rectilinear angles (14→(9→8))
- The persistence and the change of the point of view implies the order relation between 3 triangles (the more immediate at the perceptive level) and the affirmation that X,1 is not valid. ((14→(9→8)) →17→(15↔16)
- The generalisation is tied to the change of the point of view with curvilinear angles. (11↔10)

This is a confirmation of the conditions that characterise the epistemological obstacles[14]:

- the change of the point of view;
- the generalisation.

5.0 Conclusions and prospectives

1. The characterisation of the obstacles in mathematics, proposed by Duroux-Brousseau based on the works of Bachelard, allows us, in the best of the hypotheses, to recognise that knowledge is an obstacle. It does not provide instruments for research into obstacles *a priori*.

The attempt to define some criteria that will not be either historical nor didactic to identify the obstacles of epistemological nature has brought us to adopt a semiotic approach to mathematics. Identifying the obstacles from their syntactic character in mathematical languages seems to improve their being made evident.

The theory of the situations has placed the emphasis on the role of the ergonomic and informatic characters in the origin of the knowledge and hence in the didactic processes.

Using these two areas we have demonstrated that an obstacle was tied to an important character of language, associated with the important modifications of axiomatic choice. The obstacles must be researched in the first place in the change of postulates.

Other work will be necessary to confirm the nature the epistemological obstacle of the postulates and of the axiomatic systems and the more diffuse and inversely for to examine if all the obstacles experimentally identified up to today are of axiomatic origin.

2. The Postulate of Eudoxus-Archimedes is knowledge that constitutes an epistemological obstacle for the introduction of the Hypereals and maybe also to the comprehension of non-standard Analysis. For this study we have examined two new conditions that were realised in the chosen example:

- the change of the point of view;
- the generalisation

3. The statistical methods used (factorial and implicative analysis) have indicated:

- factorial analysis of the correspondence: the introduction of the supplementary individual So1 representing the profile of a pupil that possesses, as an obstacle, the PEA, has significantly put in evidence the PEA as an epistemological obstacle.
- Implicative Analysis of Gras: The implicative graph up to an intensity of 0.85 has indicated the historical path of the obstacle, whereas the implicative hierarchical tree has confirmed the conditions that characterise an epistemological obstacle and that is the change of the point of view and the generalisation.

4. There is still a lot of work to be done to better understand the role of language in the theoretical-experimental modelling for identifying the epistemological obstacles: the relation between language and point of view; the relation between mathematical language and change of strategy.

A subsequent work could also be done to refine the statistical instruments in the determination of the model.

The theoretical-experimental model for the research of the epistemological obstacles gives the possibility to researchers in the Didactics of Mathematics to verify its validity, in certain circumstances, of the epistemological obstacles.

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