Statistics as part of the Mathematics curriculum in South Africa

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1. Introduction

There is currently an educational reform movement that is affecting the Statistics community, not only in South Africa, but world-wide. New ideas have emerged about how students should learn Statistics, what topics are most important to learn and how these topics should be taught. The inadequacies inherent in using traditional forms of assessment to measure student learning have also been brought under the spotlight (Moore, 1997; Garfield et al., 1999). With the transformation of the entire education system in South Africa, outcomes-based education is currently introduced both at school and tertiary levels. At the same time, Statistics has been included as an integral part of the Mathematics curriculum in the schools in South Africa. South Africa is a multicultural society with many different ethnic groups and eleven official languages (nine African). While certain sections of the population have excellent learning opportunities, others are notoriously disadvantaged with very poor facilities, especially in the rural areas. We are therefore faced with the challenges of adopting and integrating changes in teaching methodology occurring world-wide, with the complication of adapting these to the unique situation facing us.

2. Trends World-wide

New curricular goals for students in basic Statistics, placing more emphasis on concepts and technology, are set across the world. Deemed of more importance than the theory and mechanics of statistics, is the analysis of the data, the interpretation of the data and communicating the findings. At the same time, the way in which these ideas must get across to learners and the way in which their competencies will be assessed are also addressed. One needs only to have a quick look at the titles of papers appearing in online publications such as the Journal of Statistics Education to realise that the past decade has seen a wave of change introducing exciting and novel ways of education, e.g. Learning Statistics By Doing Statistics (Smith, 1998), An Activity-Based Statistics Course (Gnanadesikan, et al., 1997), Assessment and the Process of Learning Statistics (Hubbard, 1997), Introductory Statistics Courses - A New Way of Thinking (Roiter and Petocz, 1996); Developing Material for Introductory Statistics Courses from a Conceptual, Active Learning Viewpoint (Steinhorst and Keeler, 1995), Issues in Assessing Conceptual Understanding in Probability and Statistics (Konold, 1995) and Teaching Statistics: Making It Memorable (Sowey, 1995).

It is now widely recognised that students are more easily convinced of the power of statistical reasoning if they see it applied to questions that are interesting, real and relevant to them. It is therefore vital that realistic examples from a wide variety of disciplines are employed to demonstrate to students the critical-thinking skills that can be acquired through the study of Statistics, and that can be applied to real situations every day in almost any career (Smith, 1998; Gnanadesikan et al., 1997).

One of the ways that seems especially favoured to help students develop their statistical reasoning is to incorporate active-learning strategies where students are allowed to practise Statistics as science by designing a study, collecting data, analysing the results and preparing reports, even giving oral presentations. This aids in building students' sense of responsibility for learning and turns them into active constructors of their education (Cobb, 1993). Teaching Statistics from a more conceptual, active learning viewpoint is a challenge, however several papers, e.g. Gnanadesikan et al. (1997), present numerous examples of the types of activities that work well in various classroom settings. Their experience is that the

use of activities has not only invigorated their own teaching and improved their attitude, but that an activity-based approach also enhances learning by improving the students' understanding and motivation as they are more interested and involved in the subject. Traditionally Statistics has been taught in lectures covering the mechanics of statistical methods and the theory of probability and mathematical statistics where students' involvement is limited. This type of instruction inevitably tends to reinforce passive learning, particularly in inexperienced students (Steinhorst et al., 1995). Gnanadesikan et al. (1997) recommend that the traditional series of lectures in statistics, especially in introductory courses, should be supplemented or supplanted by a program that requires the active participation of students, working individually or in groups. Steinhorst et al. (1995) agree that there has been too much emphasis in the past on rote learning and mechanics. They also provide examples of how to reformulate traditional material based on probability, descriptive statistics and sampling distributions in a conceptual way that engages students and helps them to understand the ideas of statistics without getting lost in the details.

