

## *History and Ethno-Mathematics in the interpretation of the process of learning/teaching*

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**Abstract:** In the cultural integration the relationship between mathematics languages and natural languages has a very important role.

In mathematics education there is an important problem about the relationship between Epistemology, History and communication of mathematics.

If the interpretation of phenomena learning/understanding utilises the semiotic approach of mathematics we have the instruments to analyse verbal messages and non-verbal messages.

A classification of semiotic interpretations of history of mathematics as 1) history of syntax of mathematics languages, 2) history of semantic of mathematics languages, 3) history of pragmatic languages, give the instruments of interpretations a priori of understanding/learning phenomena.

The representations historic epistemological are the possible routes to knowledge of pupils (a priori and a posterior)

In this prospective the history of mathematics is of service in research in Mathematics education: the researcher has special requests for the historian of mathematics.

In this paper we present an experience of preparation of a Chinese man in the examination in the last class of college. We compare structure of chinese language with algebra. The history of mathematics is a basic key to interpret the didactic problem.

**Key Words:** Epistemology, History of Mathematics, semiotic, semantic, syntax, pragmatic, research in mathematics education, Natural language, mathematical language, chinese language.

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### ***Introduction***

In the study of process learning/teaching an important role has the history of mathematics (Spagnolo, 2000).

In this paper we present a theoretic reference of point of didactic view and many examples of situations of teaching in different cultures.

In particular the theoretic study has as reference theory of situations of Guy Brousseau (Brousseau, 1997, 1998) with a semiotic revision in this connection with interpretations of phenomena of learning/teaching (Spagnolo, 1998).

The semiotic point of view is very important in all three components: syntactic, semantic and pragmatic (Spagnolo, 2001).

A paradigmatic example is pertinent with a particular approach of ethnomathematics in the Chinese culture.

### ***1.0 THE HISTORY IS RICH OF "SENSES": STRATEGIES OF RESOLUTION OF PROBLEMS***

A trivial example of resolution of problems in history of Mathematics in western culture.

"A table of wood, to which have been cut the fourth part, has length 135 cm. How much was the entire table?"

#### **1.1 Method of successive approximation (Babylonian)**

- We suppose that it be long 150 cm ,

$$\frac{1}{4}150\text{ cm} = 37,5\text{ cm} \quad 150\text{ cm} - 37,5\text{ cm} = 112\text{ cm}$$

2. They try so with 200cm  $\frac{1}{4}200\text{ cm} = 50\text{ cm}$   $200\text{ cm} - 50\text{ cm} = 150\text{ cm}$
3.  $150 < x < 200$ , and so ...

#### **1.2 Method of false position (Papir of Rhind)**

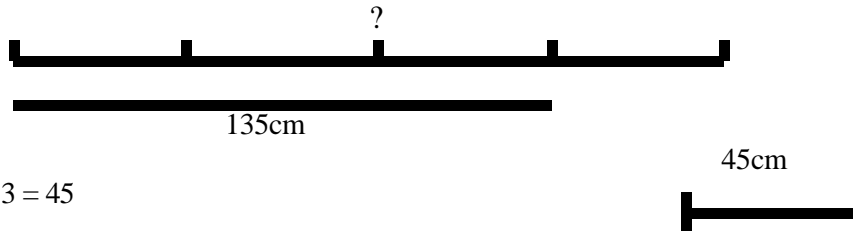
We suppose to have the solution: 150 cm.

$$\frac{1}{4}150\text{ cm} = 37,5\text{ cm} \quad 150\text{ cm} - 37,5\text{ cm} = 112,5$$

$$x : 135 = 150 : 112,5$$

$$x = \frac{135 \cdot 150}{112,5}$$

### 1.3 Geometric method (Euclid 300 a.c.)



$$135 : 3 = 45$$

$$45\text{cm} \times 4 = 180\text{cm}$$

### 1.4 Algebraic Method:

$$x - \frac{1}{4}x = 135$$

$$\frac{4x - x}{4} = 135$$

$$\frac{3}{4}x = 135$$

$$x = 135 \cdot \frac{4}{3} = 180.$$

## 2.0 HISTORY COMES TO MEET THE INTERPRETATIONS OF EXPERIENCES OF ETHNOMATHEMATICS

In the ethnomathematics approach there is the importance of language in relation with the context.

This paper analyzes an experience with 3 interpretative dimensions: languages with syntactical and semantic relations, history of mathematics and style of understanding.

### 2.1. The experimental context: The experience of Tong.

The Chinese man, Tong (age: 32)

Preparation of Mathematics should be examined in the last class of college (age: 13).

The classic problems in the Italian classroom are:

"The lateral area of right prism is  $m^2$  2016 and the base is a right-angled triangle with the hypotenuse  $m$  20 and a cathetus  $\frac{4}{5}$  of hypotenuse. To compute the volume of right prism."

