# CONNECTING MATHEMATICS PROBLEM SOLVING TO THE REAL WORLD

Erik DE CORTE\*, Lieven VERSCHAFFEL\*, and Brian GREER\*\*
\*Center for Instructional Psychology and Technology (CIP&T),University of Leuven, Belgium
\*\*Center for Research on Mathematics and Science Education, San Diego State University, U.S.A.

#### Introduction

A major argument for including verbal problems in the school curriculum has always been their potential role for the development in students of skills in knowing when and how to use their mathematical knowledge for approaching and solving problems in practical situations. The application of mathematics to solve problem situations in the real world, otherwise termed mathematical modeling, can be usefully thought of as a complex process involving a number of phases: understanding the situation described; constructing a mathematical model that describes the essence of those elements and relations embedded in the situation that are relevant; working through the mathematical model to identify what follows from it; interpreting the outcome of the computational work to arrive at a solution to the practical situation that gave rise to the mathematical model; evaluating that interpreted outcome in relation to the original situation; and communicating the interpreted results. As several authors have stressed, this process of solving mathematical application problems has to be considered as cyclic, rather than as a linear progression from givens to goals (Burkhardt, 1994; Greer, 1997; Lesh & Lamon, 1992).

For several years, it has been argued by many mathematics educators that - in contrast to the intention mentioned above - the current practice of word problems in school mathematics does not at all foster in students a genuine disposition towards mathematical modeling, i.e. treating the text as a description of some real-world situation to be modeled mathematically. According to these authors, by the end of the elementary school many pupils have constructed a set of beliefs and assumptions about doing mathematical application problems, whereby this activity is reduced to the selection and execution of one or a combination of the four arithmetic operations with the numbers given in the problem, without any serious consideration of possible constraints of the realities of the problem context that may jeopardize the appropriateness of their standard models and solutions (Davis, 1989; Greer, 1997; Freudenthal, 1991; Kilpatrick, 1987; Nesher, 1980; Reusser, 1988; Schoenfeld, 1991; Verschaffel & De Corte, 1997a). However, evidence supporting this claim was, until recently, rather scarce, except for some oft-cited examples of striking evidence of "suspension of sensemaking" by students when confronted with the well-known problem "How old is the captain?" (IREM de Grenoble, 1980) or the buses item from the NAEP in the U.S. (Carpenter, Lindquist, Matthews, & Silver, 1983).

This chapter first reviews briefly a series of recent studies that provide robust empirical evidence showing the omnipresence and the strenght of the phenomenon of disconnecting word problem solving from the real world. Next it is argued that major features of the current mathematics classroom practice and culture are largely responsible for this phenomenon. Finally, a radically different approach to the teaching of mathematical problem solving, based on the so-called "modeling perspective", is proposed, and exemplarily illustrated by a brief review of a design experiment. A much more detailed review and discussion of the theoretical and empirical work summarized in this paper, is given in Verschaffel, Greer, and De Corte (2000).

## Omnipresence and robustness of suspension of sense-making: The empirical evidence

The extent to which students ignore relevant and familiar aspects of reality in answering word problems was simultaneously studied in a systematic way among 13-14-year-olds in Northern Ireland (Greer, 1993) and 10-11-year-olds in the Flemish part of Belgium (Verschaffel, De

Corte, & Lasure, 1994). ). In both studies, pupils were administered in the context of a typical mathematics lesson a paper-and-pencil test consisting of matched pairs of items. Each pair of items consisted of:

- A standard item (S-item) that can be solved uncontroversially by applying the most obvious arithmetic operation(s) with the given numbers (e.g., "Chris made a walking tour. In the morning he walked 8 kilometers and in the afternoon 15 kilometers. How many kilometers did Chris walk?")
- A parallel problematic item (P-item) for which the appropriate mathematical model is less obvious, at least if one seriously takes into account the realities of the context evoked by the problem statement (e.g., " Bruce and Alice go to the same school. Bruce lives at a distance of 17 kilometers from the school and Alice at 8 kilometers. How far do Bruce and Alice live from each other?").

Pupils' reactions to the P-problems were categorized either as "realistic" or "non-realistic" based on the activation and use of real-world knowledge and realistic considerations about the problem context. Whenever a pupil gave an answer to the problem that took into account the context or when (s)he produced a non-realistic answer that was accompanied by a realistic comment, his(her) overall reaction to that particular problem was scored as a "realistic reaction". Reactions without any manifest trace of the activation and use of the critical real-world knowledge, were scored as "non-realistic".

