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Spagnolo • Di Paola (Eds.)

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I really enjoyed reading this book and learned a lot from its compelling analysis.
Shuhua An, Associate Professor and Director of Graduate Program in Mathematics Education, California State University, Long Beach



**European and Chinese Cognitive Styles and
Their Impact on Teaching Mathematics**

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European and Chinese Cognitive Styles and Their Impact on Teaching Mathematics

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Preface

Recently, mathematics educators have shown enthusiasm about conducting comparative studies involving East Asian countries, primarily because students of East Asian, such as Chinese students, show outstanding performance in various international assessments. Culture, teachers' knowledge, instruction, curriculum, and assessment have been identified as factors contributing to the achievement gap in various studies. However, the authors in this book argued that cognitive processes in the socio-cultural context play an important role in the differences in student mathematics learning. They analyzed the cognitive processes based on the comparison of Italian and Chinese cultural contexts both in historical and in epistemological aspects.

The investigation of the nature of the difference in student learning between Italy and China is the focus of this book. For so many years, international comparative studies in mathematics education focused on the difference in external aspects in student learning, and less attention was on the differences in the linguistic language structure in a socio-cultural context. The authors of this book broke new ground by contributing to a better understanding of how Chinese students acquire algebraic reasoning from an epistemological and linguistic aspect. By thoroughly examining the representative text of ancient Chinese mathematics culture (*The Jiuzhang Suanshu, Nine Chapters on the Mathematics Art*) and investigating the linguistic structure of the Chinese written language, the authors identified differences in the hierarchies of the reasoning models generally used by Chinese students and Aristotelian-Euclidean mould by Italian students in five experimental situations. The deeper study of the Chinese cultural origin led to an analysis of historical aspect of mathematics and specifically the evolution of algebraic thoughts over the different cultural traditions.

Notably, both qualitative and quantitative methods were well-employed in this study. The instrument was well-designed based on the purpose of this study and based on European and Chinese cultural and historical contexts. The authors used a great deal of effort on the data collection which used multiple sources, such as survey and semi-structured interviews, documents, observations, to enhance the validity of the design. The five sessions of the research experiments were sequential, building upon one another. Cohesive hierarchical implicative classification was used appropriately for quantitative analysis of experimental data.

The analytical narrative and discussion in the presentation focused on the diversity of cognitive styles, solution strategies, and learning methodologies as opportunities in mathematics. The book provides an excellent model in which it identifies linguistic, historical, neuro-scientific, psycholinguistic, and the didactic aspects in study of acquisition of algebraic competences in mathematics for the

students as it is shown in discussion and analysis of the Chinese particular system of graphemic fixation and specific characteristics of Chinese writing. The charts and tables of the participant demographic information were very helpful and informative, and the statistical illustration in writing used various figures and tables, which organizes the information of the study in a visually meaningful way and provides readers with a deep understanding of the study.

The authors used rigorous and unique methods to analyze and draw conclusions from the data by structuring data analysis in different types and chapters. More commendable, treating cultural diversity as an opportunity in mathematics by deeply studying Chinese mathematics from the arithmetic to the algebraic content in the historical aspect-cultural puzzle, is excellent approach in culture-related studies.

The book provides strong evidence that research on the cognitive processes, from arithmetic thought to algebraic thought, should take into consideration the socio-cultural context. It is an important contribution to the literature on linguistic structure in comparative studies related to Chinese student mathematics learning.

This book not only makes a great contribution to research in mathematics education, but the findings of this study also addressed insightful approaches and thoughts of understanding the development of algebraic thinking in cultural contexts for classroom teachers. Using written Chinese language from different theoretical references provided wonderful approaches for understanding student algebra cognitive development in a different way and calls educators to pay special attention to an epistemological and linguistic view of algebraic development. The findings inform classroom teachers that the cultural context plays an important role in student learning mathematics. A typical analysis of the cognitive dimension involved in some in the historical and cultural contexts is a great resource for classroom teachers.

I really enjoyed reading this book and learned a lot from its compelling analysis. I am fascinated by the topic and the method of analysis of data from historical and cultural perspectives. The authors' efforts in investigating student cognitive process from arithmetic to algebraic are extraordinary, especially the connection to linguistic, historical, the neuro-scientific, psycho-linguistic, and the didactic aspects. This study is very valuable to the mathematics education field and a remarkable contribution to international related studies.

