Using conceptual maps and semi-structured interviews in teaching mathematics.

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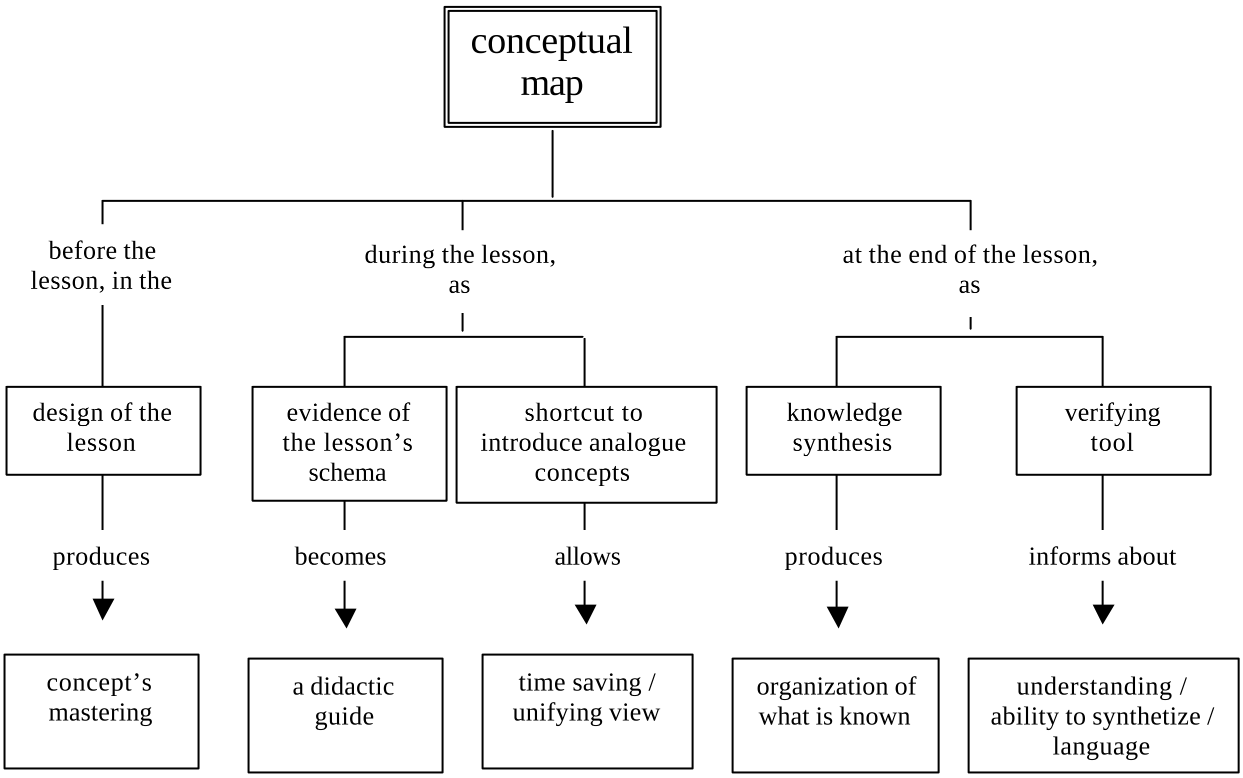
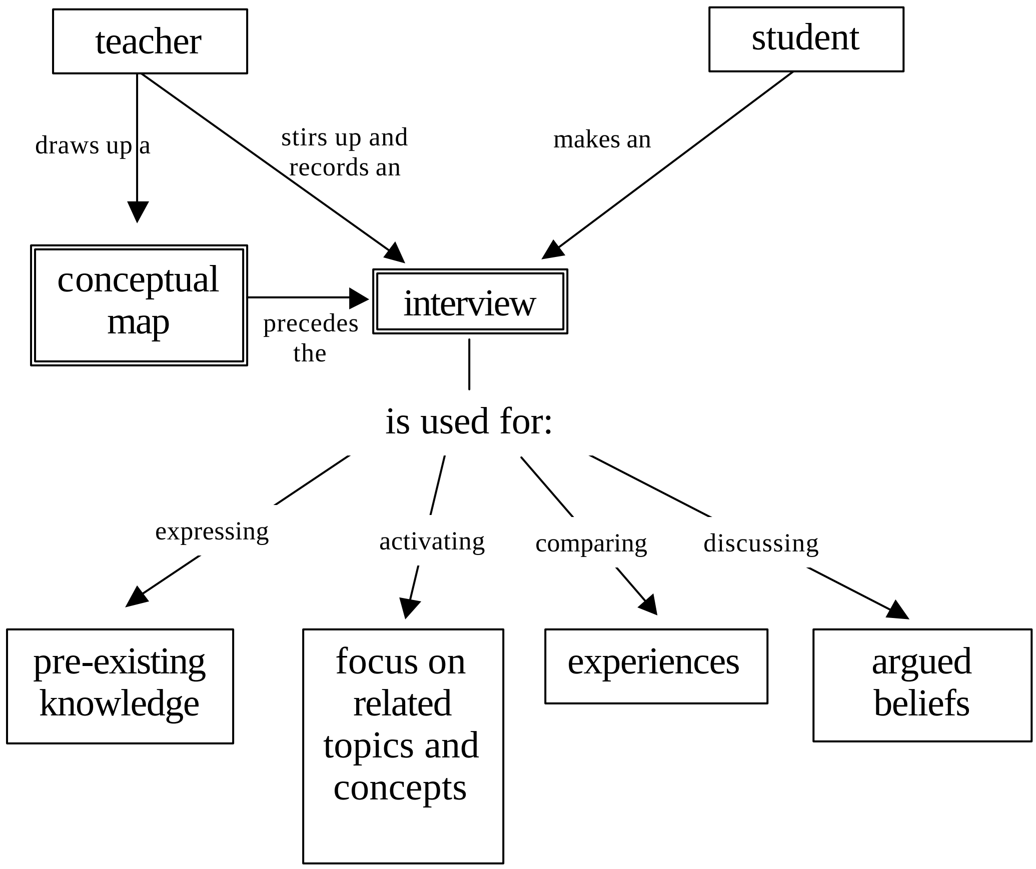
The aim of the activities reported in this paper was to assess the pedagogical effectiveness of conceptual maps and semi-structured interviews in teaching mathematics in primary and secondary schools. These tools come from the pedagogical approach developed by prof. E. Damiano, known as "didactics through concepts", and used by our research group in actual teaching of mathematics. The conceptual map is a scheme contrived to represent the knowledge that a person (or a group) has about a given concept, in terms of its properties and relationships with other concepts.

