VOLUME MEASUREMENT AND CONSERVATION IN LATE PRIMARY SCHOOL CHILDREN IN CYPRUS

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Introduction

The objective of this presentation is to demonstrate that understanding of the concept of volume has more than one related aspects therefore, teaching practices should observe that certain individual skills are developed before abstract methods of calculation of volume are introduced. This study is concentrated on the understanding of the concept of volume of rectangular solids at late primary school children (age 10-12 years), that is before they enter secondary education. The two aspects of the concept which are going to be examined is measurement and conservation.

The SOLO Taxonomy theory served as the theoretical basis for the study. Earlier research, in the 1960's and the 1970's, confirmed children's progressive understanding of the different aspects of conservation of volume as proposed by Piaget et al (1960) and Lunzer (1960). Criticisms of the Piagetian stage theory stemming from its failure to explain the problem of decalage lead to a reconsideration of both the notion of intelligence and the primacy of the underlying cognitive structure in more recent theoretical models. One such theory is the SOLO-Taxonomy theory which was originally developed by Biggs and Collis (1982) and later modified and further developed by Collis and Watson (1991) and Biggs and Collis (1991). By separating the underlying hypothetical cognitive structure from the observed level of response (to a variety of content specific materials within the school curriculum), the theory allows for external factors such as language, specific learning experiences, motivation as well as underlying cognitive factors to mediate so as to produce the observed levels of response.

Methodology

The sample comprised of 90 primary school children from three different schools in Cyprus. Groups of 30 children consisting of an approximately equal number of boys and girls were selected from each school, half of them attending grade five (15 children) and the other half attending grade six (15 children). The children from each class were selected randomly from the register.

The tasks presented for the purposes of this paper were chosen among a larger set of test items that was used for a wider research. All the tasks were presented both in the form of an interview and in written form except for the final task concerning conservation of displacement volume. This task was presented only in written form following a pilot study where children's responses to both versions of the task did not show any marked differences. During the interview the material available were mainly unit cubes and the different physical objects necessary for the contact of the test. The tasks used here are grouped under two general headings: measurement of volume and conception of volume.

A. Measurement of volume: The material consisted of unit cubes that the children could use. Each child was successively presented with the following rectangular objects and asked to calculate the volume in terms of unit cubes and provide an explanation for her/his answer.

- an empty 3x3x5 cartoon box
- a 3x4x5 construction made out of unit cubes
- a solid 2x3x4 wooden box

B. Conception of volume:

(1) Conservation of interior volume: This is the well known Piagetian task in which the child is presented with a block and is asked to build "a house of the same room on a smaller island". It was adopted with the aim of testing conservation of interior volume.

(2) Conservation of displacement volume: This task was adopted with the aim of testing children's understanding of all the aspects of volume conservation. The child was presented with a block and a container half filled with water and asked what would happen to the water when the block was put in the container. Then she/he was told that the block was broken down and all the cubes were to make a new block which was also presented (taller but thinner). The child was asked what would happen to the water if the new block was immersed in the container.

Results

A. Measurement of volume: The methods used by children to measure volume appeared to follow the same pattern for different test items. Responses were initially categorised into successful (1), unsuccessful (2). Within these categories responses are listed in decreasing order of "operational complexity" in terms of the SOLO Taxonomy levels.

11 Relational level of SOLO**:** Use of the multiplication formula V=LxBxH. This corresponds to the relational level of the SOLO taxonomy response for children are no longer bound by the external aspects of each construction and can integrate the three Euclidean dimensions to calculate volume.

12 Multistructural level of SOLO: Sequential processing of two dimensions (e.g. sequential addition or multiplication by number of layers, sequential addition or multiplication by number of rows or columns) or counting visible plus invisible cubes in an organised manner which is structurally correct.

21 Unistructural level of SOLO-transitional: Counting visible and sometimes invisible cubes in an organised but structurally incorrect manner. These answers show some signs of transition towards a multistructural level of response but are however bound by perception of the external aspects of the block.

