Conceptualisation, registers of semiotic representation and noetic in mathematical education

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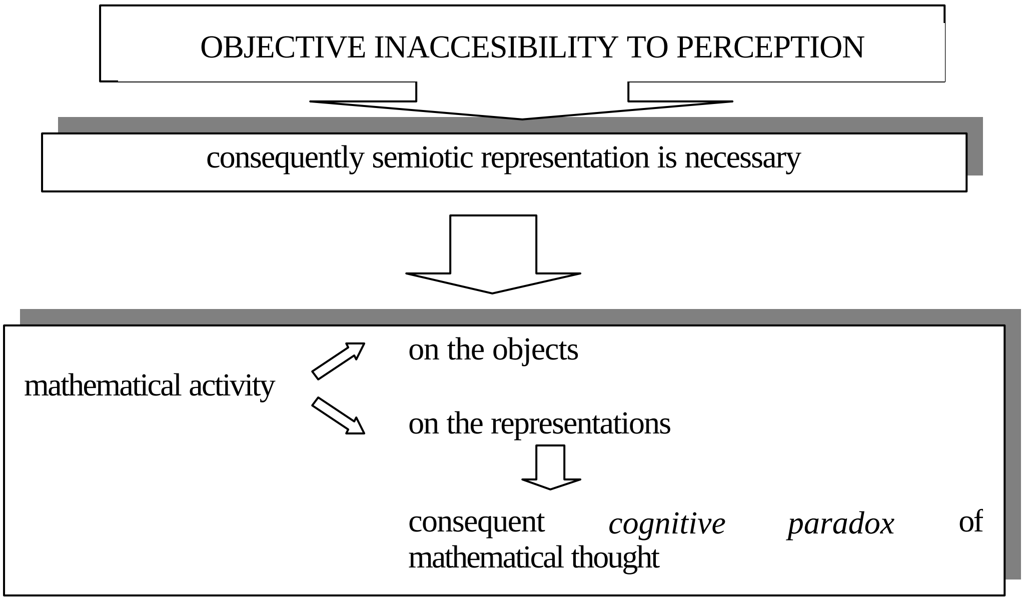
ABSTRACT. This study derives inspiration from the original discussions of Raymond Duval (1988a,b,c 1993) , and forms part of the research being done by the NRD of Bologna University. It attempts to draw out and to substantiate the diverse hypotheses that lie at the foundations of unsuccessful devolution (Perrin Glorian, 1994), and therefore also at the foundations of the schooling of mathematical awareness (D'Amore, 1999a).

1. The "cognitive paradox"

During this conference, I want to consider the sequent schema:



Let us see, then, what this *paradox* consists of (Duval, 1993, pag. 38):

"(…) on the one hand, the learning of mathematical objects cannot be other than a conceptual learning and, on the other hand, it is only by means of

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semiotic representations that an activity on mathematical objects becomes possible. This paradox can constitute a real vicious circle for learning. In what way can subjects in their phase of learning avoid mistaking mathematical objects for their semiotic representations if they can relate only to semiotic representations? The impossibility of providing a direct access to mathematical objects, outside any semiotic representations, makes the confusion also unavoidable. And, on the contrary, how can they acquire the mastery of mathematical treatments, linked necessarily to semiotic representations, if they do not already possess a conceptual learning of the represented objects? This paradox is even stronger if one identifies mathematical activity and conceptual activity and if one considers semiotic representations as secondary or extrinsic."

In this paradox, so well underlined by Raymond Duval, can a potential cause of missed devolutions be hidden? [1]

According to the teacher, the noosphere[2] and the student himself/herself, he/she is getting in touch with a mathematical "object" but - and nobody seems to realize this - the student is getting in touch only with the particular semiotic representation of that "object". The student does not have, and cannot have, direct access to the "object" and the teacher and the noosphere tend to confuse the two things; it is as though the student was blocked, inhibited: they cannot help mistaking the "object" and its semiotic representation because they do not realize it and they're not aware of it. Therefore, when facing a subsequent conceptual need that manifests itself, for example, with the necessity to modify the semiotic representation of that same "object", the student does not have either the critical, the cultural, or the cognitive tools; the teacher and the noosphere do not understand why and accuse the student, making them feel guilty for something they do not understand.