It is inevitable that the implementation of new curricular goals and teaching methods will lead to a re-examination of the way that the skills of learners are assessed. This topic has become almost fashionable and a number of books have been written on the subject (Gal et al., 1997; Watson, 1997; Angelo et al., 1993; Birenbaum et al., 1996). Exclusively using traditional forms of assessment to measure student learning rarely manages to test whether or not students understand statistical concepts, are able to integrate statistical knowledge to solve a problem, or are able to communicate effectively using the language of statistics (Garfield et al., 1999). The latter authors describe the use of alternative forms of assessment and advocate the use of multiple methods rather than a single test to provide a richer and more complete representation of student learning. Their innovative models for classroom assessment include individual or group projects, portfolios of students' work consisting of components such as computer output for data analyses and written interpretations of statistical analyses, concept maps, critiques of statistical ideas or issues in the news and enhanced multiple-choice questions that require students to match concepts or questions with appropriate explanations.

3. The South African Situation

Education and training in South Africa has recently been restructured to reflect the values and principles of a democratic society. In view of the country's history and legacy of inequality, curricula have been developed to accommodate the widely different groups in the country. A document entitled *Lifelong Learning through Curriculum Framework* has been put forward in 1997 by the different Departments of Education in South Africa (Free State, Gauteng, KwaZulu Natal, Mpumalanga, Northern Cape, Northern Province, North West, Eastern Cape and Western Cape) that provides a framework and guidelines for the development of learning programmes incorporating the norms and standards set out by the National Education Policy Act of 1996 (No. 27).

In the *Lifelong Learning through Curriculum Framework* document, emphasis is specifically placed on shifting from the traditional aims-and-objectives approach to outcomes-based education. This paradigm shift is seen as a prerequisite for the achievement of the vision of an "internationally competitive country". Outcomes-based curriculum development starts with the formulation of the purposes of learning and teaching and uses these as the criteria for further curriculum development and assessment. It is recommended that assessment consists of a series of tasks set in order to judge a learner's competence and achievement. These will include learners' portfolios, self and peer assessment and projects. Assessment must not only include knowledge and concepts, but attitudes and skills as well, and should take place in an authentic context.

The advancement of multi-lingualism in teaching and learning is to be fostered as this is recognized as a chance to afford learners the opportunity to develop and value their home languages, cultures and literacies and also other languages, cultures and literacies in the South African multicultural and in international context. This is a very noble principle, and many children are indeed taught by their parents to read, write and count from a very young age. Television also plays an important role in exposing children to different languages and cultures. Unfortunately, in some communities the children do not even see books before they go to school and counting skills are not taught at home for various reasons - the parents have to commute to get to work and do not have enough time to spend with their children, and often are illiterate themselves. (The whole subject of Adult Education is also being addressed in South Africa at present, but falls outside the scope of this paper.) In reality, many children do not hear any other language than their mother tongue (which may be a dialect of one of the nine African languages) at home and in their immediate environment. This, of course, complicates the learning process in general which takes place mostly in English.

Mathematical Literacy, Mathematics and Mathematical Sciences has been identified by the Education Departments as one of the eight "Learning Areas" through which the new curriculum will be implemented for the General Education and Training Certificate. One of the specific outcomes identified is the "Use of data from various contexts to make informed judgements". This serves the ever-increasing need to understand data and translate it into usable knowledge in this technological age of rapid information expansion. The assessment criteria include the collection of data (methods such as interviews and sampling), the application of statistical tools and the communication and critical evaluation of findings.

The South African educational system finds itself in the unfortunate situation of having a severe shortage of qualified Mathematics teachers on the one hand, and a glut of teachers in other methodologies such as history, geography and languages on the other hand. Many of these teachers face redundancy unless they are retrained. Furthermore, since Statistics has only recently been included as an integral part of the Mathematics curriculum in the schools in South Africa, few Mathematics teachers have any training in Statistics. (This situation is not unique to South Africa. Cobb, 1993, points out that there is considerable evidence to suggest that only a small fraction of students in the USA are taught Statistics by statisticians or others with any substantial and recent training in the subject.)