22 Unistructural level of SOLO: Counting area, (that is, squares and not cubes as 'space filling') on some or all of visible and sometimes invisible faces of the rectangular construction. This type of answers can be described as unistructural for children's attention focuses only on the visible aspect of the construction.

3 Other: These strategies were not as clear cut as the ones listed above and could not be identified by a distinct category of response.

As one would expect there is an order of difficulty among the three measurement tasks compared (Table 1). There is also a progressive shift in the strategies used as children moved from the easier task of measuring capacity to the more abstract task of measuring volume of an undivided solid. The number of children using the multiplication formula progressively increased from the easier task to the more difficult one while the number of children that used a layer strategy in the capacity task also shifted to using an unsuccessful strategy involving measuring individual cubes or squares.

Table 1: Frequency and percentage of methods used by children to measure volume

	Test Items					
Method of calculation	Capacity (3x3x5) empty box		Volume of cuboid (3x4x5) construction made out of unit cubes		Solid volume (2x3x4) undivided solid	
	N	%	N	%	N	%
Successful	74	83.3	64	71.1	50	55.6
11	7	7.8	9	10	15	16.7
12	66	73.2	54	60	35	38.9
3	1	1.1				
Unsuccessful	16	17.5	26	28.9	40	44.4
21	1	1.1	6	6.7	2	2.2
22	13	14.4	17	18.9	37	41.1
3	2	2.2	3	3.3	1	1.1
Total	90	100	90	100	90	100

B. Conception of Volume

1. Conservation of interior volume:

Conservers of interior volume were those children who responded that the new construction would have to be taller. Non-conservers provided a response showing no realisation that the new house would have to be taller in order to have the same room, even after the explanation. These children did not realise that when one dimension decreases the other will have to increase for the two houses to have an equal amount of flats (cubes).

The great majority of children in our sample showed understanding of conservation of interior volume. The four non-conservers of interior volume did not show understanding of the structural organisation of the original construction in their attempts to calculate the volume of the "old house". The methods used by children to calculate the number of cubes in the original block ("old house") are directly comparable to those used by children to calculate volume in the measurement tasks.

To calculate the height of the new house, after the number of cubes in the old house had been determined, the majority of children (conservers of interior volume) used multiplication, division, rearrangement or built the new construction using as many individual cubes -one by one- as found to be contained in the original block. As expected, all students who calculated the number of cubes in the original block using the multiplication formula did not have to build the new house to find its height. They instead used rearrangement, division or multiplication. The majority of students (28 students) who used a layer method to calculate the volume of the original block also used rearrangement, division or multiplication to calculate the volume of the new construction while a considerable number of those students (20 students) still had to build the actual construction to find its height. On the other hand, students who were not successful in calculating the volume of the original block resorted almost entirely to building the new block with individual cubes to find its height.

2. Conservation of displaced volume:
Based on children's answers four categories of responses were identified:
(a) Conservers **(C)**: Those children who clearly stated that the water level will rise to the same level when the two blocks are immersed into the container and supported their answer by providing an explicit reason such as that: "The water level will rise the same because the two blocks are made of equal numbers of cubes or have the same volume or that the water displaced will be exactly the same because the volume of the two blocks is the same."
(b) Non-Strong-Conservers (**NSC**):Those children who stated that the water level will rise the same but failed to provide an adequate explanation for their view either because they were constrained by their ability to express themselves in written language or because they understood the truth of such a statement intuitively but not in formal mathematical terms. These children are considered to be able to conserve interior volume but seem unable to provide a concrete explanation or show adequate understanding of all aspects of volume conservation.
(c) Non-Strong-Conservers-Position (**NSCP**):Those children who failed to respond that the water will rise the same if the two blocks are successively immersed in water. Furthermore, those children are identified under the above category specifically because they additionally stated that the level of water will rise less, clearly because they were distracted by the positioning of the second block. They explicitly responded that the second block will not be totally immersed in the water.
(d) Non-Conservers (**NC**): Those children who stated that the water will rise more when the second block is immersed in the water. They qualified their answer by stating that the second block is larger because it is taller and therefore "bigger". Clearly those children do not conserve volume in all its aspects. In some cases not even interior volume.