As a Chinese scholar, I admire Dr. Benedetto Di Paola's brilliance in investigating the relationship of Chinese linguistic structure and student algebra thinking. I greatly appreciate his hardwork and excellent contribution to the mathematics education field. Special thanks go to Dr. Filippo Spagnolo's great effort and dedication to the mathematics education field and his wisdom of advice and support to Dr. Benedetto Di Paola's research relating Chinese culture and language. Their team has been very successful and fruitful.

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Introduction

The increase of different culture pupils represents, for the last decade, the greatest novelty in the Italian school. This growth will probably be constant for the next future.

After a first period of simple notation of the situation, the Italian scholastic system and, inside of it, the teachers begun to warn the necessity to intervene and, consequently, to consider the possibility to foresee actions that kept in consideration the change of the cultural context, but also the social one, of the classroom.

In the large majority of the cases, these actions are shaped in the form of training in-service courses on general themes of the interculturality and the intercultural pedagogy for the teachers. Parallely, absolutely inevitable, different initiatives of teacher training to the didactics of the Italian language as second language, were made.

For many years, the presence of different culture pupils in the classes was considered as a “problem” regarding only the general pedagogy, for the teachers, and regarding the language, for the pupils. Namely, the people thought that, it was not necessary for other things except to furnish these tools that are essentially communicativeness.

The reality of the classroom shown and still now show how this assumption was and is enormously wrong. It shown this to the most careful teachers and to the ones that, for different reasons, warned only recently the “problem”. Among these last ones we have to include almost the totality of the mathematics teachers!

The assumption that the “problem” of the presence of different culture pupils in the classroom and the “problem” of the multicultural classes are, exclusively, a problem of communication is nearly found on the belief of the teachers that, from one side, a specific subject is a “corpus” rigidly structured in their contents and in their formality of transmission and that, from another side, the socio-cultural background of the student does not influence, in general, his learning process if not for what it achieves from his not complete or scarce knowledge of the vehicular language, or from the different level of schooling with which he entered the classroom.

This assumption is very rooted among the mathematics teachers, from the great majority of the teachers who see the subject taught as a “universal language” and “culture free”, and so not determined and conditioned, also in its contents, from the culture of the different societies.

The research works of D’Ambrosio, since the first years 1980, and the research works of Bishop, from the beginning of the second halves of the same years, clearly shown instead as such reading and vision of the mathematics are to be considered limited because they essentially reduce the mathematics to a complex of

notions and knowledge that are transmitted in a scholastic ambit. Complex of notions and knowledge that, even if it is vast and remarkable from the academic point of view, it cannot be considered as comprehensive of all the mathematical knowledge, presented in various forms and used in different social and cultural contexts, also not scholastic. Correctly, D'Ambrosio and Bishop have shown instead, even though with different approaches, that we have to speak of mathematics or in a better way of mathematical activities, rather than of mathematics.

If it is accepted then to consider the mathematics as a complex of activities (to count, to measure, to locate, to draw, to play, and to explain), we see the mathematics as a cultural product, gotten by the exercise of such activities. On the other hand, even if such activities can be sights how common to all the societies as categories, they often differ themselves in their formalities of carrying out. In effects, these differences are not only revisable when very distant society (macro-cultures), are observed in their complex system but also when single societies are analyzed (micro-cultures).

A reading in cultural key of the mathematics inevitably asks a more punctual analysis on the single societies has developed their own cultures. It is necessary to keep in mind of different factors, beginning from the principal ones in anthropological ambit, historical ambit, linguistic ambit, religious ambit, and technological ambit.

These factors are integral part of a culture, that produces them and it is developed from them, in a continuous process. They are also integral part, therefore, of a correct and complete analysis of the mathematics, as cultural product. To these factors it is necessary to refer, therefore, also when we intend to examine how the mathematical knowledge is transmitted and receipted inside one determined society or social group.

Without doubt and meritoriously, Filippo Spagnolo and Benedetto Di Paola worked in this direction in their work of analysis and comparison, as specifically regards the phase of teaching/learning of the mathematics of two different cultural realities as Italy and China, so differ among them.