The solution of Tong:

$$\begin{aligned}
V &= \left\{ \frac{\sqrt{(20^2) - (20 \cdot \frac{4}{5})^2} \cdot (20 \cdot \frac{4}{5})}{2} \right\} \cdot \left\{ \frac{m^2 2016}{\left[ 20 + (20 \cdot \frac{4}{5}) + \sqrt{(20)^2 - (20 \cdot \frac{4}{5})^2} \right]} \right\} \\
V &= \left\{ \frac{\sqrt{400 - 256} \cdot 16}{2} \right\} \cdot \left\{ \frac{m^2 2016}{\left[ 20 + 16 + \sqrt{400 - 256} \right]} \right\} \\
V &= \left\{ \frac{12 \cdot 16}{2} \right\} \cdot \left\{ \frac{m^2 2016}{48} \right\} \\
V &= m^2 96 \cdot m 42 = m^3 4032
\end{aligned}$$

I showed an other problem:

"The bases of right trapezium measure cm 32, cm 20 and the oblique side cm 13. To compute the area of surface."

The resolution, accurately transcribed, was:

$$\begin{aligned}
\overline{AB} &= cm20 \\
\overline{DC} &= cm32 \\
\overline{BC} &= cm13 \\
A_s &= \frac{(cm32 + cm20) \cdot \sqrt{(cm13)^2 - (cm32 - cm20)^2}}{2} \\
A_s &= \frac{cm52 \cdot \sqrt{(cm13)^2 - (cm12)^2}}{2} \\
A_s &= \frac{cm52 \cdot cm5}{2} = cm^2 130.
\end{aligned}$$

(I have collected 50 problems solved of Tong.)

In synthesis the procedure was :

- What are the things that I know (AB, BD, BC)?
- What are the things that I do not know (EC, BE)?
- What formula must I to utilize?

Write the resolute formula after having totally thought like equation deduced by literal identity.

In Italian schools analogous problems solved with the way of argue that could call "Euclid's". This way consists, step by step, in obtaining the elements for the resolution in the final formula.

For example the above-mentioned problem could be solved in the following way:

$$\begin{aligned}
\overline{AB} &= \overline{DE}, \\
\overline{EC} &= \overline{DC} - \overline{DE} \\
\overline{EC} &= cm32 - cm20 = cm12 \\
\overline{BE} &= \sqrt{\overline{BC}^2 - \overline{EC}^2} \\
\overline{BE} &= \sqrt{(cm13)^2 - (cm12)^2} = cm5 \\
A_s &= \frac{(B+b) \cdot h}{2} \\
A_s &= \frac{(\overline{DC} + \overline{AB}) \cdot \overline{BE}}{2} \\
A_s &= \frac{cm52 \cdot cm5}{2} = cm^2 130.
\end{aligned}$$

The method of Tong is an "algebraical" way.

Why the synthetic solution ?

Why the solution was not in many steps like the western pupils did ?

## ***2.2 The Context of official book***

In the chinese books the resolution is with "Euclid's method".

## ***2.3. The context of structure of chinese language***

Needham (1959) says that the mental mechanism for building and recognition of ideograms for association is a "mental equation".

" More primitive elements of Chinese language were generally pictograms, that is pictures reduced to the essential, made conventional, at the end very stylised. Naturally, concrete objects as the heavenly bodies, animals, plants, implements and instruments could be easier pictured. We reported some in the first part of the list two, having them origins in a Haluon's short popular leader. You will note, the most part of them, in the course of the time, have been included in radicals, (you can see later); but, it isn't always like that: hsiang (elephant) isn't a radical, but it has been classified under the radical number 152 (shih, pig) on the other hand, hu (wine's recipe) has been classified under the radical 33 (shid, studios). This depended on decisions made by lexicographers of successive ages.

In this way, the writing's fan extended and includes indirect symbols by different types of metaphoric substitution, like the part for the whole, the attribute for thing, the effect for the cause, the instrument for the activity, the gesture for the action, and so on. The list shows as the word chin, go up, derives from the picture of two footprints turning up; and as the word fù, that means "summit" derives from the ancient pot's picture.

A third characters class has composed of semantic combinations of two or more than two pictograms, making those called compound by association. In this way fù, wife, has composed by women's signs, hand and broom, fù, father, by the ancient signs of hand and stick; hao, to love, or good, combines signs of women and child.

A particularly interesting example, is the word that means male or man, nan, that includes the radicals of plough and field, indicates " who uses his force in the fields". Obviously, the sounds of the different elements lose themselves in the sound " that result", because this sign existed before that, to represent it, the scribes associated signs having other sounds, So, we have a sort of equation: li + thine = nan.

These equations make up a mental half-conscious foundation for people acquiring familiarity with the language" (Needham, 1981).