As one would expect the four children who were identified as non-conservers of interior volume could not conserve true volume either. There were 19 children who were identified as conservers of interior volume but not displacement volume. This seems to agree with Piaget findings that there is a sequential mastery of conservation: interior volume then occupied and finally displaced volume.

The results shown in Table 2 describe the responses of the children to the conservation task in terms of the response category they gave to each of the measurement tasks and the measurement part of the task involving conservation of interior volume. These results provide two clear observations specifically concerning the two extreme groups C and NC: First, children identified as conservers are all successful at calculating volume in the different measurement tasks. Second, the non-conservers although some of them (not the majority) are successful at calculating volume in the measurement tasks, they do not use the multiplication formula for the calculation of volume but resort rather to a layer or column strategy. Clearly, therefore, non-conservers at best produce a multistructural response in their attempts to calculate the volume of a rectangular solid but do not reach the relational response level which appears to be related to the use of the multiplication formula.
With reference to the remaining two response categories (**NSC** and **NSCP**) identified in the conservation of displacement volume task children's performance on this task does not seem to bear

direct relationship to their performance on the different volume measurement tasks.

Table 2: Response categories on the conservation of displacement volume task by methods used for volume measurement tasks.

	Method of calculation	Conservation of Displacement volume				
Task		**C**	**NSC**	**NSCP**	**NC**	Total
Capacity						
	1	23	21	14	16	74
	11	5	1	1		7
	21				1	1
	22		6	1	6	13
	3		2			2
Total		23	29	15	23	90
Volume of cuboid		**C**	**NSC**	**NSCP**	**NC**	
	1	23	18	12	11	64
	11	4	4	1		9
	21		2	1	3	6
	22		7	1	9	17
	3		2	1		3
Total		23	29	15	23	90
Solid volume		**C**	**NSC**	**NSCP**	**NC**	
	1	23	10	10	7	50
	11	7	2	5	1	15
	21		1		1	2
	22		18	5	14	37
	3				1	1
Total		23	29	15	23	90
Transformation task		**C**	**NSC**	**NSCP**	**NC**	
	1	22	17	13	12	64
	11	6	2	4		12
	21		1		1	2
	22		7	2	7	16
	3	1	4		3	8
Total		23	29	15	23	90

Discussion of results

Children's responses to different tasks presented in physical form involving measurement of capacity, volume of a separated block and volume of an undivided block, clearly show that these tasks become progressively more difficult. The decrease in correct answers probably reflects the degree of children's awareness of the structural complexity of each task. Although the number of correct responses decreases as we move from the capacity task to the solid task the number of students using the multiplication formula steadily increases. This leads us to conclude that through these tasks some students became increasingly aware of the structural organisation of a rectangular construction in general resorting to a more abstract method of calculation (volume formula) while other students lose sight of the structural organisation of the construction as the visual clues disappear and resort to a lower response level strategy.

Analysis of the responses to the task on conservation of displacement volume showed that there is a strong association between conservation of true volume and understanding of the structural complexity of the blocks in the measurement tasks, leading to correct calculation of volume for all conservers. Additionally among students in the group identified as conservers of displacement volume the multiplication formula was used more frequently than in the other groups, while among students identified as non-conservers the formula was hardly used.

The results presented in this study seem supportive of the view that there are specific skills necessary for children to develop before we can expect meaningful use of the multiplication formula. First children need to practice with concrete tasks of increasing structural complexity through which they can acquire personally constructed views of the organisation of the three dimensional rectangular arrays made of individual cubes before engaging with pictorial representations of divided or undivided rectangular solids. Second children have to master conservation and guided through transformation tasks come to a realisation of volume in terms of its metrical continuity doing away with distraction imposed by shape or positioning of objects.

Finally teaching practices must observe these individual skills are adequately developed and well integrated leading to the use of the multiplication formula rather than making the formula the starting point of teaching volume in late primary school. The latter could lead to rote use of the volume formula and to its mechanical use through the end of primary and well into the secondary school handicapping children's understanding not only in mathematics but also in various science subjects were use of the formula becomes increasingly necessary.

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