The deep difference of these two cultures is also warned evidently in clear way from the teacher, who often see these represented more and more and in meaningful way in his/her own classroom. Such difference often means, also for the teacher of mathematics, an element of difficulty in comparison to the use of the usual didactic methodologies. A teacher of mathematics, aware of this difficulty, will find in this research work elements of knowledge and sprouts of reflection that should allow them to better facing the novelty of the didactic intercultural situation. Reality in which they operate and to make more profitable, for the whole class, the process of teaching /learning of the mathematics subject.

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Contents

Introduction	1
1 A General Framework and Theoretical References	3
1.1 Principal Objective of the Book	3
1.2 The Experimental Epistemology of the Mathematics as Paradigma of Reference	3
1.3 The Differences and the Analogies of the Carrying Cultural Coordinates.....	4
1.4 What the References with the Mathematics, with to Deduce and to Conjecture (and to Conceptualize)	5
1.5 Experimental Epistemology, Didactics of Mathematics, Communication of Mathematics, Semiotics: Possible Theoretical Interpretation	6
1.5.1 Conclusions	10
2 The Chinese Written Language as Tool for a Possible Historical and Epistemological Reflections on the Mathematics and the Impact of Teaching/Learning of Mathematics.....	13
2.1 The Written Chinese Language as Possible Tool to Acquire Algebraic Competences in Mathematics.....	13
2.1.1 The Point of View of the Neurosciences	14
2.1.2 Origin and Development of the Chinese Writing: Some Fundamental Step	15
2.1.3 The Pinyin and the Chinese Language in the Chinese School	17
2.1.4 Observations Cognitive on the Chinese Language	19
2.1.5 Chinese Language and Mathematics	22
2.1.6 Rules of Composition of a Chinese Character	24
2.1.7 The Classification of the Chinese Characters	25
2.1.7.1 Constitutions of Ideograms and Algebraic Thought	27
2.1.8 The Research of the Characters on the Dictionary.....	30
2.1.9 The Parametric Structure of the Written Chinese Language.....	30

2.1.10	The Meta-rules of the Language and the Mathematics	32
2.2	Can the Natural Language Influence the Educational System of the Area Logical-Mathematics? Which the Impact in the Teaching of the Mathematics?	34
2.2.1	Contents and Objectives in the Historical Evolution of the Teaching of the Mathematics in the Chinese School: A Brief Panning	34
2.2.2	The Teaching of the Mathematics in the Today's Didactic Practice	39
2.3	A Resume Chart on the Relationship Natural Language Mathematics.....	44
3	The Meta-rules between Natural Language and History of Mathematics.....	47
3.1	The Algebra between the History and the Didactics, a Variety of Perspectives	47
3.1.1	The Algebraic Tradition in Antiquity: An Overview.....	52
3.1.2	The Elements of Euclid as a Canon of Reference for the Western Mathematical Tradition	57
3.1.3	An Initial Comparison between the “Elements” of Euclid and the “Jiuzhang Suanshu”	58
3.1.4	One Possible Comparison between Diofanto and Babylonians	59
3.1.5	The Algebraic Tradition among the Arabs and the Indians	60
3.1.6	Significant Developments for the Symbolic Algebra	69
3.2	A Look at the Ancient Chinese Mathematics	73
3.2.1	The Historical-Cultural	73
3.2.2	The Jiuzhang Suanshu	75
3.2.3	The Rule of Fangcheng as Meta-rule for the Chinese Algebra	80
3.2.4	Argue, and Demonstrate in Conjecturing Jiuzhang Suanshu an Example through the Algebra	84
3.3	Conclusion	87
3.4	How to Summarize the Role of History?	89
4	Common Sense and Fuzzy Logic.....	91
4.1	Fuzzy Logic, Fuzzy Thinking and Linguistic Approach.....	91
4.2	Fuzzy Sets and Their Representations	92
4.3	The Representative Point of View of Kosko	95
4.4	Some Epistemological Reflections on the Approaches to the Fuzzy Logic	98
4.5	Some Experimental Observations on Common Sense and Fuzzy Logic	99
4.5.1	Common Sense and Fuzzy Logic, M. Ajello and F. Spagnolo ..	99
4.5.2	Fuzzy Logic and Complexity.....	100
4.5.3	The Hypothesis	101