Actually: in this paradoxical phase, no one can really understand what's happening because every figure involved in this adventure has a different perception of the problem.

On the other hand the analysis of the representations is a new approach in the studies of cognitive processes, although it's not new for strictly philosophical studies.

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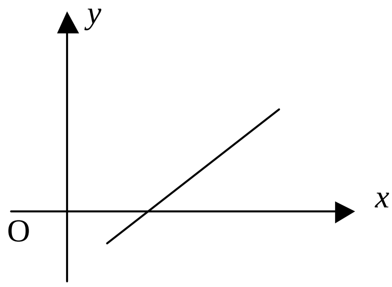
[1] By "devolution" I mean the act with which the teacher delegate to the student the direct overburdening with the responsibility of the construction of his/her own knowledge. In some cases the student accept and the learning process becomes possible; in some other cases the student doesn't accept to take it upon himself, and then the learning process becomes impossible. The bigger part of the European studies on mathematical education are based on this matter. The world "devolution" is derived from law studies: it is the passing by of goods from a person to another one. (Perrin Glorian, 1994; D'Amore, 1996b)

[2] By "noosphere" I mean everything surrounds the school's world and then, directly or not, influences the didactic action, that is influence on the triangle: teacher – student – knowledge. For example: parents, working world, school managers, external opinions etc.

Let us consider, for example, the step from the figural to the algebraic register in the analytical geometry:
The step from:



to:
x-2y-2=0

is not a banal change of register to be dominated by a 14-15-years-old student.
Nor the first neither the second case *is* the "object" "straight line", but they're both just semiotic representations.
Another example is the step from the decimal to the figural register in the representation of numbers:
many 11-12-years-old students find very difficult to represent decimal numbers as 1.75 or 1.8 on the rational numeric line; the difficulty is due to the change of semiotic register they can't domain. This change of register in some way makes someone claim that 1.75>1.8 (here an ambiguous interpretation of the decimal writing must be added).
Several times I've been using the verb "to learn"; it's difficult to be defined, but I think it's necessary to at least clarify it.
By "to learn" I mean a more or less personal construction, but always submitted to the need of "socialisation", which takes place obviously by a communicative instrument (which may be the language) and which in Mathematics will always definitely be conditioned by the choose of the symbolic mediator, i.e. the semiotic register of representation chosen (or imposed, in some ways, by the circumstances).

2. Semiotic e noetic in mathematical learning

In Mathematics the conceptual acquisition of an object necessarily passes trough the acquisition of one or several semiotic representations.

Duval himself claims this, when he first presented the question on the registers, in the famous articles of 1998 published on *Annales* (1988a, 1988b, 1988c) [a first attempt of synthesis of

them is the work of (1993); but Duval has published works on this matter also in 1989 e 1990]; Chevallard (1991), Godino and Batanero (1994) confirm it.
So, borrowing the expression from Duval: **there's no noetic without semiotic**.
In order to clarify the terms, but not pretending to do complete exposition since this words are not always used in the same sense, I prefer to explicit the meanings I'm using:

semiotic $=_{df}$	acquisition of one representation realised by signs
noetic $=_{df}$	conceptual acquisition of an object[3]

From now on I'll mean:

$r^m =_{df}$	semiotic register (m = 1, 2, 3, …)
$R^m_i(A) =_{df}$	i-esimal semiotic representation (i = 1, 2, 3, …) of a content A in the semiotic register r^m

To be noted that, according to this choices, if we change the semiotic register we necessarily have to change also the semiotic representation, but the vice versa is not as much necessary; in fact we can change the semiotic representation and maintain the same semiotic register.
I want to use again a graphic to illustrate all the question, since I think it's more effective:[4]

characteristics of the semiotic	{ *representation* *treatment* *conversion*	these are three different cognitive activities

[3] According to Plato, the noetic is the act of conceiving trough the thought; according to Aristotle, it's the conceptual comprehension act itself.
[4] I'm still referring to Duval (1993).