In both studies, pupils demonstrated a very strong overall tendency to exclude real-world knowledge and realistic considerations when confronted with the problematic items. For instance, in Verschaffel et al.'s (1994) study, only 17% of all reactions to the P-items could be considered as realistic. For only two out of the ten P-items was a considerable number of realistic answers and/or comments observed, 49% and 59 %, respectively; for the other problems the percentage of realistic reactions ranged between 0% and 17%.

The findings of Greer (1993) and Verschaffel et al. (1994) have been replicated in several countries, namely Belgium (Verschaffel, De Corte, & Lasure, 1999), Germany (Renkl, 1999), Japan (Yoshida, Verschaffel, & De Corte, 1997), Northern Ireland (Caldwell, 1995), Switzerland (Reusser & Stebler, 1997a), and Venezuela (Hidalgo, 1997).

In some of these replication studies (e.g., Caldwell, 1995; Hidalgo, 1977) individual interviews with pupils who had solved the paper-and-pencil test provided (anecdotal) evidence that at least in some of them a non-realistic response to P-items was accompanied by the explicit belief that there is a gap between the artificial world of school arithmetic word problems, on the one hand, and the real world outside school, on the other. For instance, in Caldwell's (1995, p. 39) study, one 13-year-old pupil reacted as follows to the interviewer's question as to why she did not make use of realistic considerations when solving the P-items: "I know all these things, but I would never think to include them in a math problem. Math isn't about things like that. It's about getting sums right and you don't need to know outside things to get sums right."

Obviously the outcomes of the previous series of investigations provided robust evidence for pupils' strong tendency to exclude real-world knowledge when doing word problems in the restricted context of school arithmetic. However, it remained unclear whether this tendency was really due to a deep-rooted and resistant belief about the nature of (solving) school arithmetic word problems among these pupils, or if it was merely an artefact of the experimental setting. Specifically, the formulation of the P-items (similar to the S-items), the way in which they were presented to the students (mixed with S-items), and the accompanying instructions (e.g., the lack of any warning that this was not a standard test, the absence of an explicit invitation to provide alternative answers or even to criticize the problems), all may have contributed to the expectation among the students that this was a test involving only problems that had to be conceived, handled, and solved in the standard

manner. Moreover, the use of paper-and-pencil tests may have concealed the fact that some students have effectively thought about the problematic or unsolvable nature of some Pitems, but finally decided not to incorporate these considerations into their written responses, because they assumed that they had to approach the problems in the "usual" way. To investigate the plausibility of this alternative interpretation, several follow-up studies were carried out that involved minimal interventions taking the form of providing pupils explicit hints that some of the problems needed careful scrutiny and/or giving them direct and explicit help to consider alternative responses taking into account the realities of the problem context. For instance, in studies by Reusser & Stebler (1997a), Yoshida et al. (1997), and Verschaffel et al. (1999) some pupils received before the test a written or oral warning that some of the problems would be difficult or even unsolvable because of some complexities or unclarities in the problem statement, and they were explicitly invited to mark these problems on their test sheet and to explain why some of them were unsolvable. The results of these studies showed that these minimal interventions intended to make pupils more alert, to sensitize them to the consideration of aspects of reality, and to legitimize alternative forms of answer produced, at best, only weak effects. Apparently, pupils' disposition toward non-realistic modeling of school arithmetic word problems is very strong, and those interventions are not powerful enough to break their belief that real-world knowledge is irrelevant when solving such problems.

A further set of follow-up studies focused on the impact of another kind of interventions intended to elicit children's activation of real-world knowledge, namely increasing the authenticity of the problem situation by presenting P-items in a more realistic, performancebased setting, for instance, in the context of a group discussion and/or in the presence of concrete material and performance-based goals. Representative in this respect is a study by DeFranco and Curcio (1997). They compared upper primary school pupils' reactions to the well-known buses item ("328 senior citizens are going on a trip. One bus can sit 40 people. How many buses are needed so that all senior citizens can go on the trip?") presented in two different ways: as part of a traditional arithmetic test, or as embedded in a more realistic setting of making a phone call using a teletrainer to order minivans to transport sixth graders to a party. Whereas in the traditional test condition only two of the 20 children gave a realistic answer, in the real-world setting 16 out of the 20 students gave an appropriate response. Similar other studies (e.g., Reusser & Stebler, 1997b; Wyndhamn & Säljö, 1997) have confirmed that a more realistic presentation of P-items - in contrast to the mini-interventions of the preceding series of investigations - results in more substantial improvements in children's performance, more specifically, in their inclination and capability to include the realistic considerations they were so reluctant to activate under the previous, more restricted, testing conditions.