This problem is recognised by statisticians and professional bodies, and several attempts are being made to address it. The South African Statistical Association (SASA), mouthpiece of the majority of professional and practising statisticians in South Africa, is actively involved in the teaching of Statistics in schools via their Education Committee: a subcommittee has been appointed with the specific brief of School Education and Conference Workshops aimed at school teachers. This committee is also building up a collection of games, projects, newspaper articles, etc. which can be used by Mathematics teachers to teach Statistics in stimulating ways and has called for contributions country-wide via the association. Contact has also been made with the Association for Mathematics Education of South Africa (AMESA) with a view to closer collaboration. It is envisaged that SASA members will take an active part in future congresses and meetings of AMESA, as well as contributing to the AMESA newsletter. Further illustrating the commitment of SASA to the teaching of Statistics, South Africa will host ICOTS-6, the sixth international conference on the Teaching of Statistics, in Durban, South Africa, during July 2002, under their auspices.

At Unisa we are trying to help alleviate the problem by offering one-year certificates in the teaching of Mathematics, Physical Science and Computer Science. By this means teachers

will be able to upgrade their knowledge and skills in order to become good Mathematics teachers. In the process we as statisticians will play a dual role, namely enabling these learners to acquire knowledge and skills in Statistics as well as classroom practice to achieve the outcomes required in the school syllabus. A basic kit consisting of random experiments and a selection of newspaper clippings, which is being planned, will be useful as an aid to the teacher in the classroom.
Because Statistics textbooks, especially those coming from first world countries such as the USA, are less than ideal for African students with examples in such textbooks often based on experiences and situations which the learners do not understand at all, the need for suitable teaching materials and a good introductory Statistics textbook for sub-Saharan Africa is of crucial importance. It is sad but true that learners are often unable to solve problems, not because they do not know how to solve problems, but because they fail to identify with, or understand the problem settings.

4. Training the Teachers in South Africa: Classroom Practice

4.1 Using newspaper clippings for classroom discussions

Many news items in the daily press contain statistical information that may be used for discussion in the classroom. The average teacher will need some training in how to do this and how to spot suitable items in the daily press and television news. The Science Faculty at Unisa hopes to make a contribution in this respect by means of teacher training, notably by upgrading the skills of existing teachers.

One of the skills that need to be taught is recognising fallacious statistical arguments. These include the mistake of deducing a causal relationship when a correlation has been found and extrapolating data to a population when the sample is not representative of the population.

Examples of confusing correlation with causation are numerous. The most common type occurs in the medical field where, for example, a person who was very fond of carrots developed tooth decay and then it is concluded that carrots are bad for your teeth. More subtle examples regularly appear in the press, particularly about various substances being linked to cancer or heart disease, and may be discussed in the classroom, especially finding alternative explanations for the correlation.

Another good example occurred in the field of child psychology. A researcher investigated the relationship between the time spent in front of the television set by school children and various personality traits. One of the strongest relationships was found between the number of hours spent watching television and aggression. This was a sensational finding that reached the front pages of the newspapers: watching television causes children to become aggressive. However, alternative interpretations are possible: aggressive children get to watch more television or the children were aggressive due to lack of sleep.

Topics for discussion in the classroom could be:

- What do the children believe to be the reason for the relationship between number of hours spent watching television and aggression?

- What would it take to prove a causal relationship (e.g. a randomised experiment on identical twins)?

HIV/AIDS is a serious problem in South Africa, and the newspapers regularly carry items about it. The proportion of people affected varies widely, depending on the source of the information. This may be the topic of a discussion in the classroom: how can the proportion

of people infected by HIV be estimated? Along with this, there are the social, medical and economic consequences of HIV/AIDS that may be discussed.