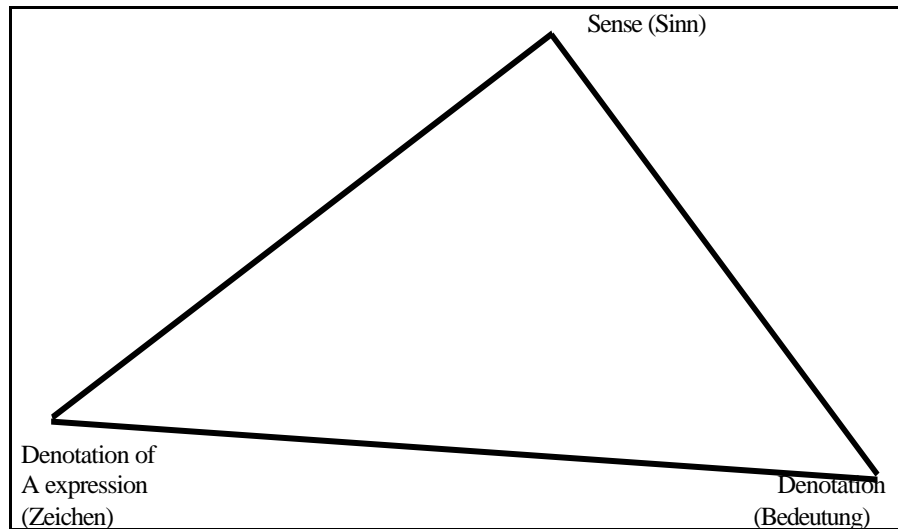
### ***2.4. The point of view of history***

The point of view for communication of Mathematics: The mathematics are languages with semiotic interpretation (Spagnolo, 1995, 1998).

What is a semiotic interpretation of mathematics languages?

- Syntactic point of view: syntax of mathematics languages, Bourbaki and structuralism, Formalism;
- Semantic point of view:
  - In the mathematics languages are the "set theory" as base of structure. For example: The group is defined in the which-ever set: the group of Integers, of symmetry, etc.
  - In the algebraic language, for example:  $4x+2$  and  $2(2x+1)$  different norm (sense) but they denote the same function (same set of ordinate couples).  $(x+5)^2=x$  and  $x^2+x+1=0$  they denote the same object (empty set) but have a different sense (Arzarello et alii, 1993).
  - In the physics language:  $F =ma$  and  $F=ma^2/a$  are syntactically correct but the second relation does not have sense in the physics language.
  - The relation of mathematics language as an interpreter of way of mind: the Gauss problem:  $1+2+3+4+5+6+7+8+9+10$   $(n+1)n/2$  and  $n+n/2 +(n/2 - 1)n$  are equivalent but the sense is different.
- Pragmatic point of view: communication point of view, didactic point of view

### Use of Frege triangle



### 2.5 In the history of mathematics

- History of Syntax of mathematics languages: Bourbaki (History of mathematics (Bourbaki, 1963)): Evolution of Algebra:
- Law of composition: Egyptian and Babylonian have a complex system of norms calculation on Natural numbers  $>0$  and Rational numbers  $>0$ , Commutativity of product of rational numbers (Euclid, Theory of magnitude), Diofanto, - 2 pages - , XVII century law of composition in algebra (Gauss), theory of substitutions (Lagrange), Galois (groups of substitutions, XIX century (2 pages). (9 pages)
- History of semantic of mathematics languages: Are the books with titles "History Mathematical Thought"<sup>2</sup>. The history of thought scours mathematics languages analysing the "senses" attributes to mathematics concepts, before organically they could play the role in mathematical language organized.
  - In this way is the book "History of mathematics, history of problems (The inter-irem commission, Ellipse, Paris)
  - The history of function concept (also in Piaget, Epistémologie et psychologie de la fonction, Etudes d'épistémologie génétique, 1968, Presses Universitaires de Frances). The point of view of psychology privileged. Every study of mathematics concept is completed with history study.


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<sup>2</sup> Examples: M. Kline, Mathematical thought from Ancient to Modern Times.

- Morris Kline, *Mathematical Thought from Ancient to Modern times*, 1972. The history of semantic and syntax are not completely separated in this occasion.
- The history of Eudoxe-Archimede Postulate (see Spagnolo, *Les obstacles epistemologiques: Le postulate d'Eudoxe-Archimede*, 1995);
- History of pragmatic of mathematics languages: There is the history of communication of mathematics.
  - What was to Know (Savoir, Sapere) in a determinate historical period?
  - What was the real knowledge of students?

In this perspective they are many important historical sources: books, official curriculum, register of teachers, reviews of mathematics and reviews of mathematics education. (In Italy there are reviews of mathematics education since 1870. In Palermo the review "Il Pitagora" (1874-1919)).