## Traditional mathematics education: The cause of children's suspension of sense-making?

Taken as a whole, the results of the preceding studies suggest that it is not so much a cognitive deficit that causes pupils' abstention from sense-making when doing arithmetic word problems in a typical school setting. To the contrary they are rather acting in accordance with the "rules of the game" which they belief to regulate the interactive ritual in which they are involved. Several authors have analysed the "hidden" beliefs that seem to be used by pupils to make the "word problem game" function efficiently (see e.g., De Corte & Verschaffel, 1985; Reusser & Stebler, 1997a; Schoenfeld, 1991). Examples are the following:

- Every problem presented by the teacher or in a textbook is solvable and makes sense.
- There is only one (precise and numerical) correct answer to every word problem.

- The answer can be obtained by performing one or more mathematical operations with the numbers in the problem, and almost certainly with all of them.
- The problem contains all the information needed to find the correct solution.
- Persons, objects, places, plots, etc. are different in a school word problem than in a real-world situation, and don't worry (too much) if your knowledge or intuitions about the everyday world are violated in the situation described in the problem situation.

It should be acknowledged that this set of beliefs about word problems is basically a hypothetical construct, for which researchers have not come up yet with much direct and compelling empirical support (apart from some anecdotal data as the example from Caldwell's (1995) study cited earlier).

Anticipating more convincing evidence for the existence and the impact of these beliefs among pupils, we can ask the question: Which aspects of mathematics teaching/learning processes induce and foster the development in pupils of these beliefs about the game of word problems, especially about the unrealistic nature of these problems? In this respect, it is plausible to assume that this development is mainly due to two aspects of current instructional practice and culture in which children learn to solve mathematical application problems:

- The impoverished and stereotyped diet of standard word problems occurring in mathematics lessons, textbooks, and tests, which can almost always be modeled and solved by carrying out one or more operations with the given numbers;
- The current teaching practice and culture relating to word problems, especially the lack of attention to the modeling perspective.

Until now there is only scarce direct research evidence for these assumptions. However, various analyses of textbooks (e.g., Stern, 1992) and assessment instruments (e.g., Cooper, 1994) provide indirect support for the first assumption. Similarly, a study by Verschaffel, De Corte, and Borghart (1997) favors indirectly the second assumption. This investigation showed that pupils' misbelief about the role of real-world knowledge during problem solving is paralleled by a similar tendency in a group of 332 student teachers. Indeed, 52% of their own answers on seven P-items were also unrealistic; moreover, when asked to evaluate realistic and non-realistic pupil answers on P-items, they had a clear inclination to score nonrealistic responses more frequently as correct (56% of the cases) than they scored realistic answers as correct (47%); only 18% of the non-realistic answer were evaluated as totally incorrect. Thus, student teachers' overall evaluation of non-realistic answers to P-items was substantially more positive than for realistic answers. Of course, this study does not yield direct evidence that teachers' own conceptions and beliefs are responsible for children's strong tendency to exclude real-world knowledge and realistic considerations from their problemsolving activities. However, taking also into account the recent literature on mathematics teaching (Fennema & Loef, 1992; Thompson, 1992), it is plausible that these teacher cognitions and beliefs on the role of real-world knowledge on word problem solving have a strong impact on their actual teaching behavior and, consequently, on their pupils' learning processes and outcomes.

## Fostering realistic mathematical modeling of problem situations: An intervention study

Nothwithstanding the remaining gaps in our understanding of the nature of pupils' beliefs about word problem solving, and of the instructional factors underlying their origin and development, some researchers have already started to move from purely descriptive studies to design experiments aiming at fostering more realistic modeling of word problems in pupils.