One of the interesting recent news items in the South African newspapers concerned a pressure group campaigning for free anti-AIDS medication for rape victims. They claimed that, since 1 in 7 people in South Africa are HIV positive, and the 23 cabinet ministers in parliament are representatives of South Africa, it means that 3 or 4 cabinet ministers must be HIV positive. This could lead to a discussion about a number of issues:

- What it means for any group to be representative (vs. representatives);
- By this argument, how many pupils in the class must be HIV positive?
- How many members of my immediate family can be expected to be HIV positive?
- What is the flaw in this argument?
- How was the ratio of 1 in 7 determined?
- Can you believe information distributed by a group with an agenda?

Graphs are important communication aids, but many people do not know how to interpret them. Bar charts, pie charts, histograms and trend lines are used particularly in the business sections of the newspapers. Learners must be taught to interpret such graphs, and examples of graphs published in the newspapers are good material for class discussions. Companies or pressure groups like to use graphical displays to create an impression that may not be completely accurate. This includes not naming the axes or not specifying the units on the axes. Another strategy is to truncate the axes from below, which exaggerates any increasing or decreasing trend. The learners should be encouraged to look out for such misleading graphs and to bring them to school for discussion in the classroom.

4.2 Sample Surveys

The concept of random sampling should be explained to the learners, and with it the idea of random variation. A simple example may be constructed by writing down particular information about each learner in the class on a card (e.g. their weekly allowance, their marks in the mathematics test or the size of their families), and placing the cards in a box. Each pupil in turn selects a sample of size 10 from the box, writes down the numbers appearing on the cards, returns the cards to the box and calculates the average of the 10 values in the sample. A histogram of the averages obtained by everybody in the class may then be compared to a histogram of the original values. Biased samples may be created by first removing a few of the cards with high values, and repeating the exercise. Points for discussion in the class:

- Comparison of the histograms of the original data and the (unbiased) random samples; the fact that their means are about the same but the spread of the sample means is smaller than the spread of the original values; why this is the basis of random sampling methods;
- Comparison of the histograms of the biased and unbiased sample means; the means are different but the spread is about the same; why, in general, biased samples give rise to misleading conclusions.

An example of a survey that is suspect as far as the randomness and representativeness of the sample are concerned, appeared in a local newspaper, the *Pretoria News*, recently. It was stated (quite sensationally, also on the newspaper posters) that 20% of the people in the Greater Pretoria area are HIV positive. This figure is based on information gained about people who visited provincial hospitals in other areas (mainly Johannesburg) in the Gauteng province (the province in which Pretoria is located). These provincial hospitals are rarely visited by people who belong to a medical scheme, and this immediately gives rise to a suspicion of bias. In addition, it is stated that no survey had been done in Pretoria because of the risk that the statistics might be skewed by the smaller numbers (of residents in Pretoria?).

Points for discussion in the classroom:

- Is this information based on a random sample?
- Are people who visit provincial hospitals representative of the population of Gauteng?
- Are the people who visit provincial hospitals even representative of the sick people in Gauteng?
- Is it safe to extrapolate HIV figures for Johannesburg to Pretoria?
- Could there be an association between poverty and the prevalence of HIV infection?
- How can statistics be "skewed by smaller numbers"?

An interesting subject for planning a sample survey appeared recently in the local newspapers. The south-western coast of South Africa is famous for the crayfish living there, and it provides for a thriving industry. From time to time there is a plankton problem ("red tide") and then the crayfish walk out of the ocean in their millions, only to die on the beach. Pictures of these beached crayfish are quite spectacular. An estimate of the amount of lost crayfish might be "between 1 and 2 tonnes". The following discussions might be appropriate:

- Some of the learners may never have seen a live or real-life crayfish, and some discussions may be held about what it is, what its utility to people is and about the size of such an animal;
- How would one estimate the amount of crayfish on the beach if asked to do so (at the site)? One possibility would be to measure the stretch of beach involved, then on a random sample of 1 metre sections of the beach count the number of crayfish per metre in order to estimate the total number of crayfish; finally obtain a random sample of crayfish in order to obtain an average weight per crayfish. The product of these two estimates will be an estimate of the total weight of the crayfish;
- The cost per crayfish or per weight of crayfish may then be used to estimate the monetary loss due to the beaching of the crayfish;
- Other issues, such as pollution of the beach, the smell, the bonanza for birds of prey and other predators, may increase the general interest of the discussion.