content A to be represented

⇓

chose of the distinctive characteristics of A

⇓

REPRESENTATION $R^m{}_i(A)$ in a given semiotic register r^m

↷ transformation of representation (*TREATMENT*) ⇓

new of representation ($i \neq j$) $R^m{}_j(A)$ in the *same* semiotic register r^m

↷ transformation of the register (*CONVERSION*) ⇓

new representation ($h \neq i$, $h \neq j$) $R^n{}_h(A)$ in *another* semiotic register r^n ($n \neq m$)

($m, n, i, j, h = 1, 2, 3, \ldots$)

In mathematical education the conversion process must have a central rule as regards the other functions, and in particular as regards he one of treatment, which is instead considered crucial from a mathematical point of view by most of people.

The construction of the mathematical concepts depends strictly on the capacity to use *several* registers of semiotic representations of the same concepts:

⊇ to *represent* them in a given register

⊄ to *treat* these representations within the same register

⊂ to *convert* these representations from a given register into another

These three elements and the above considerations draw attention to the deep connection existing between noetic and constructivism:

"construction of knowledge in mathematics" may be seen as the uniting of those three "actions" on the concepts, i.e. the expression itself of the capacity

to *represent* the concepts

to *treat* the obtained representations within a given register and

to *convert* the representations from a register into another.

We're specifying the basic-operations which together define the "construction"; it is otherwise a mysterious and ambiguous term, available at any kind of interpretation, also metaphysics.[5]

The student's giving up of devolution (obviously unconscious) and the student's inability (as a result of negative outcomes in previous attempts) to get involved into a direct and personal responsibility for the knowledge's construction, in a school context, are linked to the inability (sometimes only supposed) to *represent* or to *treat* or to *convert*, because of the lack of a previous specific didactic action. The teacher may actually don't worry about the individual components of the construction since he regards semiotic and noetic as the same thing. This identity is very spread between teachers' thinking, especially between the ones who never have had the chance to think about this question, or who consider it non-essential.[6]
All the above may bring the student to a renunciative choice and then to the schooling of knowledge (D'Amore, 1999a).[7]

According to me, to all the above another question must be added.
The everyday language is available between the semiotic registers for mathematics; the language, as acquired by the student in the first school years and as used by him in not-schooling contexts, has several and complex functions:
designation function
sentences expression function
speaking enlargement function
reflecting function (or metalinguistic).
All these functions can be found in the complex relational game concerning the learning of mathematics, but most of the times they're present not in a spontaneous way; the student, in fact, adapts his mathematical language to the one he hears from the teacher, the one used in the text books, the one used by school fellows who have success in mathematics classes.
We have then the following paradox: to use the semiotic register, which is supposed to be the most natural and spontaneous, appears to actually be the most complex to be controlled by the student.
The "natural" language stops to be actually natural and becomes a specific register which the student can't control and dominate.
At the end the student speaks an unnatural language, made by stock phrases, heard and not actually constructed, which he can't dominate anymore (Maier, 1993; D'Amore, 1996).

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[5] This consideration is, of course, peculiar for Mathematics, as well as all this paper; I can't say how they may be extended to a theory of concepts or even to a real gnoseology.
[6] This refers to a quite more general question, the one about the implicit believes of the teacher, deeply, systematically and often treated in (Speranza, 1997).
[7] «With the terms "schooling of knowledge" I here want to refer to the act, largely unconscious, by which the student, at a certain point of his social and scholastic life (but nearly always during the Primary School), delegates the School (seen as Institution) and the school teacher (representing the Institution) to *select for him the significant knowledge* (the one which is socially significant, for a status recognised and legitimated by the noosphere). With this act the student gives up to make himself directly responsible for the choice, rejecting any kind of personal criteria (such as taste, interest, motivation,…). Since this schooling means the recognition of the teacher as the keeper of the socially important knowledge, we obviously have, roughly at the same time, the schooling of interpersonal relations (between student and teacher and between student and school fellows) and as well of the relation between student and knowledge: it is what (…) we call "schooling of the relations".» (D'Amore, 1999a).

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[T*raduzione di Margherita Stefanini]*