For instance, Verschaffel and De Corte (1997b) set up a small-scale intervention study in which they tried to change fifth graders' beliefs about the role of real-world knowledge in problem solving, by immersing them into a different classroom culture in which word problems were explicitly used as exercises in realistic mathematical modeling. The pupils

from an experimental class participated in a program on realistic modeling consisting of five teaching/learning units of about 2 1/2; the two control classes followed the regular mathematics curriculum. Major characteristics of the intervention were as follows:

- Use of a set of more realistic non-routine problems as compared to the traditional textbook word problems. The tasks were specifically designed to stimulate pupils to pay attention to the complexities involved in realistic mathematical modeling. Each teaching/learning unit focused on one prototypical problem of mathematical modeling (e.g., interpreting the outcome of a division problem involving a remainder, modeling the union or separation of sets with joint elements, etc.).
- A varied set of highly interactive instructional techniques was deployed, especially small-group collaborative work followed by whole-class discussions.
- An attempt was made to establish a new classroom culture by explicitly negotiating new social norms about the role of the teacher and the students in the classroom, and new sociomathematical norms about what counts as a good mathematical word problem, a good solution procedure, and a good response (see Cobb & Yackel, 1998; Schoenfeld, 1991)

The results of the pupils from the experimental class and of the two control classes on a pretest and a post-test consisting of both learning items and transfer items, as well as on a retention test were compared. This warranted a positive conclusion about the feasibility of altering fifth graders' beliefs about the role of real-world knowledge in problem solving, and of fostering in them a disposition toward realistic mathematical modelling of word problems.

A replication of this study with German pupils by Renkl (1999) yielded similarly promising results. Moreover, several other design experiments with a much broader aim and scope, and with a higher degree of ecological validity, have documented more convincingly how by immersing students in a fundamentally changed learning environment, they can acquire more appropriate conceptions about and strategies for mathematical modeling (e.g., Cognition and Technology Group at Vanderbilt, 1997; Lehrer & Schauble, in press; Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaerts, & Ratinckx, 1999).

## Mathematical modeling as a vehicle for connecting problem solving to the real world

A large body of research summarized in the first part of this article, shows clearly children's tendency to disconnect mathematical problem solving from the real world. Rather than functioning as realistic contexts that invite, or even force, pupils to use their common-sense knowledge and experience about the real world in combination with acquired mathematical knowledge and skills, school mathematics word problems seem to be perceived by children as a capricious kind of school tasks that are separated from the real world and have to be solved by means of certain operations on the given numbers, ignoring real-world knowledge and even accepting conditions about the problem context that are empirically false.

Initial evidence has then been discussed supporting the hypothesis, that flaws in current instructional practices relating to mathematics problem solving may account for the development in students of a tendency to neglect their real-world knowledge when solving word problems. The available research has revealed flaws in the nature of the problems used in current mathematics textbooks and assessment instruments, but also in the way these problems (and pupils' responses to them) are considered and treated by teachers. Both aspects of the instructional environment may sanction pupils' tendency towards absence of realistic modeling. Taking all this into account, it is obvious that the phenomenon of suspension of sense-making cannot be considered as a cognitive deficit in pupils, or a foolish and blind behavior. To the contrary, as a result of schooling, their behavior is pragmatically functional: it mostly leads to right and expected solutions (from the point of view of the learner) and to

appropriate and expected feedback (from the perspective of the teacher). The strategy, thus, has its rational core in the socio-cognitive setting of traditional school mathematics.

Given this analysis of the state-of-the-art, it is obvious that attempts to foster in pupils more realistic mathematical modeling of word problems requires subtantial modifications in current classroom practices. In the last section of the article a representative example of a design experiment was briefly reviewed, showing that by immersing pupils in an innovative learning environment that constitutes a radical departure from traditional classroom practices, they can learn more realistic beliefs about and strategies for mathematical modeling, and, thus, for connecting word problem solving to the real world.

Taking into account the findings of this, but also the other design experiment referred to above, some major principles for the innovation of the learning and teaching of mathematical word problem solving can be put forward to conclude this article.

First, the quality, the diversity, and the authenticity of the word problems used in teaching problem solving should be increased in order to avoid the development in children of incorrect "hidden" beliefs such as the ones listed above (e.g., Every problem presented by the teacher or in the textbook is sovable and makes sense).

Second, word problems should be conceived as exercises in mathematical modeling by systematically adopting the modeling perspective in the teaching-learning environment. Figure 1 schematically represents this mathematical modeling approach which was already briefly described in the frist paragraph of this article.