These maps were used at first to set up the lessons' path and then in the actual teaching sessions, in order to frame the topics taught, introducing analog concepts and assessing the effectiveness of learning, as shown in the following scheme:



The interviews were organized according to the related conceptual map and aimed to bring out any previous knowledge of the pupils and the mental images they associate to the chosen concept. As a side effect, the interviews helped to catch the pupils' attention, to compare their experiences and to discuss their different opinions, as summarized by the following scheme:



The topics involved in our didactic research were: space geometry in primary school (one teacher), geometrical transformations, vectors, equations and introduction to the axiomatic method in secondary school (nine teachers). In the following pages, we shall report in part some of the experiences made by our teachers, while a complete account of the research can be found in [36].

Parallelism in a third class of the primary school (age 8).

The map of this concept (parallelism) has been devised before the lesson by the teacher, putting in evidence the parallelism as an “equivalence relation” between straights (planes), linked with “direction” and “distance”. During the lesson, the teacher asked: <<What comes to your minds when I say “parallel”?>>. Some answers:

<<The ski’s tracks>>

<<Two things near to each other, starting from the same point and arriving at the same point, not a little here or there>> …

<<A thing laying on the same plane of another>>

<<My mother says: put yourself parallel to your sister, so I can see how much you have grown>>

<<When two persons run and remain always near each other>>

It is clear that in the pupils’ mind parallelism is related to neighborhood, contemporary and not necessarily straight routes, as often occurs in common language.

As a consequence, the teacher laid down specific activities to make clear the meaning of parallelism in geometry: a couple of pupils took two long sticks or large cardboard, and disposed themselves parallel to each other. The teacher then moved away one of the pupils, keeping him parallel to his companion. After a lively discussion between them, the pupils understood that time and proximity where not of interest in defining geometric parallelism.

Geometric transformations in the first class of secondary school (age 14).
Laying down a conceptual map for geometric transformations, the teachers found out “equality”, “invariant” and “group” as the main concepts linked with this topic. An interview on “equality” fired an animated discussion between the students:
<<equality depends on something, e.g. two chairs are equal because they are two chairs, but may be different in some detail>>
<<the chairs must be superimposed >>
<<if equality depends on some characteristic, how may I know which one I have to take into account?>>
<<in mathematics there must be only one point of view>>
<<6 + 4 = 10 are equal in value but not in form>>
<<two squares may be different in their dimensions but equal in shape>>
<<in mathematics “to be equal” means to be equal in every feature>>
<<then there aren’t two things that can be said to be equal>>
<<mathematics cannot be an opinion>>
<<it has fixed rules>>
<<but what is then essential in mathematics?>>
After discussions like these, the students’ minds are ready to afford a formal lesson about invariants in geometric transformations and finally to accept the Erlangen Program as the formalization of a problematic experience made by themselves.

In another class the interview concerned the word “transformation” and the students said that <<there must be something that changes and something that doesn’t>>. By this way, it was spontaneous to study also geometric transformations through variant or invariant properties. The teachers gave to the students some simple geometric figures, shaped in a cardboard, and they found, first by themselves and then in group, variant and invariant properties of the figures and the corresponding shadows, cast by a lamp or by the sun. An analogue inquiry was made on orthogonal and oblique symmetries.
Finally, the results of these observations have been compared and laid down in a scheme, and the students realized by themselves that the invariance of the circle’s shape and the angles’ measurement are the consequence of the invariance of the distance.
At this point the teacher could start the lesson on isometric transformations.
These activities took two hours of lesson, but gave a real economy of time in the following of the course, along with a good consciousness of the students in studying definitions and properties.

Equations and inequalities in the first class of the secondary school (age 14).
In the last lesson on equations the teacher asked the class to make up a conceptual map on the topic and so this was a good opportunity to organize and summarize the related knowledge.
The students were then requested to modify the map in order to represent no more equations, but inequalities instead. These activities took two hours of lesson, with some discussion about intervals of solutions and principles of equivalence. By this way, the students realized that they could not only understand but also make mathematics.

Conclusions.

The teachers that experienced the tools of didactics by concepts found that:

1. The conceptual maps are a tool for teachers, to catch the hard core of the topic, and for students, to evolve from a sequence of notions and rules to a well organized knowledge. To reach these results, it is important the map is made up by teachers and students and discussed together.

2. The semi-structured interview improved the relationship with their students, giving rise to an active attitude to mathematics. Their answers offered a lot of significant examples and counterexamples for the subsequent lessons. Sometimes the students appeared uneasy with the interview, so the teacher began with a written question or some simple activity (e.g. to draw a soccer field as seen from above or the side, build some tangram figures), able to set into motion the oral discussion with the whole class. The difficulty to set up a good interview depends on the fact that the teacher must not teach, but learn from the pupils, perceiving their knowledge and experience.

As a final conclusion, we can say that – in our experiences - these pedagogical tools have made the learning process more attractive, speedy and long-lasting.

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