To acquire experience in conducting a survey, some of the questions which might be researched by the learners are the following:

Estimating the proportion of people who are left-handed:

- By asking a random sample of learners whether they are left-handed;
- By asking a random sample of learners to fill in a simple form, and observing whether they use their left hands.

Additional points for discussion are:

- Estimating the total number of left-handed people in the country;
- Discussing whether it would pay an entrepreneur to manufacture left-handed scissors, egg lifters, etc.;
- Asking a left-handed learner to tell the class about the difficulties experienced by left-handed people.

Estimating the number of learners in the school who smoke (or who have experimented with drugs):

Points for discussion in the class:

- The problem of dishonesty in answering a question in a survey, especially if the interviewee may feel that he/she may be compromised by answering truthfully;
- The problem of non-response and bias which may result from it;

- Will it help if measures are taken to ensure confidentiality?
- The health dangers of smoking (or using drugs);
- The cost of smoking (or using drugs);
- Why it is not a good idea to smoke (or use drugs).

4.3 Statistical Experiments

A statistical experiment is one way of showing a causal relationship. Some experiments which may be done in the class:

Determining the effect of different nutrients on the growth of plants:

A number of containers (plant pots) are prepared. Seeds of fast growing plants such as barley are planted in each container. The containers must be identical and the same number of seeds must be planted in each pot. The pots are randomly allocated to the treatments, the latter being different nutrients. The response variable will be the height of the plants after three weeks. The following may be discussed in the classroom:

- The importance of random allocation of containers to treatments;
- The importance of replicates (more than one container being allocated to the same treatment);
- Identical conditions for all the containers (watering, light etc.);
- How the data may be represented;
- What conclusions may be drawn from the results of the experiment.

Comparing paper planes:

A number of paper planes are made from two different types of paper and with two tail lengths. A number of learners take turns launching the planes and the distance travelled by the plane is measured. Discussions about the experiment:

- The importance of a balanced experiment;
- The importance of replicates;
- Representation of the data;
- Conclusions which may be drawn from the results.

4.4 Understanding probability

The concept of probability can easily be explained by means of simple experiments involving coins, dice and bowls with coloured beads. An interesting real-life application involves the weather forecasts. A probability of rain is provided as part of the forecast, along with the area for which the probability holds. Few people know what these probabilities mean. They are derived from a mathematical model, sometimes with the forecaster's intuition added in. One way of trying to understand these probabilities is to keep a record of the forecast for the area in which the learners live as well as the outcome each day (whether or not it rained and, if a rain gauge is available, the amount of rain). After a season the class may make an evaluation: lumping all days in the season with the same probability forecast, and calculating what proportion of the days actually had rain and the amount of rain recorded on days with the same probability.

5. Summary

The emphasis that is placed on the understanding of statistical concepts and the integration of statistical knowledge to solve problems (i.e. the analysis of the data and the interpretation of the data and the results) as well as the skill to communicate effectively about data and chance using the language of Statistics are intrinsic to the transformation that is taking place in South Africa's entire education system where outcomes-based education is now the goal both at school and tertiary levels. This is in accordance with trends world-wide, but is complicated

by the unique situation of learners in this country. With our different cultures, linguistic complications and contrasting socio-economic conditions, we in South Africa are facing the 21st century with the serious challenges of upgrading our teachers' mathematical skills as well as providing statistical training to these Mathematics teachers, enabling them to effectively produce competent learners that will emerge from their school education as "literate, creative and critical citizens leading productive, self-fulfilled lives". We share the sentiments of Cobb (1993) that "We must be realistic in judging how fast we can change, but at the same time be ambitious in our choice of direction and ultimate destination". The latter seems to be embedded in the new approach to education in South Africa, and the first will be propelled by our commitment and enthusiasm.

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