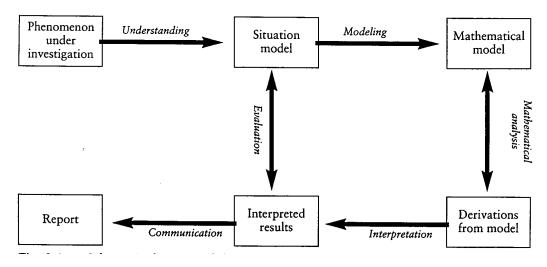


Figure 1. Schematic diagram of the process of modeling

Compared with the traditional approach to application word problems, the modeling perspective offers major advantages. The process of modeling constitutes the bridge between mathematics as a set of tools for describing aspects of the real world, on the one hand, and mathematics as the analysis of abstract structures, on the other; as such it is a pervasive aspect of mathematics. Given the increasing mathematisation of social as well as physical phenomena, an understanding of this aspect of mathematics is essential for informed citizenship. Also, taking aspects of reality into account in modeling the situations described in word problems is a potentially powerful way to connect pupils' mathematics problem solving to the real world, and to modify the belief and feeling of many children that mathematics is irrelevant in relation to their everyday experiences.

Third, more authentic word problems approached with the modeling perspective have to be embedded in a learning environment and a classroom culture that are radically different from traditional mathematics education. Major needed changes in this respect relate to the following components of the instructional context

- ❖ a move away from problem solving as a purely "solo activity" toward problem solving, and even problem posing, as a collaborative endeavour involving small-group work as well as whole-class discussion;
- ❖ the negotiation of novel social norms for regulating interaction and discussion among pupils and among teacher and pupils), and new sociomathematical norms that are − in contrast to the domain-independent social norms specific to children's mathematical activity and relate to such aspects as how they perceive mathematics problems, what counts as an efficient solution procedure, or as an good explanation, etc.

#### Address for correspondence:

Erik De Corte, Center for Instructional Psychology and Technology (CIP&T)

University of Leuven, Vesaliusstraat 2, B-3000 Leuven, Belgium

Tel.: int.-32-16-326248 Fax.: int.32-16-326274

e-mail: <a href="mailto:Erik.Decorte@ped.kuleuven.ac.be">Erik.Decorte@ped.kuleuven.ac.be</a> URL: <a href="http://www.kuleuven.ac.be/~p1486000/">http://www.kuleuven.ac.be/~p1486000/</a>

#### References

Burkhardt, H. (1994). Mathematical applications in school curriculum. In T. Husén & T. N. Postlethwaite (Eds.), <u>The international encyclopedia of education</u> (2nd ed.) (pp. 3621-3624). Oxford/New York: Pergamon Press.

Caldwell, L. (1995). <u>Contextual considerations in the solution of children's multiplication and division word</u> problems. Unpublished undergraduate thesis, Queen's University, Belfast, Northern Ireland.

Carpenter, T. P., Lindquist, M. M., Matthews, W., & Silver, E. A. (1983). Results of the third NAEP mathematics assessment: Secondary school. <u>Mathematics Teacher</u>, 76, 652-659.

Cobb, P., & Yackel, E. (1998). A constructivist perspective on the culture of the mathematics classroom. In F. Seeger, J. Voigt, & U. Waschescio (Eds.),

Cognition and Technology Group at Vanderbilt. (1997). <u>The Jasper project: Lessons in curriculum, instruction, assessment, and professional development</u>. Mahwah, NJ: Lawrence Erlbaum Associates.

Cooper, B. (1994). Authentic testing in mathematics? The boundary between everyday and mathematical knowledge in National Curriculum testing in English schools. <u>Assessment in Education</u>, 1, 143-166.

Davis, R. B. (1989). The culture of mathematics and the culture of schools. <u>Journal of Mathematical Behavior</u>, <u>8</u>, 143-160.

De Corte, E., & Verschaffel, L. (1985). Beginning first graders' initial representation of arithmetic word problems. Journal of Mathematical Behavior, 4, 3-21.

DeFranco, T. C., & Curcio, F. R. (1997). A division problem with a remainder embedded across two contexts: Children's solutions in restrictive versus real-world settings. Focus on Learning Problems in Mathematics, 19(2),58-72

Fennema, E., & Loef, M. (1992). Teachers' knowledge and its impact. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (00. 147-164). New York: Macmillan.

Freudenthal, H. (1991). Revisiting mathematics education. Dordrecht, The Netherlands: Kluwer.

Greer, B. (1993). The modeling perspective on wor(1)d problems. Journal of Mathematical Behavior, 12, 239-250.

Greer, B. (1997). Modelling reality in mathematics classrooms: The case of word problems. <u>Learning and Instruction</u>, 7, 293-307.

Hidalgo, M. C. (1997). <u>L'activation des connaissances à propos du monde réel dans la résolution de problèmes verbaux en arithmétique</u>. Unpublished doctoral dissertation, Université Laval, Québec, Canada.