## 2.6. *The comparative schema: history, languages, Mathematics*

History of Mathematics in China: Algebraical Thought	Chinese Language	History of Mathematics in Western: Geometrical Thought	Natural Languages in Western
<ul style="list-style-type: none"> <li>• Chu Chang Suan Shu: Roots, false position, solution of particular systems. 300 before C.-200 B.C.</li> <li>• Chao Chun Chin, Commentary to Chou Pei: 200-300 After C.</li> <li>• Wang Hsiao</li> </ul>	<p>≈1000 after C. Chinese language like is in present. In the Chinese hieroglyphic language the 80% of hieroglyphics are for association.</p>  <p>This hieroglyphic MAN is composed in two parts STRENGTH and FIELD (a mental equation:</p>	<p>Euclid: Axiomatic geometry 300 before C.</p> <ul style="list-style-type: none"> <li>• Geometry as first interpretation of physical world;</li> <li>• Geometry as science of to argue.</li> </ul> <p>Fibonacci 1200 introduce the algebra in western "Liber abaci".</p> <ul style="list-style-type: none"> <li>• Geometry as hypothetical - deductive</li> </ul>	<p>The Aristotle's Logic like a model of western languages. In particular the Euclidean Geometry like a model of Aristotle's Logic.</p>



<p>Tung, Equations of 3° degree. 625 a.C.</p> <ul style="list-style-type: none"> <li>• Chhin Hiu-Shao, Numerical solutions of equations of degree&gt;2. 1247 b.C.</li> <li>• Ricci and Hsu Kuang- Chhi, Tradution of Euclide Geometry. 1607.</li> </ul> <p>Lorfi Zadeh: Fuzzy sets (1965)</p>	<p>says Needham)</p>	<p>system (Hilbert, Grunlagen der Geometrie, 1899)</p>	
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The hieroglyphic language is also bearer of concept of variable. For example in the following two hieroglyphics

采 cǎi (tsǎi) cogliere	保 bǎo (bǎo) proteggere
丨 丿 ㇇ ㇇ 亠 乎 身	丨 丨 亠 亠 亠 但 保
采	保 保

To pick (cai) and to protect (bao) have the same part "person":



This part is "variable". Sometimes the variable part is in semantical context, sometimes in syntactical context.

B. Kosko (1993) says that the Fuzzy Logic (Fuzzy Thinking ) is a base of Eastern Thinking. They are the relationship between the development of Fuzzy Logic in Japan, for example, and technology that applies Fuzzy Logic. Kosko says that the Fuzzy Thinking is the way of Eastern people approaching to solve problems.

What is the relationship between Fuzzy Thinking, hieroglyphic languages and algebraic language?

Enrica Lemut (2000) sustains that technology and mathematics are the mediators of Systemic Thinking. The action of Systemic Thinking is globally, but acting locally: "A system's components are not only the individual elements composing it but also the network of interactions among them and their purpose" (Lemut, 2000, pg. 183).

Is algebraical thinking in the systemic thinking?

E. Lemut agreed.

***The schema of relationship between the Aristotle's Logic, Fuzzy Logic and languages.***

<b>Eastern: Algebraical Thinking (Systemic Thinking)</b>	<b>Western: Geometrical Thinking</b>
Fuzzy Logic (Polyvalent Logic)	Aristotle's Logic (Bivalent Logic)
Systemic Thinking	
Algebraical Thinking	Geometrical Thinking
Eastern Languages	Western Languages

The history allows the semiotic interpretation of phenomena of learning/teaching:

- Meditation about the structure of natural language (syntax);
- Meditation about the Logic;
- History way to construction of languages (semantic);
- Pragmatic: study of process of learning/teaching.

### ***3.0 Conclusions***

#### ***History - Pupils - Teacher - Research in Mathematics Education***

**Point of view of Researchers in Mathematics Education (Communication of mathematics).  
By to argue the researches and for a possible reproducibility.**

- **History of syntax: conceptions that they use to syntactical adjustment of mathematics language.**
- **History of semantic: conceptions that to use to a-priori analysis of behaviour of pupils.**
- **History of pragmatic: study of phenomena of teaching concerning "didactic transposition".**

**Point of views of Teacher:**

- **For to restatement (focus) significative "didactics situations" for teaching/learning.**
- **For the epistemology of teacher.**

- **History of syntax: personal use of teacher but also to check the analysis a-priori and/or "didactic transposition".**
- **Chronological history concerning mathematics languages.**
- **Thematic history: history of demonstration (to argue, to deduce, to infer), history of limit concept, of infinity, of rigour, of function, of natural number, etc.**
- **The recovery of knowing of discipline in the point of view more general: historical-philosophical**

**Point of view of pupil**

- **He inserts the study of mathematics languages in cultural dimension**
- **He inserts a temporal dimension in the construction of mathematics languages**

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