4.5.4	The Assignment a Computer Science Activity (Time: 3 Hours)	102
4.5.5	A Priori Analysis	103
4.5.6	Implicative Analysis Using the Chic Program.....	104
4.5.7	From the Pupil Records	105
4.5.8	A Possible Interpretation Key	106
4.5.9	Conclusions.....	107
4.5.10	Open Problems.....	108
	Conclusions	117
5	The Experimental Epistemology as a Tool to Observe and Preview Teaching/Learning Phenomena.....	119
5.1	The Experimental Context	119
5.1.1	Choice of the Problematic Situations in Accordance with the Theoretical Framework and the Hypotheses of Search.....	119
5.1.2	The First Experimental Investigation: "Sudoku Magic Box"	127
5.1.2.1	A Priori Analysis.....	130
5.1.2.2	Quantitative and Qualitative Analyses of the Data	131
5.1.2.3	Second Experimental Investigation: "The Problem of Fermat", "Varying Questionnaire and Parameter in Contexts Different Semiotic"	136
5.1.2.4	A Priori Analysis: "Fermat Problem"	145
5.1.2.5	Quantitative and Qualitative Analysis: "Fermat Problem"	146
5.1.2.6	A Priori Analysis: "Questionnaire Variable and Parameter in Different Semiotic Context"	151
5.1.2.7	Quantitative and Qualitative Analysis: "Questionnaire Variable and Parameter in Different Semiotic Context"	154
5.1.2.8	Third Experimental Investigation: "The Sequence", " The Grid of Numbers"	165
5.1.2.9	A Priori Analysis "The Sequence"	170
5.1.2.10	Quantitative and Qualitative Analysis of "The Sequence"	171
5.1.2.11	A Priori Analysis of "The Grid of Numbers"	174
5.1.2.12	Qualitative Analysis of "The Grid of Numbers"	174
5.2	From Natural Languages to Logical Linguistic Aspects: Cognitive Styles of European and Chinese Students, M. Ajello and F. Spagnolo.....	185
5.2.1	Some Reflections on "Arguing, Conjecturing and Demonstrating" in Chinese Culture with Relation to Occidental Culture	186
5.2.2	The Algorithm as Fundamental Element of Arguing and Demonstrating?.....	186

5.2.3	What Are the Stable Reasoning Patterns in Chinese Culture?.....	187
5.2.4	How Were the Situations/Problems Chosen?	189
5.2.4.1	The a Priori Analysis of the Situations. We Report the a Priori Analysis of the Hypothesized Behaviours.....	190
5.2.4.2	Presentation of the Experimental Work in the Italian Classes	191
5.2.4.3	The Quantitative Analysis: Implicative and Factorial.....	191
5.2.4.4	Implicative Analysis and of the Similarities	192
5.2.4.5	General Considerations on Factorial Analysis, Implicative Analysis and of the Similarities	193
5.2.5	Interviews with Two Chinese: Qualitative Considerations.....	194
5.2.6	The Experience in the Chinese Classes.....	195
5.2.6.1	Factorial Analysis	196
5.2.6.2	Supplementary Variables	196
5.2.6.3	Considerations on the Analysis of the Data Relative to the Chinese Sample	197
5.2.7	General Conclusions and Future Perspectives	197
6	Strategy and Tactics in the Chinese and European Culture: Chess and Weich'I , G. D'Eredità and F. Spagnolo	201
6.1	Introduction	201
6.2	Strategy and Tactics.....	203
6.3	Notes on the Conceptions of Strategy and Tactics in the Orient and the Occident	205
6.4	Historical Games of Strategy in the Orient and Occident: Chess, Wei-ch'i and the Different Conceptions of Strategy and Tactics	206
6.5	Connection to Didactics and Open Problems	211
6.6	Conclusions	212
	References	212
7	Rhythm and Natural Language in the Chinese and European Culture, D. Galante and F. Spagnolo.....	219
7.1	Rhythm and Natural Language in the West	219
7.2	Rhythm and Natural Language in China	222
7.3	Conclusions	227
	References	227
8	Conclusions.....	229
8.1	The Conclusions of the Experimental Work.....	229
8.1.1	Topics Such as Research Agreements?.....	230
8.2	The Experimental Epistemology of Mathematics: Concluding Observations on the Experimental Investigations Discussed.....	231
8.1.2	The First Experimental Investigation.....	232
8.3	The Second Experimental Investigation	234

8.4 Conclusions and Open Problems Related to "Strategy and
Tactics"238

8.4.1 Links with Education239

8.5 Conclusions and Open Problems Related to Music.....240

8.5.1 The References to Mathematics, the Reasoning and the
Conceptualisation240