Institut de Recherche sur l'Enseignement des Mathématiques (IREM) de Grenoble (1980). <u>Bulletin de l'</u> <u>Association des professeurs de Mathématique de l'Enseignement Public, no. 323, pp. 235-243.</u>

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. H. Schoenfeld (Ed.), <u>Cognitive science and mathematics education</u> (pp. 123-147). Hillsdale, NJ: Lawrence Erlbaum Associates.

Lehrer, R., & Schauble, L. (in press). Modeling in mathematics and science. In R. Glaser (Ed.), <u>Advances in instructional psychology</u> (Vol. 5). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., & Lamon, S. (1992). Assessing authentic mathematical performance. In R. Lesh & S. Lamon (Eds.), <u>Assessment of authentic performance in school mathematics</u> (pp. 17-62). Washington, DC: American <u>Association for the Advancement of Science.</u>

Nesher, P. (1980). The stereotyped nature of school word problems. For the Learning of Mathematics, 1(1), 41-8.

Renkl, A. (1999, August). <u>The gap between school and everyday knowledge in mathematics</u>. Paper presented at the Eighth European Conference for Research on Learning and Instruction, Göteborg, Sweden.

Reusser, K. (1988). Problem solving beyond the logic of things: Contextual effects on understanding and solving word problems. <u>Instructional Science</u>, 17, 309-338.

Reusser, K., & Stebler, R. (1997a). Every word problem has a solution: The suspension of reality and sense-making in the culture of school mathematics. <u>Learning and Instruction</u>, 7, 309-328.

Reusser, K., & Stebler, R. (1997b, August). <u>Realistic mathematical modeling through the solving of performance tasks</u>. Paper presented at the Seventh European Conference on Learning and Instruction, Athens, Greece.

Schoenfeld, A. H. (1991). On mathematics as sense-making: An informal attack on the unfortunate divorce of formal and informal mathematics. In J. F. Voss, D. N. Perkins, & J. W. Segal (Eds.), <u>Informal reasoning and education</u> (pp. 311-343). Hillsdale, NJ: Lawrence Erlbaum Associates.

Stern, E. (1992). Warum werden Kapitänsaufgaben "gelöst"? Das Verstehen von Textaufgaben aus psychologischer Sicht. *Der Mathematikunterricht*, 28 (5), 7-29.

Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of research. In D.A.. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.

Verschaffel, L., & De Corte, E. (1997a). Word problems. A vehicle for authentic mathematical understanding and problem solving in the primary school? In T. Nunes & P. Bryant (Eds.), <u>Learning and teaching mathematics:</u> <u>An international perspective</u> (pp. 69-98). Hove, England: Psychology Press.

Verschaffel, L., & De Corte, E. (1997b). Teaching realistic mathematical modeling and problem solving in the elementary school. A teaching experiment with fifth graders. <u>Journal for Research in Mathematics Education</u>, 28, 577-601.

Verschaffel, L., De Corte, E., & Borghart, I.(1997). Pre-service teachers' conceptions and beliefs about the role of real-world knowledge in mathematical modelling of school word problems. Learning and Instruction, 4, 339-59.

Verschaffel, L., De Corte, E., & Lasure, S. (1994). Realistic considerations in mathematical modelling of school arithmetic word problems. <u>Learning and Instruction</u>, 4, 273-294.

Verschaffel, L., De Corte, E., & Lasure, S. (1999). Children's conceptions about the role of real-world knowledge in mathematical modeling of school word problems. In W. Schnotz, S. Vosniadou, & M. Carretero (Eds.), New perspectives on conceptual change (pp 175-189). Oxford: Elsevier.

Verschaffel, L., De Corte, E., Lasure, S., Van Vaerenbergh, G., Bogaerts, H., & Ratinckx, E. (1999). Design and evaluation of a learning environment for mathematical modeling and problem solving in upper elementary school children. <u>Mathematical Thinking and Learning</u>, 1, 195-229.

Verschaffel, L., Greer, B., & De Corte, E. (2000). <u>Making sense of word problems</u>. Lisse: The Netherlands: Swets & Zeitlinger.

Wyndhamn, J., & Säljö, R. (1997). Word problems and mathematical reasoning: A study of children's mastery of reference and meaning in textual realities. <u>Learning and Instruction</u>, 7, 361-382.

Yoshida, H., Verschaffel, L., & De Corte, E. (1997). Realistic considerations in solving problematic word problems: Do Japanese and Belgian children have the same difficulties? <u>Learning and Instruction</u>, 7, 329-338.